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# TRANSACTIONS

OF THE

# FACULTY OF ACTUARIES.

VOLUME I.—1901-1903.

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## ABBREVIATIONS USED.

B.L.O.E.—Mortality Experience of British Life Offices (1863-1893).
J.I.A.—Journal of the Institute of Actuaries.
T.A.S.E.—Transactions of the late Actuarial Society of Edinburgh.
T.F.A.—Transactions of the Faculty of Actuaries.

## ERRATUM.

Page 47, line 22.—For $\pi_x$, read $\pi_x^{(m)}$.

[*Entered at Stationers' Hall.*]

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No. 1. Vol. 1.

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Inaugural Address, Session 1901-1902

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## PREFATORY NOTE.

The circumstances in which the Faculty of Actuaries have begun this series of *Transactions* are explained in the following Address.

It is intended to issue in this form, from time to time, the Papers read before the Faculty, with the Discussions thereon, and any other matter that may seem suitable for publication. The price of each number (which may vary with its importance and with the cost of production) will be printed on the outside cover.

EDINBURGH, *January* 1902.

# *The History and Present Position of the Faculty of Actuaries in Scotland.*

*4th November* 1901.

I COUNT myself specially fortunate in that the great honour of being elected to this Chair, for however transitory a period, fell to me at a time when the minds of all or most of us had already ripened for a change which it has long been my desire and ambition to see accomplished.

So far back as eighteen years ago, in an address to the members of the late *Actuarial Society of Edinburgh*, I ventured to make the following observations:—"I cannot help thinking the time "may come when the Faculty of Actuaries will take into its own "hands some of the work that is now performed here—will begin "to hold meetings for the discussion of actuarial topics, and will "perhaps publish, under its own immediate auspices, papers such "as those it now aids us with the means of bringing out in our "name. The course of events seems to me to tend in that direc- "tion. There are not a few members of the Faculty who, in years "past, made useful contributions to this Society's proceedings, but "who have outgrown their connection with you and have ceased to "attend your meetings, and there are still on your roll of active "membership a number of gentlemen who are well fitted to take "part with their older brethren in the Faculty in discussing "advanced questions of practical and scientific interest. As the "numbers of those two classes increase, the want will be more "and more felt of an organisation under which they could meet "for such an interchange of views." And I added—"Should the "Faculty hereafter move in this direction, there is no doubt that "those who have been most active in the proceedings here will be "found the chief contributors to those of the Faculty."

The views then suggested, which were shared by a certain number of members in both Societies, have since found general acceptance among us.

Not only has the Faculty decided to take up and carry on the important functions which had been left to the junior Society, but, by the unanimous consent of its members, the existence of the Society has become merged in that of the Faculty.

We are met to-day to inaugurate the new era, and in order to do so profitably we cannot, in my opinion, do better than take a survey of our past history as a professional body, of our present position, and (so far as we can) of the outlook that is before us.

The Faculty of Actuaries in Scotland is linked historically, not only with the Institute of Actuaries, but with a still older body—the Managers' Association, or, as it is now called, "The Associated Scottish Life Offices." The Managers' Association appears to have had at first no stated constitution, but to have gradually evolved itself out of occasional meetings held by the Managers of Life Assurance Companies in Scotland to confer with each other on matters affecting the common interest. In the volume of resolutions and proceedings issued some years ago, we are told that in 1841, after such meetings had become an established custom, a set of Resolutions was framed, in order "that the tacit general "understanding which has hitherto subsisted in regard to their "objects should now be put in a more formal and tangible "shape."

Some time after this, namely in 1848, when an Association of the kind had fully established its usefulness in the minds of the Scottish managers, communications were made by one of their number, the late Mr. William Thomas Thomson, to certain members of the insurance profession in London, representing the beneficial results that had attended the Association here. The idea commended itself, and at a meeting of actuaries and others connected with the management of Life Assurance Institutions in London it was agreed to be "desirable that those connected with "the management of Life Assurance Institutions should have occa-"sional opportunity of meeting together and consulting on subjects "of mutual interest." A committee appointed to consider the best mode of carrying out this object reported that in their opinion it was not expedient at that time to lay down rules for the formation of a society of any description, but that occasional meetings of the kind referred to might be held with advantage if this should, from experience, be found to establish uniformity in dealing with points of practice, and they suggested certain questions as deserving of immediate consideration. These were all of a practical business character, and were not on strictly actuarial subjects. This recommendation, however, was afterwards set aside in favour of a resolution "to establish a scientific and "practical Association among the Actuaries, Secretaries, and "Managers of Life Assurance Societies of Great Britain."

Instead of an association for considering business principles and points of practice, the new society was to be first of a scientific, and secondly, of a practical character. And further, instead of being an association of managers and heads of offices in London,

it was to consist primarily of Actuaries, and was to embrace the officials of Life Assurance Societies throughout Great Britain.

We have no record of the reasons why it was not thought desirable to carry out the suggestion made by Mr. Thomson, which at first appears to have been received with favour. Possibly they arose in part from a difference between the conceptions entertained in London and in Scotland respectively as to the place and functions of the chief officers of Life Assurance Institutions. Doubtless also, it was felt that an association merely for the discussion and settlement of points of practice would be too limited in its scope. Such an association, to be practically efficient, must have so far resembled the Scottish Managers' Association as to consist of head officials, one representing each office, whereas other functions had been conceived for the proposed association, in which all qualified actuaries could share. In the plan of the Institute, as ultimately adopted, objects of a practical business nature were indeed included, but they were subordinated to the actuarial and scientific.

I shall not weary you with the details of the formation of the *Institute of Actuaries.* It is enough to say that the scheme was heartily taken up in London and as heartily in Scotland. We are told by Dr. Sprague, in one of his addresses as President of the Institute, that the members resident in Scotland and connected with Scottish Assurance offices, formed in 1853 more than one-third of the whole body of members.

Reviewing the brilliant career of the Institute of Actuaries, one can hardly now entertain any regret that its actual aims and objects have proved other than they were originally expected to be; but apparently the heads of the profession in Scotland had to reconcile themselves to some measure of disappointment, that the hope they had cherished of having an association in London, representative of the offices there, and with which they could communicate on subjects of common interest, was not realised.

Whatever influence this feeling may have had, it is clear that the new arrangement did not prove entirely satisfactory so far as Scotland was concerned. The Institute had been only five years in existence when pressing difficulties began to be felt, and the Scottish members thought it necessary to suggest certain changes as indispensably necessary to a continuance of their cordial support. Their representations were met in a fair spirit by the Council. In particular, it was arranged that members residing more than twenty miles from London should have the opportunity of voting by proxy on any proposed changes; that the Scottish members should be allowed to nominate six Fellows, one as Vice-President, and the remaining five as members of the Council; and that the examinations of candidates resident in Scotland should

take place in Edinburgh. The points on which the Council were not able to afford full satisfaction were, (1) that the Scottish members were not, save for the one purpose above mentioned, to be recognised as a separate section of the Institute; and (2) that it was not proposed to apply any part of the funds towards promoting the objects of the Institute in Scotland. The Scotsmen, however, rested content for the time with the concessions they had obtained, especially recognising the difficulty of a sum being set apart as a grant for Scotland out of the funds of the Institute as these then stood, but they recorded their opinion, that "if the efficiency of "the Institute is to be promoted in Scotland, some practical "operations must speedily be commenced here, for which funds "will be necessary."

But ere long another and more crucial difficulty arose. It has been stated that the ultimate severance of the Scottish members was due to "differences of opinion as to the principle upon which "the office-bearers of the Institute should be selected." From a perusal of the printed correspondence, it appears to me more accurate to say that the difference arose upon the question whether professional conduct should be taken into consideration as bearing upon a member's qualifications for office, or whether personal talent and acquirement alone should be considered. The Scotsmen held the former view, while their brethren in London adhered to the latter. The circumstances which brought into prominence this divergence of views have long since passed away, and are not likely ever to recur. If they did, I believe the views of our professional brethren in London would be found more nearly to resemble our own, but on the occasion in question the difference served to precipitate a separation which had probably been long impending. The Scottish members withdrew from the Institute and formed themselves into a separate body under the title of the *Faculty of Actuaries in Scotland.*

Looking back now on what took place, one cannot help feeling that it was from the first inevitable, and that if the establishment of a separate Scottish body had not occurred at the time and in the circumstances which actually brought it about, the same result would ere long have arisen on other grounds. The truth is that it was impossible for the Scottish Managers and Actuaries, living at a distance of four hundred miles, to keep in sufficiently close touch with their colleagues in London, to feel thoroughly united with them in one body inspired by a common sentiment. They were accustomed and disposed to united action among themselves, but they were so situated as to be unable fully to enjoy the privileges of membership in a society whose meetings took place in London, and whose organisation centred there. It is not surprising, therefore, that after they had become familiar with the idea of

an Association that should not merely bring together the heads of insurance offices for business intercourse, but should be a professional body with educative functions, and with power to make its membership a guarantee of professional standing and attainment, the Scotsmen set about forming such a body for themselves.

The original members were thirty-eight in number, comprising "the present Managers, Actuaries, and Secretaries in Scotland of "Life Offices, and professional gentlemen practising there as "Actuaries (although not connected with Life Offices) who have "been Fellows or Official Associates of the Institute of Actuaries "of Great Britain and Ireland." Four of those gentlemen still remain connected with the Faculty, and bear names that are honoured among us: Mr. T. G. Dickson, Mr. D. Clunie Gregor, Mr. James Howden, and Mr. John M. M'Candlish.*

The Faculty was constituted in January 1856. After a year the Council were able to report that "the Constitution and Laws "have been framed. In addition to the thirty-eight original "members, eighteen names have been added to the lists as Fellows, "Associates, Non-resident Members or matriculated students; "twelve other gentlemen have applied for admission as Associates, "and are waiting the arrangements for examination as required by "the laws, while the Council have reason to know that many others "are engaged in the necessary studies with a view to next year's "examinations. Two important courses of lectures are now being "delivered; an examination of students has taken place; and a "commencement has been made to the formation of a library, for "which purpose, among others, a special fund has been raised, to "which the various Insurance Offices have kindly contributed. "The Faculty is thus actively and successfully carrying out the "various objects for which it was instituted, promoting in an "important manner the study of those subjects with which the "Actuary ought to be familiar, and associating the members in a "professional body, consisting exclusively of gentlemen who are "either following the profession of an Actuary, or whose education "is being conducted with reference to it."

The question of applying for a Royal Charter of Incorporation appears to have very early engaged the consideration of the

* Not many days after this address was delivered, Mr. M'Candlish died at a ripe age. This is not the place to recount his services to his profession and to the community in which he lived, but the Faculty of Actuaries will always remain under a deep debt of gratitude to him. He was one of its first two Honorary Secretaries (the late Mr. Robert Balfour, a man greatly esteemed and honoured in his day, was the other), and throughout his whole active business life took a warm practical interest in its welfare. He was many times a Member of Council, and when the office of President was instituted, he was the first to be elected to it.

Council; for in April 1857, at a General Meeting of the Faculty, a Committee was appointed to inquire and report on the subject. Nothing, however, came of the proposal at that time, and it was not until fully ten years later that the Council reported definitely in favour of steps being taken to obtain a charter. At first the project met with some discouragement. It was stated by the Parliamentary Solicitor consulted that the Board of Trade were not disposed to entertain applications for Royal Charters, seeing that the recent Companies Act had provided for the incorporation of Societies constituted for other objects than those of profit. The legal adviser here, however, considered that the Faculty could not be brought within the provisions of that Act; and even if it were, that incorporation of that kind would not confer the advantages which the Faculty sought to obtain by a Royal Charter. He recommended the Faculty to proceed with their petition, disregarding the difficulty placed before them. The expected objections by the Board of Trade were raised, but they were ultimately overcome, and in September 1868 it was reported that Her Majesty had signed the warrant for a Charter incorporating the Faculty.

In this important step the Faculty anticipated by sixteen years the Institute of Actuaries, whose Charter of Incorporation was not obtained until 1884. The nature of the main hindrance which had so long stood in the way of the Institute has been more than once alluded to in the pages of their *Journal*.

After the first abortive proceedings towards Incorporation, the next important step taken by the Faculty was the promotion of the Actuarial Society of Edinburgh in 1859. I have given some account of the origin and history of that Society in the address already alluded to, which is printed among its Transactions. I shall therefore not trouble you now with details, but I would ask your special attention to these two points—(1) That the Society was not of spontaneous origin among the junior members of the profession, but was originated by the Faculty in its desire to promote their studies and professional attainments; and (2) that the objects the Faculty had in view were mainly of an educative character. This is shown by the following extract from a report made by the Council at the time when they were engaged in promoting the formation of the Society:—"One of the main "objects for which the Faculty was instituted was to promote the "studies of gentlemen looking forward to this profession. The "Council are extremely anxious to afford them all the advantages "which other professions offer for the prosecution of their pro-"fessional studies and for the general cultivation of their minds, "so as to make them more valuable assistants in their respective "offices and to qualify them for higher positions, and thus to "promote their advancement in life. With this view it has

" appeared to the Council, that besides the education which each " must gain for himself in the course of his daily employment, " much may be done by attendance at classes, by the preparation " for such examinations as may be instituted by the Faculty, and " by the studies and mutual intercourse promoted by such a " Society as is now in the course of formation."

How the Society in time outgrew its merely educative functions, as these were conceived at the outset, we all know. Perhaps one chief cause of this was the growth of technical and instructive Actuarial literature, including the *Journal* of the Institute and the publications of the Society itself, which furnished the student with material for reading and study at home. Another cause why our students ceased to look for their early instruction to the Actuarial Society, was that in course of time gentlemen were to be found who, having passed their examinations and been admitted to the Faculty, were willing for the sake of revising their own studies to give private instruction to their younger brethren. Still more recently the publication in 1887 of the Text-book of the Institute of Actuaries has helped to place the study of Actuarial Science on a new and systematic basis, independent of such extraneous aids as attendance at an occasional meeting could afford.

The work done by and for the Actuarial Society in the valuable series of papers which form its published Transactions will always be a monument of the skill and industry of a section of its members; but we have all come to think that that work should now pass into other hands and be carried on by ourselves as the representative body of the profession in Scotland.

Mention of the Actuarial Society leads me to refer for a moment to one whose name will always be indissolubly associated with the history of actuarial science in Scotland—our friend Mr. Meikle, the first contributor to that Society's proceedings, an unwearied supporter during its whole career of forty-one years, and the last who read a paper before it. Among the many important services he has rendered to the profession, many of us will be disposed to give a high place to his *Rationale of Life Assurance Premiums*, and his *Analysis of the Profits of Life Assurance*. If I mention these specially, it is to express my own gratitude—shared I doubt not by many here—for assistance derived from them in grasping the principles of actuarial calculations. We are rejoiced to have Mr. Meikle among us, active as ever in the discharge of his duties, and ready as ever to undertake work for the benefit of the profession at large.

The work accomplished by the Faculty has as yet no such tangible record as that of the Actuarial Society, but I venture to think that its usefulness must be rated very highly. The Faculty has differentiated our profession from that of the Accountant, with

which it was long associated in the public mind, has brought the Actuaries of Scotland together into a corporate body, has stimulated and assisted the junior aspirants to the profession, has tested their qualifications, and afforded them the stamp of official recognition, and has furnished the business of insurance with many men whose attainments have secured for them positions of influence and usefulness, both in Scotland and elsewhere.

Up to the present time the Faculty has granted its diploma or certificate of competency to 156 gentlemen who had gone through the requisite course of study and successfully passed the prescribed examinations. Many of these have left Scotland to fill official and other positions elsewhere, and some, alas! have gone from us to their final rest. Our active membership, including original members and Fellows elected without examination, now numbers 162, of whom 148 are Fellows and 14 are Associates. About two-thirds of us are resident in Scotland, and the remainder are to be found in England, Ireland, India, China, Australasia, the Cape of Good Hope, and the United States. Since its membership was first constituted, the Faculty has steadfastly adhered to the principle that no one should be admitted as an Associate who has not satisfied the Council of his fitness to be called an Actuary, by passing the necessary examinations; and only those have been admitted as Fellows who have either passed through the degree of Associate, or who, from their professional position and attainments, have been considered desirable acquisitions to our membership.

Besides the remaining original members, we have now nineteen Fellows who were elected without examination, and whose connection we feel to be an honour to us and to themselves. About one-half of them are Fellows also of the Institute of Actuaries; and they all, without exception, either occupy or have occupied leading positions in the Insurance world.

I think it is worthy of note how rapidly our numbers have increased of late years, and especially since the introduction of the Text-book of the Institute of Actuaries, which has done so much to assist the actuarial student, and to give precision and definiteness to his course of instruction. At 31st December 1870 the membership of the Faculty, including Fellows and Associates, numbered 62; ten years later it had increased to 79; by the end of 1890 it had further increased to 109; and now, as already stated, the number is no less than 162. For the reason already referred to, the number of Associates does not greatly increase. We have now only 14, and in 1870 we had 12. As a rule, gentlemen having passed their examinations remain Associates only during the short probationary period prescribed by the rules, and then pass on to the grade of Fellows.

By our Constitution gentlemen who are pursuing their studies with a view to examination require to be enrolled as students of the Faculty. This year's list contains the names of no fewer than 174 students in various stages of progress. Of these, 70 have only to pass the final examination in order to be admitted Fellows; 52 have two examinations to undergo; and the like number have to pass through the whole series of three examinations. It is true that a certain number of these students give little promise of finishing their course of study and qualifying themselves for membership. About ten per cent. of the whole have been on our roll for ten years and upwards; but as an annual fee is exacted for continued enrolment, we may take it that in the great majority of cases the students are still preparing themselves for examination, and we may hope that the bulk of them will in due time come in to Associateship and Fellowship.

So much for our history and for our present position in point of numbers. We are a large body, all of us more or less thoroughly versed in Actuarial Science, and most of us presumably learned in its latest developments. And now as to our outlook for the future. Are we to continue as a separate body, doing the work in Scotland which we have hitherto been carrying on, and undertaking new functions as necessity or opportunity presents them to us, or are we to look forward to becoming part of a wider organisation?

Suggestions have at various times been made towards a closer union of the Faculty and the Institute of Actuaries; and it is worthy of note as showing the extent of goodwill subsisting between us that these have on two occasions been coincident with conjoint action in carrying out a common object.

So early in our history as in 1864, when the project of a collective mortality experience of Life Assurance Offices was being entered upon—the experience which afterwards formed the basis of the Institute of Actuaries Life Tables—we find evidence of such a suggestion and of the most friendly relations having been established. Here is an extract from a Report of our Council at that time:—

"Your Council were given to understand that a feeling existed "among leading members of the Institute of Actuaries in London "favourable to a more intimate association between their body "and the Faculty. The Council having a unanimous and cordial "desire that the most friendly relationship should exist between "the members of the profession in Edinburgh and those in London, "appointed Messrs. Thomson, Ramsay, and Raleigh, with the "Honorary Secretary, Mr. M'Candlish, a Committee to confer "with any like Committee that might be nominated by the In-"stitute of Actuaries. A Committee having been appointed by

" the Institute, several conferences were held in London, through-
" out which the most cordial feelings prevailed on both sides, and
" shortly afterwards the Institute made some changes on its
" Constitution and Laws, intended, it is believed, to facilitate the
" introduction of individual members resident in Scotland.

"Your Council having received an Interim Report from their
" Committee to the above effect, are not prepared at present to
" make any practical suggestions on the subject, but are in hopes
" that the friendly correspondence and personal intercourse which
" have taken place will lead the two bodies to co-operate to a
" greater extent than hitherto in their efforts to advance the
" objects for which they were formed."

Again we are engaged on a similar undertaking—this time of a more extensive and important character, and on a more definite basis of co-operation—and again we have the same idea of closer union suggested.

The present President of the Institute of Actuaries, in his inaugural address a year ago, said, "As a bold suggestion I throw
" out the satisfaction it would be to many of us if the Institute
" of Actuaries and the Faculty of Actuaries in Scotland could
" be united."

I may say for myself, that if such a project were ever seriously entertained, I could wish nothing better for both bodies, than that it should be carried out under the auspices of one like Mr. Higham, a gentleman whose courtesy and tact, and whose extreme consideration for his northern brethren have been no small factors in cementing the singularly happy and cordial relations which at present subsist between the Institute and the Faculty. If any way can be pointed out by which the Institute and the Faculty can be still more closely bound together in furtherance of their common objects, I for one will rejoice, and I am sure that no one among us will be backward in lending his aid to bring about so desirable a result. But, so far as my judgment goes, I agree with those who think that any such step will have to be accomplished in such a way as to leave the integrity and independence of our two Chartered Societies undisturbed. I think we in Scotland cannot afford to give up our separate status as a Faculty if we would retain some of the most important advantages which accrue from our being associated together. We cannot now, any more than our predecessors of half a century ago, enjoy all the privileges which the Institute confers upon its members in London. We cannot make free use of its library, as we can of our own, nor can we attend its meetings, nor keep in such close touch and association with our brethren south of the Tweed as we can among ourselves. Moreover, we in Scotland live under a system of laws differing in

essential particulars from those which prevail in other parts of the kingdom, and giving rise in various directions to legal questions and actuarial problems which are not familiar to our English brethren, and it behoves us to see that these specialties receive their due share of attention from those who aspire to become Scottish Actuaries.

With the further suggestion thrown out by Mr. Higham, that there might be a joint scheme of examinations, so that (to use his words) "there may be but one standard of admission to the pro-"fession in the United Kingdom," I confess myself in cordial sympathy. Due allowance being made for the special features in Scottish practice to which I have alluded, I think that nothing but good could come from establishing such a common standard. If opinion in London shall ripen in this direction, I may promise our friends there that the matter will receive careful consideration on the part of the Faculty, and I see no great reason to anticipate any difficulty if the subject be approached in the spirit of mutual regard for each other's views that marks our intercourse on other matters.

While on the subject of examinations, I cannot forbear expressing some anxiety lest in our desire to admit none but qualified men to the profession we allow ourselves—Institute and Faculty alike—to make the tests of proficiency more stringent than is really necessary, and lest we narrow our view of what is desirable in the equipment of an Actuary too much to the merely technical and scientific. In the business of life it is not always the clever student who proves most successful. More often it is the man who, having a moderate but sufficient acquaintance with technicalities, is possessed of the wide grasp and the clear view that enable him to deal efficiently with the many-sided problems that arise in the conduct of business. If our course of training and our examinations are such as to encourage only the one class and to exclude the other, we may indeed train our students to a high degree of technical efficiency, but we shall assuredly not advance the dignity and status of our profession. It will be an evil day for us if men of business and directors of Insurance Companies come to regard the Actuary as a man of figures and formulas rather than as a skilful man of affairs.

I take it that the circumstances under which we are assembled to-day are a confirmation of the views which I have stated as to our separate existence and work, and an assurance that they are generally shared among us. We have resolved to take upon us the new function of holding meetings to hear and discuss papers of professional and scientific interest, and we thus add another and most important sphere of usefulness to those which the Faculty has hitherto fulfilled. With a membership such as we

now have, there should be no difficulty in maintaining from year to year such a series of meetings, and I look forward with confidence to our members fulfilling in a manner that will reflect credit upon the Faculty the task that is now before us. We have no lack of ability, especially among our younger men. The Transactions of the Actuarial Society and the numerous contributions made by our members to the Journal of the Institute are ample evidence of this; and I am very sanguine that the stimulus which will now be afforded to the members, of having their productions listened to and discussed in the Faculty itself, and not merely in a junior Society, will help to bring forth a plentiful supply of work of a high standard. Already we have fair promise for the coming session, but we must be prepared for the possibility that it may take time to get the new arrangements fairly into order. I must, however, appeal to the younger men to put their energies into this matter, bearing in mind that the future of the Faculty rests with them, and that their own credit is bound up in making the new departure the success it ought to be.

And now, gentlemen, we turn from this review of our history and position as a corporate body to let our thoughts dwell for a little on two distinguished members to whom we wish specially to do honour on this occasion.

We have seen how the hand of the late Mr. William Thomas Thomson appeared in the initial steps which led to the formation of the Institute of Actuaries. The same hand was present throughout all the proceedings which led to the formation of our own Faculty; and although the name of Mr. Thomson does not appear so conspicuously in the one case as in the other, it is well known and understood that his was in reality the chief guiding hand in shaping the Faculty and moulding its destinies.

It was he, again, whose solicitude for the welfare of the junior aspirants to the profession led to his being mainly instrumental in founding the Actuarial Society. He was its first President, and during many years showed the warmest interest in its welfare. But besides all this, he was a man who conspicuously strove for the elevation of our profession and sought to inspire in its younger members a sense of the dignity and honour of their calling.

Besides his addresses to the Actuarial Society of Edinburgh, which I hope our younger men may always find time to study, he has left various evidences of his skill as an Actuary—for example, his *Actuarial Tables*, founded on the Carlisle basis, and his work on *The Pecuniary Interests of Heirs of Entail*; but perhaps the most conspicuous monument of his ability is the great Life Assurance Institution, with its world-wide ramifications,

whose foundations, we may say, were laid under his skilful supervision. To those, however, who can recall the names of even a few of the accomplished and successful men who received their training in the *Standard* Office under him, and went out to fill important positions in their profession here and elsewhere, it will always appear that his excellence showed itself not merely in what he himself accomplished, but in that rarer and greater gift—the power to recognise and develop talent in others.

It appeared to the Council that the inauguration of the new departure we are now taking would be a fitting opportunity for the Faculty to acknowledge its indebtedness to our virtual Founder, and to secure a tangible memorial of him to be handed down to future generations of our members. The proposal to obtain a bust of the late Mr. Thomson for this hall was unanimously adopted at a general meeting of the members, and, with the kind assistance of his son, Mr. Spencer Thomson, we have had executed from existing busts and portraits the one which you now see. Those of us—already a diminishing number—who have personal recollection of the late Mr. Thomson, are agreed in thinking that the sculptor, Mr. D. W. Stevenson, R.S.A., has succeeded in catching the likeness and expression in a very satisfactory degree. His work recalls the refined features and the courtesy and dignity of manner of the original, and we congratulate the Faculty on possessing so excellent a representation of one whom it should always delight to honour.

The other gentleman whom we now seek to honour is happily still with us, and his labours as an actuary are so extensive, so well-known, and so recent, that any detailed reference would be out of place here. When Dr. Sprague retired a year ago from active professional work, it appeared to the members of the profession, both here and in London, that the distinguished services he had rendered in the advancement of Actuarial Science, which had earned for him a world-wide reputation, had also entitled him to the gratitude as well as the admiration of Actuaries, and called for recognition in some tangible form. It was felt that the honour of affording such recognition belonged especially to the Institute and the Faculty. Accordingly, a committee, representative of both bodies, was formed to carry this object into effect; and having considered the best means of doing so, the Committee were of opinion that Dr. Sprague should be invited to allow his portrait to be taken in marble and that copies should be placed in this Hall, and in the Hall of the Institute. An opportunity having been afforded to the members of the Institute and the Faculty to contribute, the necessary funds were quickly raised. The work was intrusted to Mr. Pittendrigh Macgillivray, R.S.A., with the result that is now before us. The likeness is faithful

as well as artistic, and we are happy in the possession of so fitting a memorial of our distinguished Fellow. Long may he be spared to enjoy the well-earned leisure which has come to him in the evening of his days.

At the close of the address, Mr. A. H. TURNBULL said—Mr. President and gentlemen, having been a student of the Institute of Actuaries in the early fifties—I believe with the title of Associate—and having been connected with the Faculty, and with the Actuarial Society, since each came into existence, the excellent address to which we have just listened is of deep interest to me, and I am exceedingly pleased to know that a history of the origin and objects of the three bodies, prepared by so able and exact an observer as our President, will form the first contribution to the published proceedings of the Faculty of Actuaries. This year of grace 1901—surely in this hall I may call it the first year of the twentieth century?—bids fair to be known among insurance men as the year of amalgamations. In it many have been spoken of; some have been initiated, and some have been carried through. Of these last, the one in which we are interested, the amalgamation of the Actuarial Society with the Faculty, though attracting little attention, should ultimately prove of great benefit. Indeed it must do so if the members of the now united body respond to the appeal of their President, and do what lies in their power to add to that knowledge which the actuary whose heart is in his profession is ever seeking after. Our President congratulates himself on holding office at so important an epoch in our history. It appears to me that it is we who should congratulate ourselves on having in office at such a time a gentleman whose ready tact and never-failing courtesy did so much to facilitate the necessary arrangements, and whose habit of thoroughness gives all associated with him confidence that what they undertake under his guidance will assuredly be carried to a successful issue. I agree with our President that an amalgamation of the Institute and Faculty cannot be arranged on equal terms, however anxious many on each side of the Tweed may be to have it so; and if not on equal terms, then, from our point of view, an amalgamation is not desirable. Still, the relations between the two bodies can never be too friendly, and need never take the form of rivalry. Now that having absorbed the Actuarial Society we are to run on much the same lines as the Institute—with common objects in view—I feel sure that we shall rather gain than lose in the estimation of our neighbours if we exert ourselves to show that the Faculty is worthy of an independent existence. Gentlemen, I have much pleasure in moving a hearty vote of thanks to our President for his admirable address.

The motion was carried with acclamation.

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1902.

TRANSACTIONS

OF THE

FACULTY OF ACTUARIES.

*First Year's Risk.* *By* HENRY MOIR, F.F.A., F.I.A., *Actuary of the Provident Savings Life Assurance Society, New York.*

[Read before the Faculty, 2 December 1901.]

A CORRECT measure of the Rate of Mortality in the First Year of Assurance is of considerable value. It enables the actuary to calculate the limit to which initial expense may grow without incurring the risk of loss through lapse at the end of the first year; while its more obvious purpose is to estimate the immediate effect of the Selection by offices through their medical examinations and restrictive rules. In the first year we see the full force of the skill of medical officers pitted against the desire on the part of the lives assured to make their best appearance. We need not suggest that they try to deceive the doctor, but it is often necessary for him to apply tactful cross-questioning in order to elicit important information.

Deaths amongst assured lives in the first year are, with the exception of accidental deaths and those from illnesses of short duration, largely under the control of the medical examiners; and it is therefore hoped that a critical examination of certain variations in the rates of mortality may be useful to medical men, and may enable them to exercise supervision even more efficient than that heretofore shown. The examiner is understood to submit all the lives coming before him to the same test; but there is a natural Discrimination exercised by the lives themselves which constitutes an important factor in the question of early mortality.

Sometimes this Discrimination runs in harmony with the doctor's selection, but sometimes it runs counter; and the doctor might reasonably exercise greater strictness in dealing with those cases in which the counter selection is likely to prove strongest.

The subject of First Year's Risk was so fully discussed about twenty years ago that, on the basis of the statistics then available, but little more could have been said. The result was an unsuccessful attempt to base theories upon insufficient data, the final conclusion being that material from which the first year's risk could be accurately deduced was not then at hand. Now, however, with the publication of the New Combined Experience of the offices, there has been revealed a vast mass of rough material from which fairly accurate conclusions may be deduced. We must not overlook the fact that the experience of individual offices may differ from the aggregate results. This feature was clearly expressed by Mr. King (*J. I. A.*, xxi, 254), and I cannot do better than quote his remarks:—

"I do not think our efforts to ascertain the mortality in the "first insurance year will be of much avail, because I doubt if it "be a determinate quantity. After lives have been insured some "years, I think the rates of mortality among them will be found "to be nearly identical in all sound offices; but not so the "mortality of the first year. That will depend on the manage-"ment of the particular office, and will vary widely."

All minor investigations relating to the New Experience must of necessity be left to the private enterprise of actuaries; and as a small contribution to the common store of knowledge I submit the following tables and remarks, in the hope that they may lead to an interesting discussion.

I have sometimes found it difficult to confirm from the ungraduated tables for the longer periods certain deductions from the experience of the first year. This arises from the method in which the figures have been brought together. The ungraduated statistics, as published, are open to criticism in that they show a want of uniformity in matters of detail. One or two examples which have been observed in connection with the present inquiry may be cited:—(1) The *complete* expectation of life is published in the aggregate Annuity Experience, while the tables of the Assurance Experience give the *curtate*. (2) The function is not tabulated at all in the single-life female section, although published in the less important joint-life. (3) The defects of the aggregate female table make it impossible, without much trouble, to compare the mortality with that of the corresponding male experience. (4) Select tables in quinquennial groups are not given at all in the limited-premium section, although the numbers exposed to risk greatly exceed those in the female experience where such

tables are given. There are several other differences in treatment which render comparisons difficult, but they are probably already familiar to those members of the Faculty who have interested themselves in the varying rates of mortality under different classes of policies.

Dr. T. B. Sprague and Mr. George King each investigated the first year's mortality in connection with the Institute of Actuaries $H^M$ statistics, which were based upon "calendar" years. They viewed the subject from different standpoints, and accordingly the rates of mortality they submitted differed greatly. The following remarks by Mr. King (*J. I. A.*, xix, 387) show the difficulties under which these early investigators laboured. "In the "*Mortality Experience* [$H^M$] the rate of mortality for year 0 of "assurance is the annual rate for the latter half of the current "year of life in which the policy is effected, but there is no method "of finding from this the true rate for the whole year. Mr. "Higham encountered the same difficulty, and got over it by "combining the mortality of years 0 and 1. Mr. Berridge sub-"mitted another plan, which appears to be very ingenious, but "very difficult to follow; and in which he involves the mortality "of years 0, 1, and 2. I do not like either of these methods, "because, if there be any force in what is called selection against "the company, we lose sight of it by mixing the mortality in year "0 with any other. I have preferred the more cowardly course "of entirely ignoring the difficulty, and treating year 0 as a com-"plete year. Perhaps by so doing the mortality is slightly under-"rated, etc."

Mr. King, when criticising the method followed by Dr. Sprague, added:—"The truth probably lies somewhere between us." Dr. Sprague adopted quite a different line of reasoning, and endeavoured by a method of finite differences to deduce average policy years from the calendar year statistics.

In the $H^M$ Table all classes of policies were run together; and accordingly in the preliminary comparison which I have now made, all classes of the New Experience have been brought together (see Tables 1 and 2), and the graduated results placed alongside the former estimates.

All the new tables hereafter given have been graduated by the graphic process, and the usual tests have been applied with satisfactory results. Complete tables showing the relation of the graduated to the rough facts are appended; for the General Aggregate table (Table 3) and for four of the principal classes, namely, (1) Whole-life with profits (Table 4), (2) Whole-life without profits (Table 5), (3) Endowment-assurances (Table 6), and (4) Limited-payments (Table 7). All the other tables were tested in the same way; but it is undesirable to load this short

paper with heavy statistical tables. For the purpose in view, smoothness of graduation is of less consequence than close adherence to the original facts. In an investigation of this nature, where variations in the mortality naturally lead to inquiries and speculations as to the causes, a curve which follows every well-marked feature is more interesting and valuable than one which sweeps all peculiarities aside for the sake of smoothness of graduation and gives only general results.

Three small sections of the new statistics (namely, Contingent, Temporary, and Joint) have been published in quinquennial groups only, and not for individual ages. The graduation of these sections is therefore less trustworthy. In introducing them in the General Aggregate table, it will be observed that they have been placed at the central ages of each quinquennial group. As the individual ages are stated in an overwhelming proportion of the whole, the graduated results cannot have been affected to any noticeable extent. Indeed it will be observed that the business of the ordinary Whole-life class with profits is so great in proportion to all others, that the first year's risk, when all classes are combined, does not differ materially from the same function for Whole-life assurances.

COMPARISON OF OLD WITH NEW RATES OF MORTALITY—$q_{[x]}$.

| Age. $x$. | Sprague. $H^M$. | King. $H^M$. | Chatham. Gotha. | New Aggregate. |
|---|---|---|---|---|
| 20 | ·0050 | ·0036 | ... | ·0027 |
| 25 | ·0045 | ·0027 | ·0031 | ·0025 |
| 30 | ·0045 | ·0042 | ·0040 | ·0031 |
| 35 | ·0049 | ·0042 | ·0057 | ·0037 |
| 40 | ·0055 | ·0038 | ·0066 | ·0043 |
| 45 | ·0064 | ·0058 | ·0069 | ·0058 |
| 50 | ·0079 | ·0046 | ·0095 | ·0081 |
| 55 | ·0104 | ·0127 | ·0128 | ·0114 |
| 60 | ·0144 | ·0107 | ·0200 | ·0161 |
| 65 | ·0205 | ·0064 | ... | ·0228 |

The results deduced by several other actuaries were given by Dr. Sprague (*J. I. A.*, xxi, 237-242), but it is scarcely necessary to reproduce them here. I have, however, included the rates experienced by the *Gotha* Life Office as given by Mr. Chatham (*J. I. A.*, xxix, 144), as they were drawn from a policy-year experience, and are consequently in my opinion much more valuable. The heavier mortality shown by these may be partly due to differences between Great Britain and Germany, and partly to the fact that the *Gotha* experience runs further back to a time when medical selection was not so effective.

The values submitted by Dr. Sprague and Mr. King are con-

sistently higher than the new values at the early ages, up to 48; but it seems strange to find them *lower* at ages above 48. One would have thought that the more efficient Selection during the last forty years would have caused the mortality to be lower for *all* ages than the rates formerly experienced; and the clearly defined and systematic variation leads to an inquiry as to its cause. I think the cause is to be found in the assumptions which were necessary in treating the calendar year investigations. Both investigators had to assume that "year 0" represented a period of half a year, and each pointed out that this assumption was likely to be inaccurate. Not only was the assumption likely to be inaccurate, but its divergence from the truth would probably be greater at some ages than at others. A young man effecting a small policy does so before a birthday, or before getting married, or at any other odd period throughout the year when an agent happens to solicit his business. There may be a slight tendency to place such policies towards the end of the year; but this is not so clearly marked as in the case of the more mature business man, whose financial arrangements are so frequently made during the last three months, or just after the close, of the calendar year. Such a condition of affairs, combined with the more efficient medical examination which now prevails, completely explains the peculiar nature of the difference between the old and the new rates. The assumption that the young man was insured for six months in "year 0" might be reasonably accurate; while older men might be at risk for about three months only during the year of entry, although the period of exposure was assumed to be six months.

The following is the most interesting table now submitted, as it compares the rates of mortality in different classes of policies:—

FIRST YEAR'S MORTALITY, $q_{[x]}$, IN DIFFERENT CLASSES OF POLICIES.

(Lightest mortality on the left, and heaviest on the right.)

| Age. | Limited payments. | Endowment assurances. | Whole-life with profits. | Ascending scale. | Term. | Whole-life without profits. | Contingent. |
|---|---|---|---|---|---|---|---|
| 20 | ·0012 | ·0022 | ·0026 | ·0028 | ·0059 | ·0058 | ·0056 |
| 25 | ·0017 | ·0022 | ·0027 | ·0035 | ·0050 | ·0046 | ·0052 |
| 30 | ·0021 | ·0023 | ·0033 | ·0028 | ·0046 | ·0040 | ·0060 |
| 35 | ·0029 | ·0031 | ·0037 | ·0032 | ·0049 | ·0045 | ·0082 |
| 40 | ·0032 | ·0039 | ·0042 | ·0044 | ·0055 | ·0063 | ·0097 |
| 45 | ·0034 | ·0050 | ·0057 | ·0065 | ·0071 | ·0086 | ·0108 |
| 50 | ·0036 | ·0063 | ·0075 | ·0097 | ·0102 | ·0116 | ·0122 |
| 55 | ·0039 | ·0082 | ·0103 | ·0132 | ·0141 | ·0165 | ·0154 |
| 60 | ·0044 | ·0112 | ·0153 | ..... | ·0189 | ·0206 | ·0300 |

The same information is given in a more complete and simple form in Diagram 1.

There are four classes under which the statistics are sufficient to yield quite reliable results, namely,

(1) Whole-life with profits.
(2) Whole-life without profits.
(3) Whole-life limited payments.
(4) Endowment-assurances.

The indications given by the smaller classes are confirmed by these four classes, and moreover they all agree so well with what might on careful consideration be expected, that I am inclined to think the mortality rates reasonably accurate all round. One might have supposed that, as all the lives pass the same ordeal, the death-rate within so short a period would have been approximately the same in the different classes; yet it will be seen that in some classes it is nearly double what it is in others. Ordinary Whole-life policies with profits show a much lighter rate than similar policies without profits; Endowment-assurances show less than either; while Limited-payment assurances show the least mortality rates of all. I was at first surprised to find the Limited-payment policyholders occupying this pre-eminent position, and thought the situation might be accidental. But a little consideration served to show that the condition is not unnatural. Endowment-assurance policyholders combine an investment with their assurance protection, and look forward to drawing the sum assured themselves; therefore their natural Discrimination causes them to be healthier lives than those who take ordinary policies. But Limited-payment policyholders desire life assurance only, and with careful foresight keep their old age free from premium payments. They not only hope to make all premium payments; but also expect to have the policies increased by bonus additions after the premium-paying term has ceased. Further, it is not at all an uncommon circumstance for an office to accept a doubtful risk at ordinary rates, provided it be placed on the Endowment-assurance plan. It is now generally understood that this practice provides in many cases a very poor protection to the office; but the fallacy in the system was less recognised during the thirty years covered by the statistics than it is now.

The mortality rates divide themselves into three classes:—(1) Limited-payment and Endowment-assurance policies, which contain an investment feature and show light mortality; (2) Ordinary with profits, and the unimportant Ascending scale, which represent the intermediate class; and (3) Ordinary without profits, Term, and Contingent, which show high mortality rates. The first two sections contain policies taken out for "provident" purposes, while the third section contains policies taken out in connection with "financial" transactions, borrowings, and such like. The difference in the rates of mortality experienced by these three

classes is not confined to the first year, but is noticeable throughout the whole assurance term; so that the effect upon any office with preponderating business in one of the extreme classes will be considerable. The office transacting many investment policies may look for light mortality, while the office which cultivates policies through creditors and moneylenders is likely to have the opposite experience. The loss from the heavy mortality is, however, a much more important matter than the gain from light mortality; because, under the former class of policies, the reserves are small and the death-strain is consequently great, while under investment policies the opposite condition holds. Moreover, financial policies are generally for larger amounts, and the mortality statistics take no account of this factor. If, therefore, the monetary effect be considered, it will be seen that a large business of this undesirable class would prove a severe strain upon the resources of an office.

But can no steps be taken to lessen the mortality under financial policies? It seems likely that certain offices will continue, as in the past, to accept proposals submitted through moneylenders and financial agents, and even to cater for such business, undesirable though it is; but the more carefully conducted offices might take further precautions than they at present do, in order to secure an efficient Selection. All such policies should be viewed with suspicion. Their number is comparatively small, probably less than 5 per cent of the whole (though each policy may be for a considerable sum); and they might therefore be treated more rigidly when any flaw, however slight, is found in family or personal history. Not only so, but the usual inquiries might be more carefully conducted, and the medical examiner might be warned beforehand, so that he too might exercise greater caution than usual, thus submitting the life to a very stringent examination, especially on the subject of habits and moral tone. Another method of protecting the office would be by specially loading the premiums to meet the extra mortality, but offices adopting this course would get no proposals whatever, and might lose some really good business.

It will be observed that one feature of this class of policy is that the mortality shows a distinct decrease between ages 20 and 30; while any such decrease is scarcely perceptible under the other classes. Young men who anticipate their patrimony and effect policies in connection with their borrowings, can seldom be described as quiet members of society. More frequently they are wild and reckless; and, if not actually given to debauchery, have generally a foolish and careless temperament liable to periodic excess and the consequent danger to life. Now if the Selection by the office were conducted quite impartially and

without reference to the largeness of the sum assured, it would still be very difficult to detect many such cases. The loose habits have not endured sufficiently long to leave their indelible mark on the constitution, while the private friends' reports are too often a mere farce. Sometimes they are filled up by old family friends who know little of the inner life of the proposer, and at other times they are completed by boon companions who interpret the words "sober and temperate" according to their own dim lights. Be that as it may, such undesirable policies are placed upon the records of the offices. This period of youthful dissipation rapidly wears itself out between the ages of 18 and 25. Some pay the penalty of their rashness, while in others the naturally vigorous constitution is allowed to reassert itself, with the result that the mortality shows an improvement for several years. Moreover, after years of dissipation the effects are apparent to medical examiners, who rate such lives accordingly. Those who take provident policies are likely to be of a more steady disposition and less liable to run into excess, especially within so short a period as one year from the date of their policy. They too may of course be led astray; and it will be found that a similar dip in the mortality appears, though but faintly, amongst such policies, the feature being more noticeable in years subsequent to the first.

I am convinced that by devoting time and attention to the study of these mortality curves, most of the peculiar twists and turns might be satisfactorily explained. For example, I observed in the *British Medical Journal* of 17 September 1898 that Mr. Low found that extra mortality from obesity "seemed to occur entirely between the ages of 50 and 60, as both before and after that period the actual and expected number of deaths corresponded very closely". Now although the feature is scarcely perceptible in the graduated diagrams, yet I found that there was a slight increase in the progression of the mortality between these ages, and a subsequent decrease. This can only be observed by examining carefully the progressive differences. As stout men constitute only a very small proportion of the whole number effecting policies at ordinary rates, it seems almost wonderful that any difference whatever may be observed in the first insurance year. Such matters are however more curious than useful, and I therefore pass to other phases of the subject.

The next table compares female mortality during the first insurance year with the corresponding rates of male mortality:—

COMPARISON OF FIRST YEAR'S MORTALITY OF MALES AND FEMALES—WHOLE-LIFE WITH PROFITS.

| Age. | Male.* | Female.* |
|---|---|---|
| 20 | ·0026 | ·0068 |
| 25 | ·0027 | ·0057 |
| 30 | ·0033 | ·0055 |
| 35 | ·0037 | ·0056 |
| 40 | ·0042 | ·0072 |
| 45 | ·0057 | ·0076 |
| 50 | ·0075 | ·0077 |
| 55 | ·0103 | ·0110 |
| 60 | ·0153 | ·0160 |

* See Diagram 2.

The female curve is of an entirely different nature from that depicting male mortality; and, as might be expected, the rates are considerably higher. There are two well-marked maximum periods in the female curve—at ages 20-22 and 40-43; and afterwards at age 50 the curve takes its final and definite upward movement. Between ages 23 and 40 the first year's mortality is almost constant. As the same maximum periods do not occur distinctly during years subsequent to the first, they might be accidental; but, as they appear at periods when the extra risk from first pregnancy is understood to be great, and policies are frequently effected with the special object of covering that risk, there is also considerable likelihood that these maxima are peculiar to the first year's female mortality. This view is upheld by the fact that similar maximum periods show themselves under policies without participation, although the data relating to such are more meagre. It will be remembered that the female tables include all healthy lives though subjected to extra premium on account of sex. Even at older ages, above 50, the first year's mortality is heavier than amongst males, and this is doubtless due to the fact that the medical examination of females is never so effective. But for this one would expect the rates to be lighter instead of heavier.

The Discrimination shown by those effecting policies is so well defined under the various classes, that one is naturally led to compare the first year's risk under policies with the same function under annuities. Seeing that there is no medical examination whatever under the latter, one would expect to find personal Selection most apparent. The principal ages at entry are entirely different in the two sections; but there is an overlapping period which is sufficiently long for the comparison desired. From ages 50 to 60 the statistics are sufficient in all four sections to

give reliable results, and the same state of affairs exists at these as at the other ages. At first I was inclined to use the female experience because of the larger number of female annuitants; but the peculiarities of female mortality, which probably apply to assurances to a greater extent than to annuities, would have made such comparison unfair, and male mortality was therefore used.

COMPARISON OF FIRST YEAR'S MORTALITY AMONGST ANNUITANTS AND ASSURED LIVES—MALES.

| Age. | Annuitants.* | Assured Lives. | | |
|---|---|---|---|---|
| | | All Classes. | With profits. | Without profits. |
| 40 | ·0058 | ·0043 | ·0042 | ·0063 |
| 45 | ·0061 | ·0058 | ·0057 | ·0086 |
| 50 | ·0079 | ·0081 | ·0075 | ·0116 |
| 55 | ·0131 | ·0114 | ·0103 | ·0165 |
| 60 | ·0183 | ·0161 | ·0153 | ·0206 |
| 65 | ·0254 | ·0228 | ·0219 | ... |
| 70 | ·0414 | ·0324 | ·0295 | ... |
| 75 | ·0543 | ... | ·0600 | ... |

* See Diagram 3.

The mortality amongst Annuitants in the first year is slightly higher than that amongst Assured Lives when all classes are combined; but *lower* than is experienced amongst "financial" policies. All the tables therefore indicate an extent of knowledge on the part of the public regarding their prospects of longevity, which few would formerly have credited. When Discrimination is exercised against an office under financial policies, the medical examination is not so effective as to reduce the first year's mortality to that experienced by Annuitants. On the other hand, when the Discrimination by the lives runs in harmony with the Selection of the office, the mortality is considerably lighter than that caused by the Discrimination of Annuitants alone. Probably they could equalise matters if, in all cases, they consulted a doctor and took his advice; but their mortality is sufficiently light already without any such additional infliction on the offices. In subsequent years the mortality amongst Annuitants is much lighter than that amongst provident policies; but, as withdrawals come in as a disturbing influence, we cannot say whether or not the office Selection is in the long-run more effective than the personal Discrimination.

The effect of dating back policies should be kept in mind, and especially when considering the relative rates of mortality in the

first year amongst Annuitants and Assured Lives. We have no means of estimating the extent of this practice, which is common to all offices in a greater or less degree; but there is probably no doubt whatever as to its general effect. There are two principal reasons for dating back:—

(1) To secure the reduced rate of premium at an age one year less; and

(2) To secure an additional year's bonus by effecting the policy before the close of a financial year.

Both of these reasons apply to With-profit policies; the first to Without-profit policies; and neither of them to Annuities. Accordingly the mortality amongst Assured Lives is probably too light, while the Annuity mortality will be more nearly accurate. This shows that the personal Discrimination is even more effective than the figures show, so far as Annuities and Assured Lives are concerned. Amongst Assured Lives, however, it diminishes the force of Discrimination between With- and Without-profit policies. Probably Whole-life policies will not be dated back to a smaller extent than Limited-payments and Endowment-assurances, although, as actuaries, we know there is less reason for dating back investment policies. The Discrimination of Assured Lives has considerable force, but it is probably not quite so acute as to distinguish between the effect of dating back under different classes of policies—other than Participating and Non-participating. Were it not that the rates of mortality during subsequent years confirm the differences between the classes as experienced during the first year, I should have been disposed to look for other reasons in explanation of the marked differences; but the figures leave no room to doubt that, even down to the first year, pronounced differences in the mortality rates do exist; and Discrimination by the Assured seems to give the only satisfactory explanation.

In the first paragraph of these notes it is stated that one of the obvious purposes of ascertaining the first year's risk is "to estimate the immediate effect of the Selection by offices"; and, in following up this part of the subject, I have been induced to carry the inquiry beyond the scope of the paper as indicated by its title, and to compare the rates of subsequent years with those of the first year, thus tracing the deterioration year by year. The results are given in the undernoted table, and the rates of the English Life Table (Male) have been inserted for comparison. The same process of graduation has been applied to the subsequent years as was used for the first year; that is, the mortality of the second year was graduated by itself; the fifth year by itself; and so on.

MORTALITY AMONGST LIVES ASSURED FOR DIFFERENT DURATIONS, AS EXPERIENCED WHILE PASSING THROUGH DIFFERENT AGES.

| Period. | Age 40-1. | Age 45-6. | Age 50-1. | Age 55-6. | Age 60-1. |
|---|---|---|---|---|---|
| 1st year, . . . . . $q_{[x]}$ | ·0042 | ·0057 | ·0075 | ·0103 | ·0153 |
| 2nd year, . . . . . $q_{[x-1]+1}$ | ·0064 | ·0078 | ·0110 | ·0156 | ·0203 |
| 5th year, . . . . . $q_{[x-4]+4}$ | ·0080 | ·0094 | ·0123 | ·0172 | ·0218 |
| 10th year, . . . . . $q_{[x-9]+9}$ | ·0093 | ·0110 | ·0144 | ·0197 | ·0300 |
| 15th year, . . . . . $q_{[x-14]+14}$ | ·0104 | ·0122 | ·0153 | ·0204 | ·0285 |
| 20th year, . . . . . $q_{[x-19]+19}$ | ·0118 | ·0130 | ·0160 | ·0219 | ·0303 |
| 26th to 30th years, $q_{[x-n]+n}$ | ·0129* | ·0140 | ·0167 | ·0222 | ·0310 |
| English Life Table No. 3 (Male), . . . . . $q_x$ | ·0130 | ·0154 | ·0188 | ·0246 | ·0325 |
| English Life (1876-80), by Noel A. Humphreys (*J. I. A.*, xxvii, 503), $q_x$ | ·0136 | ·0160 | ·0195 | ·0268 | ·0354 |

* Estimated from curve at older ages.

It will be observed that the mortality amongst Assured Lives even from the 26th to the 30th year is slightly lower than that of the English Life Table No. 3. Of course that table relates to a period several years anterior to the earliest portion of the new tables, but it is generally understood that the subsequent mortality rates do not differ materially, and tables for later periods are not within my immediate reach.

Further, the method here used shows that there is no maximum mortality period between the 11th and 15th years, as was formerly contended by Dr. Sprague (*J. I. A.*, xv, 339); but with one slight exception there is a steady and continuous increase as the years roll on. I have no doubt whatever that Mr. Macaulay's explanation of the apparent maximum (*J. I. A.*, xxxii, 117) is the true one.

In concluding, I have a word or two to say regarding the method of graduating each year's experience by itself, as the advantages of treating the rough facts in this way are considerable. This idea was suggested to me many years ago by one whose help as a working member of the Faculty is now much appreciated. By this method we see the gradual deterioration which takes place year by year after the lives have been placed upon the books; we carry the "Policy-year" system to its logical conclusion; and, if the facts be sufficiently numerous, a mathematical process like Woolhouse's or Makeham's may be satisfactorily used. Makeham's method would of course lose the practical advantage of "Uniform

Seniority"; but I think there is a possibility of our finding something like a law for select tables by calculating the mortality constants applicable to the different years. I regret that I have not had time to do this. The comparison of these would be interesting, and would show the true effect of the Selection by offices as modified by subsequent withdrawals. The curves representing the mortality in the various years show satisfactory regularity, and one or two of them required very slight graduation.

## TABLE 1.—Combined Classes—Exposed to Risk, First Year.

| Age. | Endowment-assurances. | Limited-payments. | Ascending scale. | Contingent. | Temporary. | Joint. | Ordinary Whole-life. With profits. | Ordinary Whole-life. Without profits. | Total. |
|---|---|---|---|---|---|---|---|---|---|
| 15 | 809 | 57 | 28 | .. | .. | .. | 794 | 55 | 1243 |
| 16 | 445 | 73 | 35 | .. | .. | .. | 1122 | 87 | 1762 |
| 17 | 785 | 182 | 57 | .. | .. | .. | 1682 | 93 | 2749 |
| 18 | 1170 | 179 | 90 | .. | .. | .. | 2572 | 157 | 4168 |
| 19 | 1951 | 270 | 151 | .. | .. | .. | 3713 | 301 | 6386 |
| 20 | 3272 | 455 | 233 | 155 | 1178 | 142 | 5768 | 433 | 11636 |
| 21 | 4981 | 811 | 401 | .. | .. | .. | 10010 | 693 | 16896 |
| 22 | 6084 | 959 | 517 | .. | .. | .. | 12392 | 1005 | 20957 |
| 23 | 6865 | 1162 | 666 | .. | .. | .. | 15099 | 1131 | 24923 |
| 24 | 7542 | 1347 | 821 | .. | .. | .. | 17861 | 1237 | 28808 |
| 25 | 8161 | 1631 | 954 | 449 | 1083 | 950 | 20401 | 1411 | 35040 |
| 26 | 7919 | 1626 | 995 | .. | .. | .. | 21501 | 1425 | 33466 |
| 27 | 7817 | 1646 | 993 | .. | .. | .. | 22024 | 1496 | 33973 |
| 28 | 7516 | 1674 | 1070 | .. | .. | .. | 22249 | 1581 | 34090 |
| 29 | 7315 | 1699 | 1070 | .. | .. | .. | 22606 | 1521 | 34211 |
| 30 | 7351 | 1744 | 1021 | 528 | 1159 | 1465 | 23113 | 1469 | 37850 |
| 31 | 6405 | 1530 | 978 | .. | .. | .. | 21215 | 1446 | 31574 |
| 32 | 5926 | 1446 | 864 | .. | .. | .. | 20535 | 1359 | 30130 |
| 33 | 5404 | 1452 | 863 | .. | .. | .. | 19670 | 1381 | 28770 |
| 34 | 4941 | 1285 | 801 | .. | .. | .. | 18505 | 1356 | 26888 |
| 35 | 4533 | 1236 | 718 | 522 | 1341 | 1410 | 17790 | 1325 | 28875 |
| 36 | 3859 | 1093 | 679 | .. | .. | .. | 16510 | 1278 | 23419 |
| 37 | 3318 | 1015 | 630 | .. | .. | .. | 14944 | 1263 | 21170 |
| 38 | 3049 | 969 | 618 | .. | .. | .. | 14157 | 1299 | 20087 |
| 39 | 2714 | 802 | 598 | .. | .. | .. | 13337 | 1268 | 18719 |
| 40 | 2658 | 815 | 563 | 474 | 1417 | 1053 | 12821 | 1303 | 21104 |
| 41 | 1903 | 692 | 485 | .. | .. | .. | 11235 | 1111 | 15426 |
| 42 | 1734 | 656 | 471 | .. | .. | .. | 10434 | 1092 | 14387 |
| 43 | 1418 | 560 | 429 | .. | .. | .. | 9517 | 987 | 12911 |
| 44 | 1165 | 441 | 367 | .. | .. | .. | 8749 | 915 | 11637 |
| 45 | 980 | 410 | 354 | 355 | 1275 | 697 | 8202 | 917 | 13190 |
| 46 | 778 | 412 | 347 | .. | .. | .. | 7152 | 901 | 9590 |
| 47 | 558 | 295 | 307 | .. | .. | .. | 6466 | 844 | 8470 |
| 48 | 450 | 262 | 289 | .. | .. | .. | 5860 | 821 | 7682 |
| 49 | 391 | 236 | 255 | .. | .. | .. | 5846 | 800 | 7028 |
| 50 | 410 | 218 | 220 | 201 | 1060 | 428 | 5193 | 746 | 8476 |
| 51 | 209 | 146 | 170 | .. | .. | .. | 4045 | 711 | 5281 |
| 52 | 128 | 104 | 161 | .. | .. | . | 3670 | 701 | 4764 |
| 53 | 91 | 113 | 127 | .. | .. | .. | 3193 | 638 | 4162 |
| 54 | 80 | 94 | 102 | .. | .. | .. | 2825 | 597 | 3698 |
| 55 | 68 | 75 | 97 | 126 | 720 | 179 | 2467 | 549 | 4276 |
| 56 | 47 | 59 | 88 | .. | .. | .. | 2110 | 513 | 2812 |
| 57 | 23 | 50 | 45 | .. | .. | .. | 1700 | 440 | 2258 |
| 58 | 21 | 37 | 56 | .. | .. | .. | 1477 | 464 | 2055 |
| 59 | 22 | 39 | 35 | .. | .. | .. | 1363 | 442 | 1901 |
| 60 | 27 | .. | .. | 49 | 420 | 65 | 1348 | 422 | 2331 |
| 61 | 9 | .. | .. | .. | .. | .. | 806 | 358 | 1173 |
| 62 | 8 | .. | .. | .. | .. | .. | 712 | 295 | 1010 |
| 63 | 4 | .. | .. | .. | .. | .. | 541 | 273 | 818 |
| 64 | 6 | .. | .. | .. | .. | .. | 438 | 235 | 679 |
| 65 | 2 | .. | .. | .. | .. | .. | 357 | 214 | 573 |
| 66 | 3 | .. | .. | .. | .. | 32 | 316 | 170 | 521 |
| 67 | 3 | .. | .. | .. | .. | .. | 238 | 132 | 373 |
| 68 | 1 | .. | .. | .. | .. | .. | 176 | 116 | 293 |
| 69 | .. | .. | 80 | .. | .. | .. | 126 | 86 | 292 |
| 70 | 1 | .. | .. | 43 | .. | .. | 123 | 83 | 255 |
| 71 | .. | .. | .. | .. | .. | .. | 62 | 57 | 119 |
| 72 | .. | 118 | .. | .. | .. | .. | 40 | 44 | 202 |
| 73 | .. | .. | .. | .. | .. | .. | 43 | 43 | 86 |
| 74 | .. | .. | .. | .. | .. | .. | 20 | 30 | 50 |
| 75 | .. | .. | .. | .. | .. | .. | 14 | 17 | 31 |

## TABLE 2.—Combined Classes—Deaths, First Year.

| Age. | Endowment-assurances. | Limited-payments. | Ascending scale. | Contingent. | Temporary. | Joint. | Ordinary Whole-life. | | Total. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | With profits. | Without profits. | |
| 15 | .. | .. | .. | .. | .. | .. | 1 | .. | 1 |
| 16 | 2 | .. | .. | .. | .. | .. | 3 | 1 | 6 |
| 17 | 1 | .. | .. | .. | .. | .. | 7 | 2 | 10 |
| 18 | 3 | 1 | .. | .. | .. | .. | 8 | .. | 12 |
| 19 | 9 | .. | .. | .. | .. | .. | 9 | 2 | 20 |
| 20 | 8 | .. | 1 | 1 | 10 | 1 | 17 | 5 | 38 |
| 21 | 16 | .. | 1 | .. | .. | .. | 26 | 5 | 48 |
| 22 | 7 | 1 | 1 | .. | .. | .. | 27 | 3 | 39 |
| 23 | 16 | 4 | 2 | .. | .. | .. | 31 | 7 | 60 |
| 24 | 15 | .. | 4 | .. | .. | .. | 57 | 5 | 81 |
| 25 | 19 | 2 | 5 | 2 | 3 | 3 | 50 | 4 | 88 |
| 26 | 16 | 4 | 4 | .. | .. | .. | 51 | 8 | 83 |
| 27 | 17 | 3 | 1 | .. | .. | .. | 69 | 9 | 99 |
| 28 | 15 | 3 | 2 | .. | .. | .. | 53 | 5 | 78 |
| 29 | 14 | 3 | 1 | .. | .. | .. | 82 | 5 | 105 |
| 30 | 16 | 4 | 6 | 3 | 4 | 2 | 72 | 6 | 113 |
| 31 | 20 | 9 | .. | .. | .. | .. | 76 | 3 | 108 |
| 32 | 23 | 2 | 3 | .. | .. | .. | 75 | 9 | 112 |
| 33 | 15 | 4 | 1 | .. | .. | .. | 67 | 7 | 94 |
| 34 | 12 | 1 | 4 | .. | .. | .. | 76 | 1 | 94 |
| 35 | 12 | 4 | 2 | 5 | 8 | 12 | 71 | 5 | 119 |
| 36 | 9 | .. | 5 | .. | .. | .. | 57 | 6 | 77 |
| 37 | 15 | 5 | 1 | .. | .. | .. | 52 | 5 | 78 |
| 38 | 7 | 5 | 3 | .. | .. | .. | 53 | 9 | 77 |
| 39 | 9 | 2 | 3 | .. | .. | .. | 44 | 9 | 67 |
| 40 | 14 | 5 | 5 | 5 | 5 | 6 | 60 | 4 | 104 |
| 41 | 10 | 1 | 3 | .. | .. | .. | 42 | 12 | 68 |
| 42 | 5 | 2 | 1 | .. | .. | .. | 58 | 7 | 73 |
| 43 | .. | 1 | 1 | .. | .. | .. | 61 | 8 | 71 |
| 44 | 10 | 3 | 2 | .. | .. | .. | 51 | 6 | 72 |
| 45 | 6 | .. | 2 | 3 | 11 | 4 | 46 | 10 | 82 |
| 46 | 4 | 1 | 3 | .. | .. | .. | 43 | 8 | 59 |
| 47 | 2 | .. | 1 | .. | .. | .. | 31 | 9 | 43 |
| 48 | 2 | 2 | 3 | .. | .. | .. | 32 | 5 | 44 |
| 49 | 4 | .. | 3 | .. | .. | .. | 33 | 9 | 54 |
| 50 | 3 | 1 | 1 | 4 | 11 | 6 | 49 | 9 | 84 |
| 51 | .. | .. | 3 | .. | .. | .. | 21 | 7 | 31 |
| 52 | 1 | .. | 3 | .. | .. | .. | 37 | 9 | 50 |
| 53 | .. | 1 | 1 | .. | .. | .. | 27 | 12 | 41 |
| 54 | 1 | .. | .. | .. | .. | .. | 34 | 12 | 47 |
| 55 | .. | .. | .. | .. | 10 | .. | 17 | 10 | 37 |
| 56 | .. | .. | .. | .. | .. | .. | 22 | 7 | 29 |
| 57 | 1 | 1 | .. | .. | .. | .. | 23 | 8 | 33 |
| 58 | 1 | 1 | .. | .. | .. | .. | 23 | 8 | 33 |
| 59 | .. | .. | .. | .. | .. | .. | 24 | 10 | 34 |
| 60 | .. | .. | .. | 2 | 8 | 2 | 25 | 10 | 47 |
| 61 | .. | .. | .. | .. | .. | .. | 12 | 3 | 15 |
| 62 | .. | .. | .. | .. | .. | .. | 12 | 7 | 19 |
| 63 | .. | .. | .. | .. | .. | .. | 10 | 3 | 13 |
| 64 | .. | .. | .. | .. | .. | .. | 5 | 3 | 8 |
| 65 | .. | .. | .. | .. | .. | .. | 9 | 5 | 14 |
| 66 | .. | .. | .. | .. | .. | .. | 4 | 5 | 9 |
| 67 | .. | .. | .. | .. | .. | .. | 5 | 5 | 10 |
| 68 | .. | .. | .. | .. | .. | .. | 3 | 5 | 8 |
| 69 | .. | .. | 3 | .. | .. | .. | 5 | 5 | 13 |
| 70 | .. | .. | .. | .. | .. | .. | 3 | 3 | 6 |
| 71 | .. | .. | .. | .. | .. | .. | 2 | 5 | 7 |
| 72 | .. | 2 | .. | .. | .. | .. | 3 | 8 | 13 |
| 73 | .. | .. | .. | .. | .. | .. | 2 | 1 | 3 |
| 74 | .. | .. | .. | .. | .. | .. | 1 | 3 | 4 |
| 75 | .. | .. | .. | .. | .. | .. | 1 | .. | 1 |

## TABLE 3.—All Classes.

| Age. $x$ | Exposed to risk. | Graduated $q_{[x]}$ | Deaths. Expected. | Deaths. Actual. | Difference. (4)—(5) | Accumulated difference. |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 15 | 1243 | ·00285 | 3·5 | 1 | + 2·5 | + 2·5 |
| 16 | 1762 | ·00284 | 5·0 | 6 | − 1·0 | + 1·5 |
| 17 | 2749 | ·00283 | 7·8 | 10 | − 2·2 | − ·7 |
| 18 | 4168 | ·00281 | 11·7 | 12 | − ·3 | − 1·0 |
| 19 | 6386 | ·00278 | 17·8 | 20 | − 2·2 | − 3·2 |
| 20 | 11636 | ·00273 | 31·8 | 38 | − 6·2 | − 9·4 |
| 21 | 16896 | ·00266 | 44·9 | 48 | − 3·1 | −12·5 |
| 22 | 20957 | ·00259 | 54·3 | 39 | +15·3 | + 2·8 |
| 23 | 24923 | ·00254 | 63·3 | 60 | + 3·3 | + 6·1 |
| 24 | 28808 | ·00252 | 72·6 | 81 | − 8·4 | − 2·3 |
| 25 | 35040 | ·00253 | 88·7 | 88 | + ·7 | − 1·6 |
| 26 | 33466 | ·00257 | 86·0 | 83 | + 3·0 | + 1·4 |
| 27 | 33973 | ·00265 | 90·0 | 99 | − 9·0 | − 7·6 |
| 28 | 34090 | ·00276 | 94·1 | 78 | +16·1 | + 8·5 |
| 29 | 34211 | ·00290 | 99·2 | 105 | − 5·8 | + 2·7 |
| 30 | 37850 | ·00305 | 115·4 | 113 | + 2·4 | + 5·1 |
| 31 | 31574 | ·00320 | 101·0 | 108 | − 7·0 | − 1·9 |
| 32 | 30130 | ·00334 | 100·6 | 112 | −11·4 | −13·3 |
| 33 | 28770 | ·00348 | 100·1 | 94 | + 6·1 | − 7·2 |
| 34 | 26888 | ·00361 | 97·1 | 94 | + 3·1 | − 4·1 |
| 35 | 28875 | ·00373 | 107·7 | 119 | −11·3 | −15·4 |
| 36 | 23419 | ·00384 | 89·9 | 77 | +12·9 | − 2·5 |
| 37 | 21170 | ·00394 | 83·4 | 78 | + 5·4 | + 2·9 |
| 38 | 20087 | ·00405 | 81·4 | 77 | + 4·4 | + 7·3 |
| 39 | 18719 | ·00418 | 78·2 | 67 | +11·2 | +18·5 |
| 40 | 21104 | ·00434 | 91·6 | 104 | −12·4 | + 6·1 |
| 41 | 15426 | ·00454 | 70·0 | 68 | + 2·0 | + 8·1 |
| 42 | 14387 | ·00479 | 68·9 | 73 | − 4·1 | + 4·0 |
| 43 | 12911 | ·00508 | 65·6 | 71 | − 5·4 | − 1·4 |
| 44 | 11637 | ·00541 | 63·0 | 72 | − 9·0 | −10·4 |
| 45 | 13190 | ·00578 | 76·2 | 82 | − 5·8 | −16·2 |
| 46 | 9590 | ·00618 | 59·3 | 59 | + ·3 | −15·9 |
| 47 | 8470 | ·00661 | 56·0 | 43 | +13·0 | − 2·9 |
| 48 | 7682 | ·00707 | 54·3 | 44 | +10·3 | + 7·4 |
| 49 | 7028 | ·00756 | 53·1 | 54 | − ·9 | + 6·5 |
| 50 | 8476 | ·00810 | 68·7 | 84 | −15·3 | − 8·8 |
| 51 | 5281 | ·00867 | 45·8 | 31 | +14·8 | + 6·0 |
| 52 | 4764 | ·00928 | 44·2 | 50 | − 5·8 | + ·2 |
| 53 | 4162 | ·00993 | 41·3 | 41 | + ·3 | + ·5 |
| 54 | 3698 | ·01062 | 39·3 | 47 | − 7·7 | − 7·2 |
| 55 | 4276 | ·01136 | 48·6 | 37 | +11·6 | + 4·4 |
| 56 | 2812 | ·01216 | 34·2 | 29 | + 5·2 | + 9·6 |
| 57 | 2258 | ·01306 | 29·5 | 33 | − 3·5 | + 6·1 |
| 58 | 2055 | ·01400 | 28·8 | 33 | − 4·2 | + 1·9 |
| 59 | 1901 | ·01502 | 28·6 | 34 | − 5·4 | − 3·5 |
| 60 | 2331 | ·01612 | 37·6 | 47 | − 9·4 | −12·9 |
| 61 | 1173 | ·01730 | 20·3 | 15 | + 5·3 | − 7·6 |
| 62 | 1010 | ·01856 | 18·7 | 19 | − ·3 | − 7·9 |
| 63 | 818 | ·01990 | 16·3 | 13 | + 3·3 | − 4·6 |
| 64 | 679 | ·02132 | 14·5 | 8 | + 6·5 | + 1·9 |
| 65 | 573 | ·02282 | 13·1 | 14 | − ·9 | + 1·0 |

## TABLE 4.—Whole-Life Assurances, With Profits.

| Age. $x$ | Exposed to risk. | Graduated $q_{[x]}$ | Deaths. Expected. | Deaths. Actual. | Difference. (4)−(5) | Accumulated difference. |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 15 | 794 | ·00265 | 2·10 | 1 | + 1·10 | + 1·10 |
| 16 | 1122 | ·00264 | 2·96 | 3 | − ·04 | + 1·06 |
| 17 | 1682 | ·00263 | 4·42 | 7 | − 2·58 | − 1·52 |
| 18 | 2572 | ·00262 | 6·74 | 8 | − 1·26 | − 2·78 |
| 19 | 3713 | ·00261 | 9·69 | 9 | + ·69 | − 2·09 |
| 20 | 5768 | ·00260 | 15·00 | 17 | − 2·00 | − 4·09 |
| 21 | 10010 | ·00259 | 25·93 | 26 | − ·07 | − 4·16 |
| 22 | 12392 | ·00258 | 31·97 | 27 | + 4·97 | + ·81 |
| 23 | 15099 | ·00260 | 39·26 | 31 | + 8·26 | + 9·07 |
| 24 | 17861 | ·00262 | 46·80 | 57 | −10·20 | − 1·13 |
| 25 | 20401 | ·00266 | 54·27 | 50 | + 4·27 | + 3·14 |
| 26 | 21501 | ·00273 | 58·70 | 51 | + 7·70 | +10·84 |
| 27 | 22024 | ·00283 | 62·33 | 69 | − 6·67 | + 4·17 |
| 28 | 22249 | ·00296 | 65·86 | 53 | +12·86 | +17·03 |
| 29 | 22606 | ·00311 | 70·30 | 82 | −11·70 | + 5·33 |
| 30 | 23113 | ·00326 | 75·35 | 72 | + 3·35 | + 8·68 |
| 31 | 21215 | ·00339 | 71·92 | 76 | − 4·08 | + 4·60 |
| 32 | 20535 | ·00350 | 71·87 | 75 | − 3·13 | + 1·47 |
| 33 | 19670 | ·00358 | 70·42 | 67 | + 3·42 | + 4·89 |
| 34 | 18505 | ·00363 | 67·17 | 76 | − 8·83 | − 3·94 |
| 35 | 17790 | ·00367 | 65·29 | 71 | − 5·71 | − 9·65 |
| 36 | 16510 | ·00373 | 61·58 | 57 | + 4·58 | − 5·07 |
| 37 | 14944 | ·00381 | 56·94 | 52 | + 4·94 | − ·13 |
| 38 | 14157 | ·00391 | 55·35 | 53 | + 2·35 | + 2·22 |
| 39 | 13337 | ·00405 | 54·01 | 44 | +10·01 | +12·23 |
| 40 | 12821 | ·00424 | 54·36 | 60 | − 5·64 | + 6·59 |
| 41 | 11235 | ·00447 | 50·22 | 42 | + 8·22 | +14·81 |
| 42 | 10434 | ·00473 | 49·35 | 58 | − 8·65 | + 6·16 |
| 43 | 9517 | ·00502 | 47·78 | 61 | −13·22 | − 7·06 |
| 44 | 8749 | ·00534 | 46·72 | 51 | − 4·28 | −11·34 |
| 45 | 8202 | ·00568 | 46·59 | 46 | + ·59 | −10·75 |
| 46 | 7152 | ·00603 | 43·13 | 43 | + ·13 | −10·62 |
| 47 | 6466 | ·00639 | 41·32 | 31 | +10·32 | − ·30 |
| 48 | 5860 | ·00675 | 39·56 | 32 | + 7·56 | + 7·26 |
| 49 | 5346 | ·00712 | 38·06 | 38 | + ·06 | + 7·32 |
| 50 | 5193 | ·00751 | 39·00 | 49 | −10·00 | − 2·68 |
| 51 | 4045 | ·00794 | 32·12 | 21 | +11·12 | + 8·44 |
| 52 | 3670 | ·00842 | 30·90 | 37 | − 6·10 | + 2·34 |
| 53 | 3193 | ·00896 | 28·61 | 27 | + 1·61 | + 3·95 |
| 54 | 2825 | ·00958 | 27·06 | 34 | − 6·94 | − 2·99 |
| 55 | 2467 | ·01030 | 25·41 | 17 | + 8·41 | + 5·42 |
| 56 | 2110 | ·01115 | 23·53 | 22 | + 1·53 | + 6·95 |
| 57 | 1700 | ·01208 | 20·54 | 23 | − 2·46 | + 4·49 |
| 58 | 1477 | ·01311 | 19·36 | 23 | − 3·64 | + ·85 |
| 59 | 1338 | ·01421 | 19·37 | 24 | − 4·63 | − 3·78 |
| 60 | 1348 | ·01534 | 20·68 | 25 | − 4·32 | − 8·10 |
| 61 | 806 | ·01650 | 13·30 | 12 | + 1·30 | − 6·80 |
| 62 | 712 | ·01770 | 12·60 | 12 | + ·60 | − 6·20 |
| 63 | 541 | ·01900 | 10·28 | 10 | + ·28 | − 5·92 |
| 64 | 438 | ·02040 | 8·94 | 5 | + 3·94 | − 1·98 |
| 65 | 357 | ·02190 | 7·82 | 9 | − 1·18 | − 3·16 |
| 66 | 316 | ·02340 | 7·39 | 4 | + 3·39 | + ·23 |
| 67 | 238 | ·02490 | 5·93 | 5 | + ·93 | + 1·16 |
| 68 | 176 | ·02640 | 4·65 | 3 | + 1·65 | + 2·81 |
| 69 | 126 | ·02790 | 3·52 | 5 | − 1·48 | + 1·33 |
| 70 | 123 | ·02950 | 3·63 | 3 | + ·63 | + 1·96 |
| 71 | 62 | ·03150 | 1·95 | 2 | − ·05 | + 1·91 |
| 72 | 40 | ·03600 | 1·40 | 3 | − 1·60 | + ·31 |
| 73 | 43 | ·04000 | 1·72 | 2 | − ·28 | + ·03 |
| 74 | 20 | ·05000 | 1·00 | 1 | + ·00 | + ·03 |
| 75 | 14 | ·06000 | ·84 | 1 | − ·16 | − ·13 |

TABLE 5.—WHOLE-LIFE ASSURANCES, WITHOUT PROFITS.

| Age. $x$ | Exposed to risk. | Graduated $q_{[x]}$. | Deaths. Expected. | Deaths. Actual. | Difference. (4)—(5) | Accumulated difference. |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 20 | 433 | ·00577 | 2·50 | 5 | −2·50 | −2·50 |
| 21 | 693 | ·00559 | 3·87 | 5 | −1·13 | −3·63 |
| 22 | 1005 | ·00537 | 5·40 | 3 | +2·40 | −1·23 |
| 23 | 1131 | ·00513 | 5·80 | 7 | −1·20 | −2·43 |
| 24 | 1237 | ·00488 | 6·04 | 5 | +1·04 | −1·39 |
| 25 | 1411 | ·00464 | 6·55 | 4 | +2·55 | +1·16 |
| 26 | 1425 | ·00442 | 6·30 | 8 | −1·70 | − ·54 |
| 27 | 1496 | ·00423 | 6·33 | 9 | −2·67 | −3·21 |
| 28 | 1581 | ·00409 | 6·47 | 5 | +1·47 | −1·74 |
| 29 | 1521 | ·00400 | 6·08 | 5 | +1·08 | − ·66 |
| 30 | 1469 | ·00396 | 5·82 | 6 | − ·18 | − ·84 |
| 31 | 1446 | ·00396 | 5·73 | 3 | +2·73 | +1·89 |
| 32 | 1359 | ·00400 | 5·44 | 9 | −3·56 | −1·67 |
| 33 | 1381 | ·00409 | 5·65 | 7 | −1·35 | −3·02 |
| 34 | 1356 | ·00423 | 5·74 | 1 | +4·74 | +1·72 |
| 35 | 1325 | ·00445 | 5·90 | 5 | + ·90 | +2·62 |
| 36 | 1278 | ·00473 | 6·04 | 6 | + ·04 | +2·66 |
| 37 | 1263 | ·00506 | 6·39 | 5 | +1·39 | +4·05 |
| 38 | 1299 | ·00543 | 7·05 | 9 | −1·95 | +2·10 |
| 39 | 1268 | ·00584 | 7·41 | 9 | −1·59 | + ·51 |
| 40 | 1303 | ·00628 | 8·18 | 4 | +4·18 | +4·69 |
| 41 | 1111 | ·00673 | 7·48 | 12 | −4·52 | + ·17 |
| 42 | 1092 | ·00718 | 7·84 | 7 | + ·84 | +1·01 |
| 43 | 987 | ·00764 | 7·54 | 8 | − ·46 | + ·55 |
| 44 | 915 | ·00811 | 7·42 | 6 | +1·42 | +1·97 |
| 45 | 917 | ·00859 | 7·88 | 10 | −2·12 | − ·15 |
| 46 | 901 | ·00909 | 8·19 | 8 | + ·19 | + ·04 |
| 47 | 844 | ·00962 | 8·12 | 9 | − ·88 | − ·84 |
| 48 | 821 | ·01020 | 8·37 | 5 | +3·37 | +2·53 |
| 49 | 800 | ·01085 | 8·68 | 9 | − ·32 | +2·21 |
| 50 | 746 | ·01159 | 8·65 | 9 | − ·35 | +1·86 |
| 51 | 711 | ·01244 | 8·84 | 7 | +1·84 | +3·70 |
| 52 | 701 | ·01338 | 9·38 | 9 | + ·38 | +4·08 |
| 53 | 638 | ·01439 | 9·18 | 12 | −2·82 | +1·26 |
| 54 | 597 | ·01544 | 9·22 | 12 | −2·78 | −1·52 |
| 55 | 549 | ·01650 | 9·06 | 10 | − ·94 | −2·46 |
| 56 | 513 | ·01753 | 8·99 | 7 | +1·99 | − ·47 |
| 57 | 440 | ·01848 | 8·13 | 8 | + ·13 | − ·34 |
| 58 | 464 | ·01929 | 8·95 | 8 | + ·95 | + ·61 |
| 59 | 442 | ·01996 | 8·82 | 10 | −1·18 | − ·57 |
| 60 | 422 | ·02055 | 8·67 | 10 | −1·33 | −1·90 |

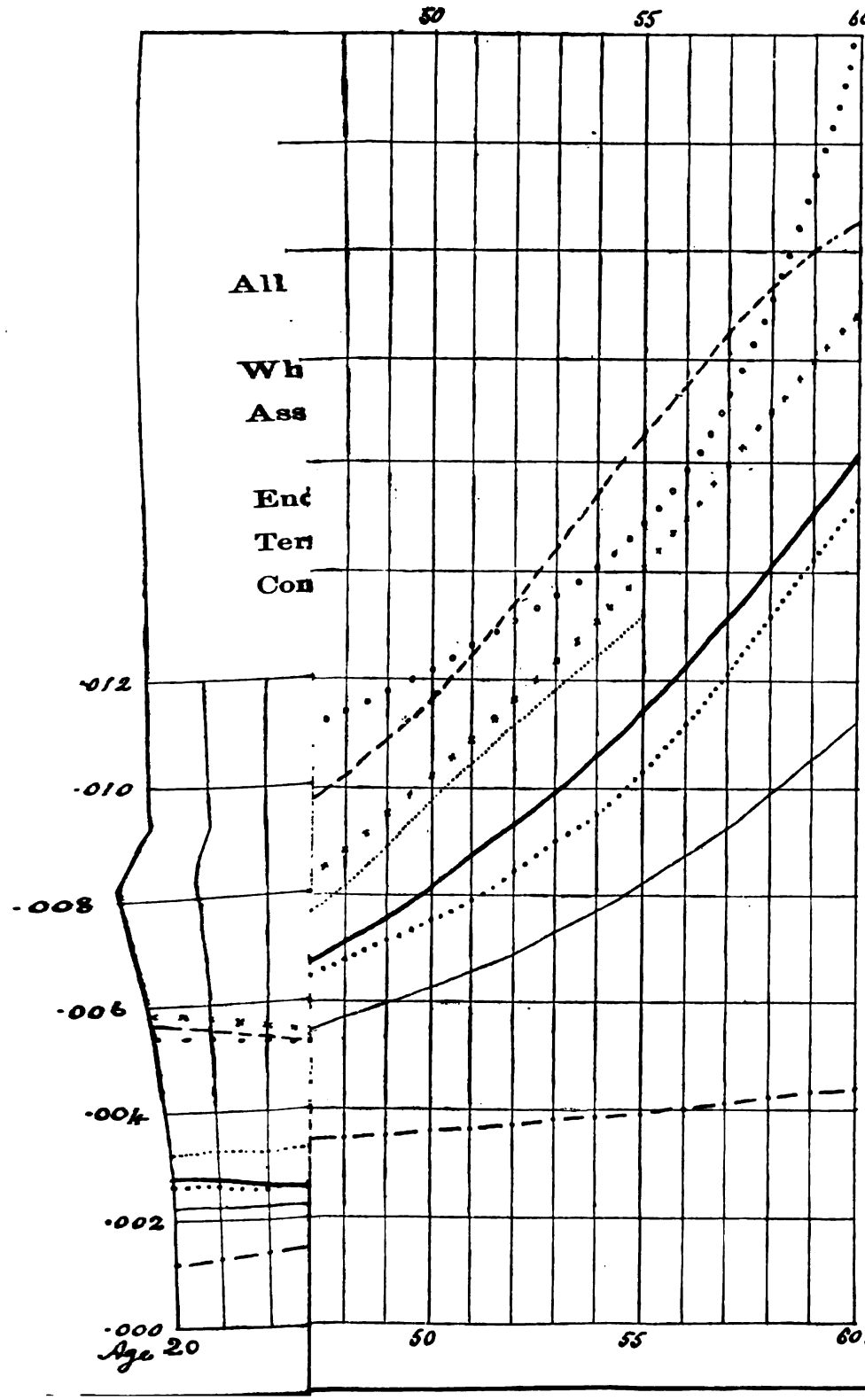
TABLE 6.—ENDOWMENT-ASSURANCES.

| Age. $x$ | Exposed to risk. | Graduated $q_{[x]}$. | Deaths. Expected. | Deaths. Actual. | Difference. (4)—(5) | Accumulated difference. |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 15 | 309 | ·00220 | ·68 | 0 | + ·68 | + ·68 |
| 16 | 445 | ·00220 | ·98 | 2 | −1·02 | − ·34 |
| 17 | 785 | ·00220 | 1·73 | 1 | + ·73 | + ·39 |
| 18 | 1170 | ·00220 | 2·57 | 3 | − ·43 | − ·04 |
| 19 | 1951 | ·00220 | 4·29 | 9 | −4·71 | −4·75 |
| 20 | 3272 | ·00220 | 7·20 | 3 | +4·20 | − ·55 |
| 21 | 4981 | ·00220 | 10·96 | 16 | −5·04 | −5·59 |
| 22 | 6084 | ·00220 | 13·38 | 7 | +6·38 | + ·79 |
| 23 | 6865 | ·00219 | 15·03 | 16 | − ·97 | − ·18 |
| 24 | 7542 | ·00217 | 16·37 | 15 | +1·37 | +1·19 |
| 25 | 8161 | ·00215 | 17·55 | 19 | −1·45 | − ·26 |
| 26 | 7919 | ·00214 | 16·95 | 16 | + ·95 | + ·69 |
| 27 | 7817 | ·00215 | 16·81 | 17 | − ·19 | + ·50 |
| 28 | 7516 | ·00218 | 16·38 | 15 | +1·38 | +1·88 |
| 29 | 7315 | ·00224 | 16·39 | 14 | +2·39 | +4·27 |
| 30 | 7351 | ·00233 | 17·13 | 16 | +1·13 | +5·40 |
| 31 | 6405 | ·00245 | 15·69 | 20 | −4·31 | +1·09 |
| 32 | 5926 | ·00259 | 15·35 | 23 | −7·65 | −6·56 |
| 33 | 5404 | ·00274 | 14·81 | 15 | − ·19 | −6·75 |
| 34 | 4941 | ·00290 | 14·33 | 12 | +2·33 | −4·42 |
| 35 | 4533 | ·00306 | 13·87 | 12 | +1·87 | −2·55 |
| 36 | 3859 | ·00322 | 12·43 | 9 | +3·43 | + ·88 |
| 37 | 3318 | ·00338 | 11·21 | 15 | −3·79 | −2·91 |
| 38 | 3049 | ·00354 | 10·79 | 7 | +3·79 | + ·88 |
| 39 | 2714 | ·00370 | 10·04 | 9 | +1·04 | +1·92 |
| 40 | 2658 | ·00387 | 10·29 | 14 | −3·71 | −1·79 |
| 41 | 1903 | ·00406 | 7·73 | 10 | −2·27 | −4·06 |
| 42 | 1734 | ·00427 | 7·40 | 5 | +2·40 | −1·66 |
| 43 | 1418 | ·00449 | 6·37 | 0 | +6·37 | +4·71 |
| 44 | 1165 | ·00472 | 5·50 | 10 | −4·50 | + ·21 |
| 45 | 980 | ·00495 | 4·85 | 6 | −1·15 | − ·94 |
| 46 | 778 | ·00519 | 4·04 | 4 | + ·04 | − ·90 |
| 47 | 558 | ·00544 | 3·04 | 2 | +1·04 | + ·14 |
| 48 | 450 | ·00570 | 2·57 | 2 | + ·57 | + ·71 |
| 49 | 391 | ·00598 | 2·34 | 4 | −1·66 | − ·95 |
| 50 | 410 | ·00628 | 2·57 | 3 | − ·43 | −1·38 |
| 51 | 209 | ·00660 | 1·38 | 0 | +1·38 | + ·00 |
| 52 | 128 | ·00694 | ·89 | 1 | − ·11 | − ·11 |
| 53 | 91 | ·00731 | ·67 | 0 | + ·67 | + ·56 |
| 54 | 80 | ·00772 | ·62 | 1 | − ·38 | + ·18 |
| 55 | 63 | ·00818 | ·52 | 0 | + ·52 | + ·70 |
| 56 | 47 | ·00869 | ·41 | 0 | + ·41 | +1·11 |
| 57 | 23 | ·00925 | ·21 | 1 | − ·79 | + ·32 |
| 58 | 21 | ·00986 | ·21 | 1 | − ·79 | − ·47 |
| 59 | 22 | ·01052 | ·23 | 0 | + ·23 | − ·24 |
| 60 | 27 | ·01123 | ·30 | 0 | + ·30 | + ·06 |

TABLE 7.—LIMITED-PAYMENTS.

| Age. $x$ | Exposed to risk. | Graduated $q_{[x]}$. | Deaths. Expected. | Deaths. Actual. | Difference. (4)—(5) | Accumulated difference. |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 20 | 455 | ·00120 | ·55 | 0 | + ·55 | + ·55 |
| 21 | 811 | ·00130 | 1·05 | 0 | +1·05 | +1·60 |
| 22 | 959 | ·00140 | 1·34 | 1 | + ·34 | +1·94 |
| 23 | 1162 | ·00149 | 1·73 | 4 | −2·27 | − ·33 |
| 24 | 1347 | ·00158 | 2·13 | 0 | +2·13 | +1·80 |
| 25 | 1631 | ·00166 | 2·71 | 2 | + ·71 | +2·51 |
| 26 | 1626 | ·00174 | 2·83 | 4 | −1·17 | +1·34 |
| 27 | 1646 | ·00181 | 2·98 | 3 | − ·02 | +1·32 |
| 28 | 1674 | ·00189 | 3·16 | 3 | + ·16 | +1·48 |
| 29 | 1699 | ·00199 | 3·38 | 3 | + ·38 | +1·86 |
| 30 | 1744 | ·00212 | 3·70 | 4 | − ·30 | +1·56 |
| 31 | 1530 | ·00227 | 3·47 | 9 | −5·53 | −3·97 |
| 32 | 1446 | ·00243 | 3·51 | 2 | +1·51 | −2·46 |
| 33 | 1452 | ·00259 | 3·76 | 4 | − ·24 | −2·70 |
| 34 | 1285 | ·00273 | 3·51 | 1 | +2·51 | − ·19 |
| 35 | 1236 | ·00285 | 3·52 | 4 | − ·48 | − ·67 |
| 36 | 1093 | ·00296 | 3·24 | 0 | +3·24 | +2·57 |
| 37 | 1015 | ·00305 | 3·10 | 5 | −1·90 | + ·67 |
| 38 | 969 | ·00312 | 3·02 | 5 | −1·98 | −1·31 |
| 39 | 802 | ·00318 | 2·55 | 2 | + ·55 | − ·76 |
| 40 | 815 | ·00323 | 2·63 | 5 | −2·37 | −3·13 |
| 41 | 692 | ·00328 | 2·27 | 1 | +1·27 | −1·86 |
| 42 | 656 | ·00332 | 2·18 | 2 | + ·18 | −1·68 |
| 43 | 560 | ·00335 | 1·88 | 1 | + ·88 | − ·80 |
| 44 | 441 | ·00338 | 1·49 | 3 | −1·51 | −2·31 |
| 45 | 410 | ·00340 | 1·39 | 0 | +1·39 | − ·92 |
| 46 | 412 | ·00343 | 1·41 | 1 | + ·41 | − ·51 |
| 47 | 295 | ·00346 | 1·02 | 0 | +1·02 | + ·51 |
| 48 | 262 | ·00350 | ·92 | 2 | −1·08 | − ·57 |
| 49 | 236 | ·00355 | ·84 | 0 | + ·84 | + ·27 |
| 50 | 218 | ·00360 | ·78 | 1 | − ·22 | + ·05 |
| 51 | 146 | ·00366 | ·53 | 0 | + ·53 | + ·58 |
| 52 | 104 | ·00372 | ·38 | 0 | + ·38 | + ·96 |
| 53 | 113 | ·00379 | ·43 | 1 | − ·57 | + ·39 |
| 54 | 94 | ·00386 | ·36 | 0 | + ·36 | + ·75 |
| 55 | 75 | ·00394 | ·30 | 0 | + ·30 | +1·05 |
| 56 | 59 | ·00402 | ·24 | 0 | + ·24 | +1·29 |
| 57 | 50 | ·00411 | ·21 | 1 | − ·79 | + ·50 |
| 58 | 37 | ·00420 | ·16 | 1 | − ·84 | − ·34 |
| 59 | 39 | ·00430 | ·17 | 0 | + ·17 | − ·17 |
| 60 | * | ·00442 | * | * | * | * |

* The ungraduated statistics are grouped from 60 onwards.


50
55
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·012
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Age 20
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THE
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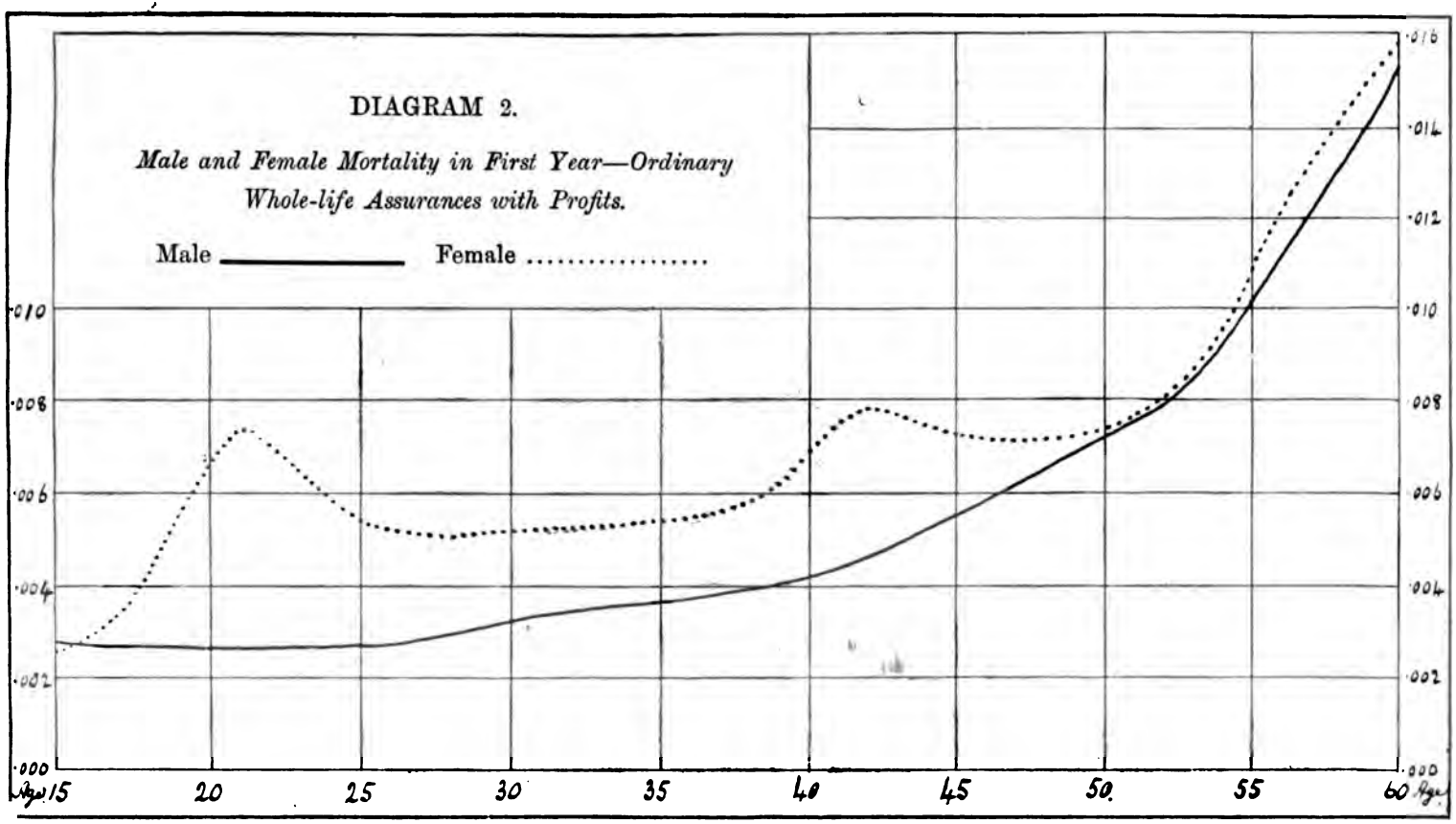
DIAGRAM 2.
Male and Female Mortality in First Year—Ordinary Whole-life Assurances with Profits.
Male
Female
·010
·008
·006
·004
·002
·000
·016
·014
·012
·010
·008
·006
·004
·002
·000
Age 15
20
25
30
35
40
45
50
55
60
Age

THE
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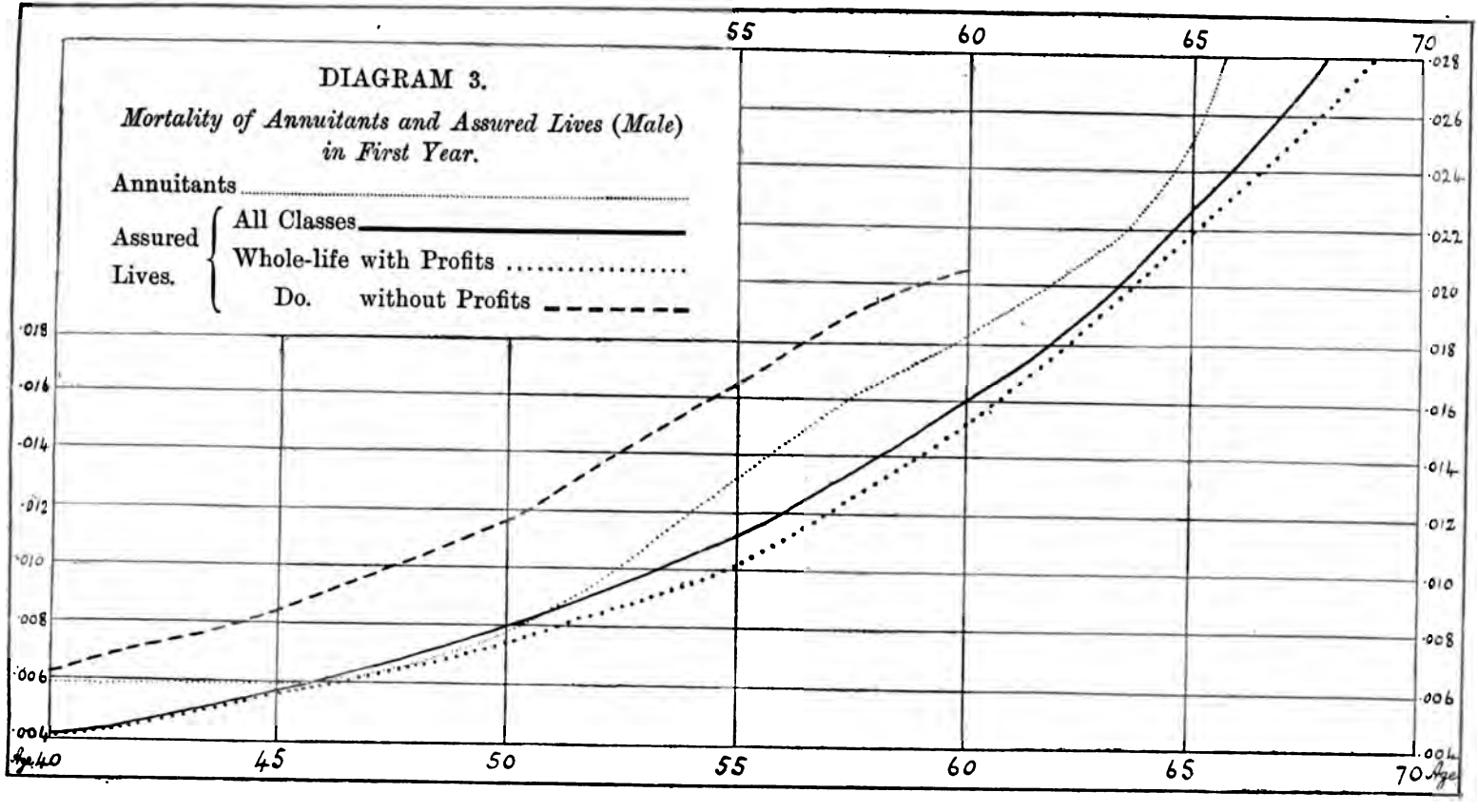
DIAGRAM 3.
Mortality of Annuitants and Assured Lives (Male) in First Year.
Annuitants
Assured Lives.
All Classes
Whole-life with Profits
Do. without Profits
·018
·016
·014
·012
·010
·008
·006
·004
Age 40
45
50
55
60
65
70
Age

## DISCUSSION.

THE PRESIDENT (Mr. G. M. LOW) said there were two observations made by Mr. Moir in the early part of his paper which he hoped they would remember, and which he hoped would bring forth fruit hereafter to the advantage of the Faculty and the actuarial profession. These were, first, that in the statistics of the New Experience there exists a vast mass of rough material from which a great deal of useful and interesting information may be got. And the other observation was that all minor investigations of the nature which Mr. Moir had undertaken in his paper, must be left to private enterprise. The investigation itself was projected on a large scale, and had been carried out with great labour and at great expense; and he thought they might take it that it was now approaching completion as far as the Committee felt that it was their duty to go. But they had furnished the material from which more extended investigations could be made on various lines, with great advantage to the profession; and he earnestly hoped that this would engage the attention of members of the Faculty, and that there might be some fruit in years to come. The truth was that, until this paper of Mr. Moir's, statistics of the first year's risk in British assurance offices had not been brought before the profession in a collective way. The approximations made by Dr. Sprague, Mr. King, and others from the $H^M$ Experience were necessarily mere approximations, from the nature of the principle upon which that table was constructed. By adopting policy years as the basis of the present investigation, the Committee had rendered it possible to trace accurately the mortality of the first and subsequent years; and he thought nothing in connection with this great investigation was more remarkable than the way in which the mortality even of the first year of assurance seemed to follow the laws which they would expect it to follow as regards the different classes of risk. That was most interestingly brought out in Mr. Moir's paper, and appeared to him a very striking fact indeed; for, while they had all expected that in the main these different classes of assurance would show different degrees of mortality following certain broad lines, they had no idea that, even in the first year of insurance, the difference would be so marked as Mr. Moir had shown it to be. He hoped they would have a good discussion, because he thought the paper lent itself to discussion to an even larger extent than might appear at first sight.

Mr. J. CHATHAM thought the Faculty singularly fortunate in having this paper submitted to them for discussion at this their first meeting held for the purpose. It was beginning at the beginning. The question submitted was one of the most interesting questions that they could have, and at the same time one of the most elementary. The discussion of the first year's risk was engaged in, as Mr. Moir had pointed out, about twenty years ago by some of the best intellects in the actuarial profession, and the results they arrived at were, as the President had said, merely approximate. He had himself approached the subject about ten years ago. Mr. Moir had been good enough to give the results in his paper, but he (Mr. Chatham) would like to point out that they were based not only on the *Gotha* Experience,

but also on the experience of the *Scottish Amicable*, with which they agreed very closely. The following first year's unadjusted probabilities would show how closely they corresponded :—

| Age. | Scottish Amicable. | Gotha. |
|---|---|---|
| 25 | ·0033 | ·0031 |
| 30 | ·0039 | ·0040 |
| 35 | ·0051 | ·0057 |
| 40 | ·0068 | ·0066 |
| 45 | ·0066 | ·0067 |
| 50 | ·0092 | ·0096 |

Mr. Moir had quoted the remark of Mr. King, that "after lives have been "insured some years, I think the rates of mortality among them will be "found to be nearly identical in nearly all sound offices, but not so the "mortality of the first year. That will depend upon the management of the "particular office, and will vary widely." He had also quoted that remark in his paper, and had added : "I think also I have shown that the rate of "mortality even in the first year will, in well-managed offices transacting "an ordinary business, be the same." He did not think that they would find the difference in the first year, but more in later years. Then, in the same table, Mr. Moir had given the results of the New Experience, and had said that the heavier rate of mortality shown by his (Mr. Chatham's) rates might be partly due to the differences between Great Britain and Germany, and partly to the fact that the *Gotha* experience runs further back to a time when medical selection was not so effective. Now, he showed in his paper that the rates of mortality in Great Britain and Germany were very similar, and the difference was certainly not due to that. Mr. Moir, however, was correct as regards medical selection. There was no question that medical selection had been better for many years than it was formerly. But there was another cause to which he thought the greater part of the difference was due, and that was the greatly improved sanitary conditions of the surroundings of the people. Take, for example, typhoid and typhus fever. He had taken out the death-rates in the population of Scotland due to these two diseases during the twenty-five years 1866 to 1890, that being within the period to which the New Experience relates, and had grouped them quinquennially to bring out more clearly the law which they follow. The deaths per 100,000 of the population were as follows :—

| Period. | Typhoid Fever. | Typhus Fever. |
|---|---|---|
| 1866-70 | 216 | 293 |
| 1871-75 | 209 | 116 |
| 1876-80 | 190 | 38 |
| 1881-85 | 135 | 22 |
| 1886-90 | 97 | 12 |

That was to say, the deaths from typhoid fever had been reduced by about one-half, and those from typhus fever to about one-twentieth of what they were. In these deaths were included the deaths of children, but he did not think that they would affect greatly the results. Then coming to Mr. Moir's most interesting table, namely the table of the first year's mortality in the different classes of policies, he thought that this was the great lesson of the New Experience, and very instructive it was. He did not think the New Experience would make much difference in reserves, in premiums even, in many cases ; but at any rate it had taught them this great lesson—the different mortality of the different classes. They found that the contingent class was the worst, which was almost to be expected. It was generally persons whose habits are not of the best description,

borrowing on the security of their reversions, whose lives were insured in that class. The endowment-assurance rate in the first year was generally less than half of the contingent rate. Then they found whole-life with-profit policies, and those under the ascending scale of premiums, a little higher than the endowment assurances; while term assurances, whole-life without-profits, and contingent assurances are all very high. Now coming to the limited-payment rates, they found that the rate of mortality given by Mr. Moir, as compared with endowment assurances, was :—

| | | | |
|---|---|---|---|
| For age 20, | ·0012 | against | ·0022 |
| „ 25, | ·0017 | „ | ·0022 |
| „ 45, | ·0034 | „ | ·0050 |
| „ 50, | ·0036 | „ | ·0063 |
| „ 55, | ·0039 | „ | ·0082 |
| „ 60, | ·0044 | „ | ·0112 |

Now he agreed to a great extent with what Mr. Moir had said as to the difference in the rates of mortality in different classes of assurance; but he could hardly concur with him that the relative magnitude of the rates of mortality was as indicated above. He did not believe that limited-payment policyholders have such a premonition of their longevity, that they can reduce their mortality in the first year by nearly one-half of that of the endowment-assurance policyholders. And why should policyholders at ages 30, 35, and 40, be denied the prevision which those at younger and older ages are supposed to have? Mr. Moir had said that the statistics are sufficient to yield quite reliable results. Well, he did not agree with him in the case of the limited-payment policyholders. If they turned to the table of the exposed to risk and deaths at the end of the paper, they would find that, with one exception, the largest number of deaths at any age was five; and further that, out of 41 years under observation, in 14 there were no deaths at all, and in 9 there was only 1 death; that is to say, in 23 years, or more than half of the whole, the number of deaths did not exceed one. It seemed to him, too, on examining the individual rates of mortality for the aggregate tables, that the mortality among endowment assurances was lighter than that among limited-payment policies; and he had grouped quinquennially the exposed to risk and the deaths, so as to remove irregularities, with the following results :—

| Ages. | Limited Payments. | Endowment Assurances. |
|---|---|---|
| 20–24 | ·0036 | ·0037 |
| 25–29 | ·0044 | ·0038 |
| 30–34 | ·0052 | ·0046 |
| 35–39 | ·0064 | ·0055 |
| 40–44 | ·0077 | ·0074 |
| 45–49 | ·0090 | ·0098 |
| 50–54 | ·0135 | ·0126 |
| 55–59 | ·0198 | ·0205 |

That is to say, in every case except three, the limited-payment policies showed a higher rate of mortality than endowment assurances. Now, that comparison was not strictly accurate, but still he thought it was sufficient to give a very good indication of what the rate was. But he had thought that the higher rate of mortality might be due to the fact that in the case of the select tables the number of lives of recent selection was larger in the case of one of the classes than the other; and he had therefore taken also the aggregate table, excluding the first ten years, as for all practical purposes selection was exhausted after that period. He had to start at

ages 30 to 34, because the numbers at younger ages were very small; and the following were the results :—

| Ages. | Limited Payments. | Endowment Assurances. |
|---|---|---|
| 30–34 | ·0088 | ·0056 |
| 35–39 | ·0081 | ·0063 |
| 40–44 | ·0093 | ·0089 |
| 45–49 | ·0102 | ·0113 |
| 50–54 | ·0139 | ·0134 |
| 55–59 | ·0208 | ·0207 |

Here again, in every case except one, the limited-payment rate was higher than the endowment-assurance rate; and he thought that the results he had obtained proved the general law which he enunciated some time ago, that the higher the rate of premium, the lower the rate of mortality. Mr. Moir had referred to financial policies, and he found himself influenced by what was said. He really thought some steps should be taken; and he had sometimes thought it would be desirable in their case to revert to the old practice of insurance companies, of making the man appear before the officers of the company. As regards the table of mortality amongst lives assured for different durations, as experienced while passing through different ages, Mr. Moir said that the method used by him proved that there is no maximum mortality period between the 11th and 15th years, as was formerly contended by Dr. Sprague; but he (Mr. Chatham) did not find anything in the table to support this, because the table gives the 10th and 15th years with nothing in between them. Then as to the method of graduating each year's experience by itself, he thought the first one to do that was Dr. Sprague in his paper "On the construction and use of a series of select mortality tables", where he gave in a diagram the first and the second year's probabilities. It is quite true that he did not form them by themselves, but took the ratios of the second to the first, and of the third to the second; but still they might practically hold that he was the first to put it down. He (Mr. Chatham) had used it very frequently. In his own paper he graduated the upward curve for each central age at entry, and then joined the first year's probabilities with cross lines, the second year's in the same way, and so on, adjusting where necessary. He thought that was the best method that could be adopted—better than graduating one set of probabilities separately. Then as to the high rate of mortality at the early ages, which Mr. Moir had given in the tables at the end of the paper—for instance, taking all classes together, the rate at age 15 was ·00285; at age 20, ·00273; and at age 25, ·00253; and there were similar differences in the whole-life with-profits class and also in other classes. Now he did not think this was really so; but the numbers exposed to risk and the deaths were so small that they could not base any conclusion upon them. It was certainly opposed to anything he had ever had in his own experience. Mr. Moir had also referred to the decrease in the short-term rate between 20 to 30, and there could be no question about that. Some little time ago he had calculated short-term rates by the New Experience just as it stood, and graduated monetary results. He had used the formula $1.1\ (\pi + \cdot 0025)$, taking $3\frac{1}{2}$ per cent interest. For one year the rate at age 20 was 18s. 3d., at age 25 it was 16s. 1d., and at age 30 it was 14s. 9d.; and for terms of three, five, and ten years the results were similar. Instead of these rates going down, he proposed to charge for ages 25 to 35, 16s. 6d. for one year. He was afraid the rate for age 20 would require to be specially considered. He thought that the duties of a referee required that the paper should be examined with a critical eye, and he had mentioned several points in which he differed from Mr. Moir; but he wished it to be understood that he was in no way

detracting from the merits of the paper. He would have liked to mention the large number of cases in which he agreed with him, but time would not permit. He thought the paper was a very valuable one indeed, and in writing it Mr. Moir had done it with credit to himself and benefit to the profession at large.

Mr. J. M. WARDEN thought Mr. Moir was to be congratulated on having the honour to follow immediately the Presidential Address to which they had the pleasure of listening last month, and to submit to the Faculty the first paper read at a sessional meeting, and the first prepared from a study of the almost inexhaustible mine of wealth for the actuarial explorer contained in the volumes of the New Experience. The Faculty might also congratulate themselves on the fact that the first ordinary paper should come from across the Atlantic, while regretting that its author could not be present to read the paper himself. One of the most important uses to be made of the results brought out in a paper like Mr. Moir's, was in the fixing of short-term rates. For a number of years after the publication of Dr. Sprague's Select Tables, there was a tendency to reduce these rates; but it should not be forgotten that his figures were based upon an experience ceasing in 1863 and largely composed of whole-life policies. It was now much better understood that the mortality experienced under policies subject to low premiums is high; and it was not surprising to find that in the New Experience the first year's risk was a good deal higher under term policies than under ordinary policies. Notwithstanding this, the term rates, as brought out by Mr. Chatham, did not differ much from the rates to which they had been accustomed—probably in consequence of the general improvement in vitality since the collection of the former experience. He did not suppose Mr. Chatham would be bold enough to publish higher term rates for age 20 than for 25. A considerable proportion of term insurances were for one year only, and it was disappointing to find that the facts were so few; for only 70 deaths occurred in the first year of insurance for entrants between 20 and 60 years of age. He doubted whether the scanty facts could be considered a satisfactory basis for short-term premiums; and thought it would be better to calculate these from, say, the whole-life experience, looking to the short-term facts for such guidance as they appear to give. Mr. Moir had compared, in the way familiar to all of them, the expected deaths according to his graduation with the actual deaths among the entrants at each age. The deviation was seldom great; and the frequent changes from positive to negative showed the goodness of the graduation as a whole. The author had added a column containing what was called the "accumulated difference"; but it seemed hardly suitable to use this term, as the differences that were added together did not relate to the same lives, as in the case of an ordinary aggregate table carried on from age to age, or of a select table tracing the entrants at one age through the various years of insurance. In one respect he thought Mr. Moir had refined too much. In what were called "financial policies", that is, whole-life without-profits and term policies, there was a decided dip in the mortality curve between ages 20 and 35; but in certain other classes the author had shown that feature also, to a much less extent it was true, but still where it appeared to be unnecessary. This was specially the case with regard to endowment assurances, where a slight dip was shown between ages 22 and 29; but if they turned to page 131 of the volume containing the endowment-assurance experience, they would find a table showing the rates of mortality for ages at entry taken five together; and from that table the first year's mortality apparently increased throughout with the age at entry. Further, the first year's deaths among endowment assurances effected at the youngest ages were very few in number. The same was the case at all

ages under limited-payment policies, and he could not but think that the striking and somewhat unexpected results brought out in the paper with regard to that class, were an accident or fluctuation, due to the paucity of the facts. There were only 75 deaths in all, which seemed too few upon which to base trustworthy conclusions. The method adopted by Mr. Moir of making the mortality for each year of insurance run smoothly by itself, undoubtedly carried the "select" principle and the policy-year system to their logical conclusions; and he thought the author had been wise in going further than the first year before publishing the results of that year; as the mortality for the subsequent years of insurance would afford a very efficient check on the correctness of the inferences made as to the first year's risk. Indeed, it seemed just as useless, in regard to select tables, to try to fix the mortality for one particular year of insurance without reference to that of the adjacent policy years, as it would be in an aggregate table to fix the rate for one age without considering the facts for the adjacent ages. Like the previous speaker, he had long been familiar with the plan of graduating the curves in a select table both upwards and across, and considered that to be the most effective method, if one was prepared for the additional labour involved. No one who had had to look into the results of "financial" policies, especially when effected by young men borrowing on their expectations, could doubt that the greatest caution is required in dealing with their proposals; and he thought they would all find very useful the facts Mr. Moir had brought out, and the judicious remarks he had made on this point. With regard to the author's idea in dealing with calendar years *versus* policy years, that perhaps the younger men were on the average insured for about six months in the calendar or office year of entry, and the older men for a less period, he had found nothing to confirm this in his own office. He had looked into the whole-life policies effected during a series of years; and, taking separately the business effected in the first and second halves of the year, he had found that the average rate of premium was the same in both. A close study of the paper would reveal many other interesting points; and he felt sure they were all agreed that it was a valuable contribution to their knowledge of the subject.

Mr. A. H. Turnbull remarked that he had not had the advantage of seeing the paper before the meeting, but it struck him when it was being read, that the reason assigned by Mr. Moir for the mortality shown by the new tables being less at ages under 45 and more at ages above, than observed by either Dr. Sprague or Mr. King, was hardly sufficient to account for the difference. Mr. Warden had pointed out that there was little in it; but another reason had occurred to him, from his having read or seen recently, possibly in Dr. Smee's diagrams of mortality in this country, that the rate had decreased very materially up to about 45, and after that it had increased. There was no doubt medical men were able to keep unhealthy people living for a longer time than formerly; but these same people have to die afterwards, and they go off rather quicker than stronger men would have done. His authority put down the increase after 45 to business worries and all that. He (Mr. Turnbull) believed it was the delicate men who years ago would never have lived to be 45, who brought the rate down now among the general population; and its existence even in the first year of insurance could, he thought, be thus explained. He was rather sorry to make any such criticism upon the paper, and to have heard so many other criticisms. He hoped some other member would direct attention to much that was praiseworthy, rather than to what could be criticised. One other matter was not very clear to him, namely, the position occupied by limited-payment policies in Diagram 1. He was very much struck with the line in the diagram showing the rate in the first year to be less than in the

endowment assurances. He certainly looked for it to be less than whole-life assurances, but not less than endowment assurances. On the other hand, all the other classes, except perhaps the ascending scale, are just in the order in which he would have put them. The paper was a very interesting one, and he was pleased to see so many members present to hear it.

Mr. D. Deuchar had very little to say, not having had an opportunity of perusing the paper beforehand; but it occurred to him that perhaps some of the members present might have put to themselves the same question that he had put to himself—do the limited-payment policies include single-payment cases? Or, do the premiums in that class, on the average, cease within a term much shorter than the average term of an endowment assurance? There might be something in that. Then he thought there was a good deal in Mr. Moir's remark about the effect of antedating the risk to secure a bonus or save a birthday; and he thought one would like to study that part of his paper. He did not know whether the author had got any facts in regard to that; but he thought it would be useful for the individual members of the Faculty to look into the facts in their own offices. He imagined the antedating had materially reduced the apparent mortality of the first year.

Mr. A. E. Sprague suggested that the low mortality in the aggregate endowment-assurance table might be due to the fact that endowment assurances had largely increased of late years, so that the bulk of the lives in that portion of the experience had been insured for a comparatively few years only. The aggregate limited-payment experience probably consisted of lives who had on the average been longer insured than those embraced in the endowment-assurance experience; and it would therefore, other things being equal, show a heavier mortality. It was therefore quite conceivable that the mortality in the limited-payment class might be lighter than that in the endowment-assurance class, notwithstanding the fact that a comparison of the aggregate tables appeared to show the reverse to be the case. It was true that Mr. Chatham had made a comparison leaving out the first ten years of assurance; and that this, as well as the comparison of the aggregate tables, showed that the endowment assurances came out better; but the effect of selection lasted for more than ten years, and this comparison would therefore be influenced in the same manner (though in much smaller degree) as the comparison of the aggregate tables. Mr. Moir had mentioned the question of a maximum rate of mortality after fifteen years or so. He quite concurred with Mr. Moir (so far as regards the whole-life with-profits class) that there is no appearance of the rate of mortality going up to a maximum and then diminishing. It seemed to him that the rate tended to run up steadily to the extreme limit of life. He had referred to the paper by Dr. Sprague, and pointed out that the maximum therein mentioned was brought out in a comparison of the select table with an aggregate table. Dr. Sprague did not say that the absolute rate of mortality reached a maximum and then diminished. It was only in the comparative rates of mortality that the maximum occurred; and the phenomenon might possibly be due to some peculiarity of the aggregate table—for example, the well-known fact that an aggregate table overstates the mortality among the older lives.

Mr. D. Paulin had satisfaction in pointing out that both the writer of this paper and the reader of it had received their actuarial education and training as members of the staff of his own office. He thought they were greatly indebted to Mr. Moir for his suggestive and able paper. He agreed with Mr. Chatham that something more should be done to discriminate the greater risk on the lives of aristocratic and other West-End borrowers, who

had "outrun the constable" and had made the acquaintance of obliging Jews. He feared they could not be persuaded to come to Edinburgh to be personally inspected by the Scottish boards; but he thought they might introduce into the proposal-forms a new question which might serve a useful purpose, and which might be something of the following nature—"Do you "prefer a short life and a merry one, to a long life on the usual humdrum "conditions?" Mr. Moir had emphasised very strongly, and evidenced by his tables and statistics, the effect of selection against the office; and also the value of the effect of selection on the part of the office. He thought they were very much indebted to him for the clear and admirable way in which he had stated these points, which were of great practical value in the conduct of business. During the delivery of the paper, he had been watching Dr. Sprague's physiognomy in marble just above him; and had found that he appeared to be greatly interested in the whole discussion, and not a bit offended when Mr. Moir disagreed with some of his well-considered conclusions, though he gave no indication that he thought Mr. Moir had come to better or wiser conclusions than he had come to and had expounded in former years.

Mr. J. J. McLauchlan had listened to the paper and to the discussion with great pleasure and interest. He thought they were very fortunate in having, as the subject of their first discussion, a paper of this character, dealing with a definite and manageable section of the results of the important investigation that had just been made into the experience of the life offices. It was a paper that even those who, like himself, were not now daily engaged in practical actuarial work, were able to follow and interest themselves in; and he hoped that the Faculty would have many other papers of the same character. He would not have expected to find that the special features pointed out in the mortality of the various classes were so strongly marked in the first year of assurance. It was to be expected that the recent medical selection would minimise the effect of the causes tending to increase the rate of mortality in certain classes. He would not have thought that female life, at the childbearing ages, was in the first year so much worse than male life. At one time it was, he believed, a common practice to postpone proposals on the lives of pregnant women until the pregnancy was over. In regard to financial cases, one would have thought the lives were good enough for a year at any rate; and if the extra mortality was so strongly marked in the first year, it was clear that in future years it would be more strongly marked still. He thought that these comparisons were fitted to impress in a remarkable way on men engaged in the business of life insurance, the practical importance of the question of the extra risk incurred in insuring certain classes of lives. He had no doubt that, if it had been practicable to include in the recent investigation into the experience of the offices, some inquiry into the effect of occupation upon life insurance risks, they would have had a number of similar and interesting differences to consider.

Mr. W. A. Sim said that no reference had been made to the agency system, and he thought it had a bearing on the class of policy taken—perhaps more so now than formerly. Some years ago an eminent American actuary expressed the opinion that one reason why the rate of mortality in America was more favourable than in this country, was probably because of the greater proportion of voluntary applications from the British public, who felt they needed insurance without urging from agents; while another reason stated as having something to do with it, was the greater favour shown in America for endowment-assurance and other high-priced policies. Mr. Moir, in speaking of a possible maximum rate of mortality, had referred approvingly to what had been put in writing by Mr. T. B. Macaulay in 1893,

to the effect that the period of maximum mortality percentage, which had sometimes been observed, might perhaps be due, in part at least, to the nature of the tables used as standards, rather than to any peculiarity in the experience which was being analysed. About six years ago the *Canada* Life Assurance Company published a volume containing its mortality experience for a period of something like forty-seven years up to 1893. The experience was made up on the policy-year method; and on page 25 of that volume there appeared this—"Part 2 of Table XX shows the rates" [*i.e.* the annual rates of mortality per cent] "by quinquennial years of assurance and quin-" quennial groups of ages (next birthday) at exposure. This table confirms " the investigations of Messrs. Sprague, King, and others, viz. :—that shortly " after entry the lives, on the average, seriously deteriorate, but afterwards " show a marked improvement. Thus, examining the rates in the above-" named table, it will be seen that, while there is a sudden rise in the rates " for the second five years, an improvement is usually shown either in the " third or fourth quinquennium. The most natural explanation of this is " that the large number of healthy lives withdrawing in the early years, " brings about a deterioration in the body of remaining lives, thus causing " the higher resulting rate in the second five years, but after the effect of " this has worn off, an improvement takes place." He merely mentioned that because, although the New Experience figures, as given by Mr. Moir, showed no such maximum rate of mortality, the author of that publication by the *Canada* Life Assurance Company appeared to be satisfied that, according to its experience, there was a maximum rate of mortality of the nature in question.

The President said he was sure that, if Mr. Moir had been present, he would have been gratified, not only with the attendance, but also with the profitable discussion which had taken place on his paper. It was to be regretted that he was not present, because some points had been stated upon which perhaps it would have been desirable to hear him in reply. It might be arranged that they should have the benefit of Mr. Moir's observations when the discussion appeared in print. It was interesting to hear Mr. Chatham's remarks about the short-period rates; because it reminded one of the old discussion which had taken place in the time of the late Mr. Spens of the *Scottish Amicable*, when that gentleman propounded what was regarded as an actuarial heresy, as to the relative values for assurance purposes of lives entering at different ages. It would be curious indeed, if, on mature investigation, it should turn out that Mr. Spens had a great deal in favour of his view. He moved that a hearty vote of thanks be given to Mr. Moir for his paper; and their thanks were also due to Mr. H. W. Brown for reading it.

## MR. MOIR'S REPLY.

Mr. Moir writes that he is gratified to observe that his contribution to the Sessional Meetings of the Faculty called forth a full discussion of a subject which he had for many years considered an interesting one. Several points, treated by speakers critically, were already answered in the essay. For example, all fluctuations in the mortality curves to which objection is taken, are covered by the sentence: "For the purpose in view, smoothness " of graduation is of less consequence than close adherence to the original " facts" (see page 20). The tests of graduation speak for themselves in this matter, whether the numbers exposed be great or small. Mr. Moir still agrees with Mr. King that the mortality of the first year may differ considerably in "sound offices", owing to temporary causes such as changes in management or in medical administration. In later years the average

may be more nearly maintained. Much was said about the relative mortality in the limited-payment and endowment-assurance classes during the first year; and this may excuse somewhat extended remarks on this subject, as the figures surprised Mr. Moir as they seem to have surprised others. It is said that the statistics relating to limited-payment policies are not sufficient to yield reliable results, the small number of deaths being specially referred to. The measure of reliability, as every one knows, is not the number of deaths but the exposed to risk. From ages 23 to 36 the numbers exposed exceeded 1000 in each year, reaching a maximum of 1744 at age 30, the total exposures between 20 and 60 being over 31,000; and facts on such a basis cannot lightly be cast aside in order to set up hypotheses based on what one might expect: the facts should rather be tested from others of a similar nature. By examining the mortality during the second and third years, the numbers exposed may be nearly tripled, and the deaths more than quadrupled. The graduated results at quinquennial ages are:—

| Age. $x$. | 2ND YEAR'S RISK— $q_{[x-1]+1}$ | | Age. | 2ND AND 3RD YEARS COMBINED. | |
|---|---|---|---|---|---|
| | Endowment-assurances. | Limited-payments. | | Endowment-assurances. | Limited-payments. |
| 20 | ·0046 | ·0032 | 20 | ·0050 | ·0040 |
| 25 | ·0037 | ·0033 | 25 | ·0038 | ·0038 |
| 30 | ·0032 | ·0038 | 30 | ·0035 | ·0036 |
| 35 | ·0040 | ·0042 | 35 | ·0040 | ·0038 |
| 40 | ·0049 | ·0047 | 40 | ·0049 | ·0044 |
| 45 | ·0057 | ·0049 | 45 | ·0065 | ·0061 |
| 50 | ·0068 | ·0054 | 50 | ·0096 | ·0088 |
| 55 | ·0089 | ·0063 | 55 | ·0132 | ·0118 |
| 60 | ·0122 | ·0080 | 60 | ·0160 | ·0150 |

From the above rates and those formerly given, it is clear that during the first three policy years the advantage lies with the limited-payment class; while Mr. Chatham's evidence regarding later years of assurance indicates the opposite. Are these conclusions at variance, or is there some active force affecting the results? Under limited-payment policies the withdrawals between 20 and 44 are about 13 per cent of the entrants, while under endowment-assurances they are 22 per cent. At the ages when entrants were most numerous, the withdrawals under the former class were about 4 per cent of the entrants during the first year, and about 8 per cent in three years: under endowment-assurances the corresponding figures are 10 per cent and 15 per cent. If therefore the withdrawals exercise a *beneficial* effect on the mortality, the two conclusions are not at variance. They rather afford further proof that discontinuance reduces the number of persons of careless temperament who experience high rates of mortality. Mr. Chatham says emphatically, as the result of his investigation, that the heavier mortality of the *Gotha* experience "was certainly not due to" differences between Great Britain and Germany. This definite statement is of special interest, as it was called forth by an indefinite remark that the variation "might be partly due to" such differences. Improved sanitary conditions probably constitute another reason for the variation; but, in addition to the fact that children, women, delicate men, and old men probably form a considerable proportion of the deaths from

the fevers named, it should be borne in mind:—(1) that the typhus and typhoid statistics relate to Scotland only; (2) that the majority of those who die from these fevers, especially typhus, are taken from the poorest classes, who would be insured in industrial offices only; and (3) that presumably the deaths should be divided by 5 in order to get an annual rate. Unless these points are kept in view, the figures are apt to assume an exaggerated importance. Mr. Warden's remarks go far towards demolishing the attempt to explain the systematic variation between Dr. Sprague's and the new rates of mortality for the first year at young and old ages, unless in the statistics he investigated there was no inducement for those effecting policies in January or February to "date back" to 31 December. The point made by Mr. Turnbull had occurred to Mr. Moir, but was not expressed, because Selection would exclude most of such delicate men: this may, however, be the true explanation of the differences, which have all the appearance of being caused in some such manner.

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# CORRESPONDENCE.

## POLICY-VALUES.

*To the Editor of the Transactions of the Faculty of Actuaries.*

SIR,—The following simple demonstration showing the relation between the values of a policy subject to a yearly premium and one subject to premiums payable $m$ times a year, or, in symbols, that ${}_nV_x^{(m)} = {}_nV_x\left(1 + \frac{m-1}{2m}\pi_x\right)$, may interest some of your readers.

We have

$$
\begin{aligned}
{}_nV_x^{(m)} - {}_nV_x &= A_{x+n} - \pi_x^{(m)}a_{x+n}^{(m)} - A_{x+n} + \pi_x a_{x+n} \\
&= \pi_x a_{x+n} - \pi_x^{(m)} a_{x+n}^{(m)} \\
&= \frac{A_x}{a_x} \cdot a_{x+n} - \frac{A_x}{a_x^{(m)}} \cdot a_{x+n}^{(m)} \\
&= A_x \left\{ \frac{a_{x+n} a_x^{(m)} - a_x a_{x+n}^{m}}{a_x a_x^{(m)}} \right\} \\
&= \frac{A_x}{a_x^{(m)}} \left\{ \frac{a_{x+n}\left(a_x - \frac{m-1}{2m}\right) - a_x\left(a_{x+n} - \frac{m-1}{2m}\right)}{a_x} \right\} \\
&= \pi_x^{(m)} \cdot \frac{m-1}{2m} \cdot \frac{a_x - a_{x+n}}{a_x} \\
&= \frac{m-1}{2m} \cdot \pi_x^{(m)} \cdot {}_nV_x
\end{aligned}
$$

$$
\therefore\ {}_nV_x^{(m)} = {}_nV_x\left(1 + \frac{m-1}{2m}\pi_x^{(m)}\right)
$$

It will be observed that the annuities-due, payable yearly and fractionally have been used throughout in the above demonstration, and that the ordinary correction $\frac{m-1}{2m}$ in the well-known approximation $a_x^{(m)} = a_x + \frac{m-1}{2m}$, is retained in the form $a_x^{(m)} = a_x - \frac{m-1}{2m}$; with the result that the proof appears to be easier to follow than that given in the *Institute of Actuaries' Text-Book*, Part II, page 345.

The annuity-due might with advantage be oftener used in such demonstrations; and, as it is so closely connected with the calculation of premiums, it is, I think, a cause for regret that, in the monetary tables based upon the mortality experiences, the values of the annuity-due and the corresponding logarithms are not tabulated, in preference to the values of the annuity payable at the end of the year. If this were done, the risk of error would be reduced, and time saved, in calculations where at present it is necessary to interpolate to obtain the annuity-due, either whole-life or temporary, for a fractional age.

Yours faithfully,

M. M. LEES.

EDINBURGH, 14 *March* 1902.

No. 3 Vol. I

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OF THE

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[Read before the Faculty, 6 January 1902.]

A MORTALITY TABLE shows the number living at each age $x$, and the number dying in the following year; and the ratio of the latter to the former, is the probability of dying in a year, and is called the rate of mortality at age $x$. This function varies from age to age by finite steps; but it is not to be supposed that the actual intensity of mortality remains constant throughout each year and changes suddenly at the end of each completed year of life. On the contrary, it is a function which varies by an indefinitely small amount with each indefinitely small interval of time; and the measure of its value at any instant, like that of any other varying rate, is the change that would be produced in a unit of time if the rate were to remain constant throughout that unit, and equal to its value at the instant in question. For example, when we speak of a railway train travelling at the rate of 60 miles an hour, we do not mean that it has travelled 60 miles in the past hour, or that it will travel 60 miles in the next; but that, if its rate were to remain the same for an hour, it would travel 60 miles in that time, the unit of time in this case being one hour. Similarly, the intensity of mortality at any instant among a large number of persons, is not measured by the number who actually die in the following year (the unit of time

in the case of a mortality table being one year), but by the number who would die in that year if all the conditions remained unchanged throughout the year; that is to say, it is measured by the number who would die if they did not deteriorate in any way with their increasing age or otherwise, and if the number of persons under observation remained unaltered, the places of those who died being constantly filled by fresh lives subject to the same intensity of mortality. The number who would die under these conditions must be divided by the number under observation, in order to get the measure of the intensity of mortality per single life; and the quotient is called the instantaneous rate of mortality, or more commonly the force of mortality, and is denoted by $\mu_x$. During the last year of a mortality table the force of mortality is so great that all the lives entering upon it die in less than a year. Hence, if this force could be applied for a whole year to a body of lives whose numbers were kept constant by the influx of new lives to replace those who died, the total number dying would be greater than the number under observation at any time. That is to say, the force of mortality must, at some time during the last year of the table, be greater than unity. At the extreme limit of the table its value is infinite; and, in general, it may have any value from zero to infinity, so that it differs essentially in its nature from the probability of dying in a year, which can never exceed unity. To ascertain its value approximately, we must find the number of persons dying in a short interval of time, $t$ say, and divide this by the number under observation, the quotient being the probability of dying in the time $t$. Dividing this by $t$, we get the probability of dying in a year on the assumption that the intensity of mortality will throughout the year remain equal to its average value during the short interval $t$. In symbols, $\mu_x$ is approximately equal to $\frac{l_x - l_{x+t}}{tl_x}$ where $t$ is small. The shorter the interval $t$, the closer will be the approximation to the true value of $\mu_x$; and finally passing to the limit when $t$ is indefinitely small, we get an indefinitely close approximation, that is to say, an exact value. In symbols, $\mu_x = \underset{t=0}{L} \frac{l_x - l_{x+t}}{tl_x}$.

Now, in the notation of the differential calculus we put $dx$ for $t$ when it becomes indefinitely small, and $dl_x$ for $l_{x+t} - l_x$, so that in this notation we have $\mu_x = -\frac{l}{l_x}\frac{dl_x}{dx}$; that is to say, the force of mortality is equal to the differential coefficient of $l_x$ with respect to $x$, with its sign changed, and divided by $l_x$. From the foregoing it appears that $\mu_x dx$ is the probability of a single life aged $x$ dying in the indefinitely short interval $dx$; and that $l_x \mu_x dx$ is the number dying in that interval.

The principal uses to which the function $\mu_x$ is put, are in connection with what are known as Continuous Benefits. The usual form of benefit is the sum of a finite number of expressions, each of which consists of a certain sum receivable at the end of a certain year, $t$ say, multiplied by the probability of its being received at the end of that year, and discounted for $t$ years. Thus, for example, the present value of an ordinary assurance, namely of an assurance of 1, receivable at the end of the year of death, is

$$=v\frac{d_x}{l_x}+v^2\frac{d_{x+1}}{l_x}+v^3\frac{d_{x+2}}{l_x}+\ \ldots\ +v^t\frac{d_{x+t-1}}{l_x}+\ \ldots\ =\sum v^t\frac{d_{x+t-1}}{l_x};$$

where $\frac{d_{x+t-1}}{l_x}$ denotes the probability of dying in the year $t$, and $\Sigma$ denotes the sum of the expression following it for all integral values of $t$ from unity upwards. On the other hand, an assurance of 1, receivable at the moment of death, is a Continuous Benefit. It is the sum of an infinite number of indefinitely small terms, since there are an infinite number of moments at which the benefit may be received, and the probability of its being received at any one of these moments is indefinitely small. The probability that the benefit will be received at time $t$, is equal to the probability that the life will survive till that time, namely $\frac{l_{x+t}}{l_x}$, multiplied by the probability of dying immediately thereafter, namely $\mu_{x+t}dt$; and, the present value of 1 receivable $t$ years hence (where $t$ may be integral or fractional) being $v^t$, it follows that the present value of a Continuous Assurance is the sum of the expression $v^t\frac{l_{x+t}\mu_{x+t}dt}{l_x}$ for all values of $t$, integral or fractional, between zero and the end of the mortality table; or, what comes to the same thing, between zero and infinity. This sum is denoted by $\int_0^\infty v^t\frac{l_{x+t}\mu_{x+t}dt}{l_x}$; and this expression is, it will be seen, strictly analogous to the expression for an ordinary assurance.

The exact value of this function cannot be found unless the mathematical form of $l_x$ (and therefore of $\mu_x$) be known; but it can be found with sufficient accuracy for practical purposes by various formulas of approximate summation, if we have a table of $\mu_x$. Several of these formulas are demonstrated in the *Text-Book* (part 2) and *Journal* of the Institute of Actuaries; but a few remarks on them may not be out of place here.

If we wish to find the sum of a number of terms which proceed with regularity, it will obviously give a rough approximation if we select, say, every tenth term, add these together, and multiply the total by ten. By proceeding in this way we give equal

weight to each selected term; and a closer approximation will be got if, instead of doing this, we multiply each of the selected terms by a coefficient corresponding to its importance, thus giving greater weight to the more important terms. The calculation of these coefficients requires considerable knowledge of higher mathematics; but, if their values are given, any one with slight mathematical knowledge can apply them, and thus find, with comparatively little labour and quite sufficient accuracy, the values of complicated benefits, the calculation of which by any other process would involve a prohibitive amount of labour. Take, as an example, Hardy's formula given in the *Text-Book* (part 2, page 488),

$$\int_0^\infty u_t dt = n\{\cdot 28u_0 + 1\cdot 62u_n + 2\cdot 2u_{3n} + 1\cdot 62u_{5n} + \cdot 56u_{6n} + 1\cdot 62u_{7n}\}.$$

Here all we have to do is to calculate six values of $u$ corresponding to the integral values of the variable 0, $n$, $3n$, $5n$, $6n$, and $7n$, multiply each of these by the corresponding coefficient, add the resulting products together, and finally multiply the total by $n$. This formula is a particular case of the more general formula given in the same place:—

$$\begin{aligned}\int_0^\infty u_t dt = n\{&\cdot 28u_0 + \cdot 56(u_{6n} + u_{12n} + u_{18n} + \ldots) \\ &+ 1\cdot 62(u_n + u_{5n} + u_{7n} + u_{11n} + \ldots) \\ &+ 2\cdot 2(u_{3n} + u_{9n} + u_{15n} + \ldots)\}\end{aligned}$$

when $u_{9n}$ and all higher terms vanish; and, therefore, when applying it, $n$ must be so chosen that $u_{9n} = 0$, or, in other words, falls beyond the limits of the tables employed in the calculation. It must be carefully borne in mind that the determination of $n$ does not necessarily depend on the age of the oldest life involved in the expression $u$. For example, one formula for a continuous reversionary annuity to a life aged 20 after the death of a life aged 80, is

$$\bar{a}_{80|20} = \int_0^\infty v^t \frac{l_{80} - l_{80+t}}{l_{80}} \cdot \frac{l_{20+t}}{l_{20}}\, dt$$

where $\frac{l_{80} - l_{80+t}}{l_{80}} \cdot \frac{l_{20+t}}{l_{20}}$ is the probability of the annuity being payable at time $t$, and $dt$ is the sum receivable at that time in respect of the indefinitely small interval of time $dt$. In this case $n$ must not be determined with reference to the older life. Thus, if the *Text-Book* table, whose limit is age 102, be employed, it would not be correct to take $n = 3$, because this would not make $u_{27}$ vanish, although the life aged 80 would in 27 years have passed beyond the limit of the table. The annuity will be payable so long as the life now aged 20 survives, and therefore $n$ must

be determined with reference to that life. We may take $n=10$; because in 90 years the life now aged 20 will certainly be dead, and therefore $u_{90}=0$. At 3 per cent interest the true value of the reversionary annuity is 18·438. By Hardy's formula, taking $n=10$, the approximate value is 18·969, which is fairly close for such an extreme case; but, if $n$ be taken $=3$, the approximation is only 11·743, which is hopelessly wide of the truth. This very large error shows the necessity of choosing $n$ properly.

It should be noted that the determination of $n$ depends not only on the form of the function to be evaluated, but also on the particular formula employed. Thus the above-mentioned reversionary annuity may be found by the following formula :—

$$\bar{a}_{80|20}=\int_0^\infty v^t \frac{l_{80+t}\mu_{80+t}}{l_{80}} \cdot \frac{l_{20+t}}{l_{20}} \cdot \bar{a}_{20+t}\,dt,$$

and in this case $u_n$ will vanish if $n$ be large enough to make the older life pass beyond the limit of the table; so that, if we use the latter formula, $n$ may be taken $=3$.

Again, it may happen that $n$ must be determined with reference to the limit, not of the mortality table, but of some other table entering into the calculation. For example, in finding the values of benefits depending on the birth of issue, the limit of the table of probabilities of issue will generally be much lower than the limit of the mortality table, and $n$ must be determined with reference to this lower limit. The rule for determining $n$ may therefore be stated as follows :—Let $\omega$ be the limiting age of the table (namely, the age which none of the lives attain, or, in an issue table, the age at which the probability of issue vanishes), and $x$ the age of the life with reference to which $n$ must be determined; then $n$ must be taken $=\frac{\omega-x}{9}$ if this be an integer, and, if it be a fraction, $n$ must be taken as the next higher integer. In certain cases this will have the effect of making $u_{7n}$ also pass beyond the limit of the table, so that it need not be calculated, and the work is thereby shortened. When using Woolhouse's "nugget" formula

$$\int_0^\infty u_t dt=n\{\cdot 28092u_0+1\cdot 619u_n+2\cdot 1849u_{3n}+1\cdot 8304u_{5n}\}$$

(*J. I. A.*, xxvii, 154), it must be remembered that that formula is based on the assumption that $u_{7n}$ and higher terms vanish, and therefore the divisor in this case must be 7 instead of 9. These rules give the minimum value of $n$ which can be employed in the calculation; but there is no objection to using larger values, provided only that $n$ be not taken large enough to make too many of the terms vanish, as in that event the result will depend on too few terms to be trustworthy.

When applying a formula of approximate summation, the work can frequently be shortened by the omission of the first term. This will be the case when $u$ contains a factor of the form $l_x - l_{x+t}$, as this factor, and therefore also $u_0$, vanishes when $t=0$. Again, when the function to be found is an immediate annuity, whether on single or joint lives or the last survivor of two or more lives, the first term is of the form

$$v^t \frac{l_{x+t}\, l_{y+t}\, l_{z+t} \cdots}{l_x\, l_y\, l_z \cdots},$$

which is equal to unity when $t=0$. It is therefore unnecessary to go through the various steps to calculate $u_0$ and multiply it by its coefficient—all that need be done is to add the coefficient to the total of the other terms, after they have been multiplied by their respective coefficients. It is therefore advisable, before commencing the calculation, to see whether it is necessary to calculate the term $u_0$ or not.

One of the most important applications of such formulas of approximate summation, is the calculation of premiums for unusually complicated benefits; but as there are no published tables of $\mu$ for the Select Tables, they cannot, in general, be employed.

It is generally admitted that Select Tables should be employed in the calculation of office premiums, because an aggregate mortality table makes the net premiums too low at the younger ages and too high at advanced ages. In the case of a survivorship assurance on a young life against an old one, the effect of using an aggregate table is therefore to introduce two errors in the same direction, and thus to underestimate the risk to an appreciable extent; and in one special case which came under my notice, the error was so marked as to lead me to attempt the construction of tables of $\mu$, by means of which the true values of such benefits could be readily approximated to. The present paper is the result.

The chief difficulty in finding the force of mortality in a select life table is to ascertain its value at the commencement of the insurance. This is evident from the following standard formulas demonstrated in the *Text-Book* and *Journal*:—

$$\mu_x = \frac{l_{x-1} - l_{x+1}}{2l_x}, \tag{1}$$

or more generally

$$\mu_x = \frac{l_{x-n} - l_{x+n}}{2nl_x},$$

$$\mu_x = \frac{8(l_{x-1} - l_{x+1}) - (l_{x-2} - l_{x+2})}{12l_x}, \tag{2}$$

all of which are particular cases of the general formula

$$\mu_x = \frac{l_{x-1} - l_{x+1}}{l_x} \frac{(|n)^2}{|\underline{n+1}\ |\underline{n-1}} - \frac{l_{x-2} - l_{x+2}}{2l_x} \frac{(|n)^2}{|\underline{n+2}\ |\underline{n-2}} + \cdots \tag{3}$$

(see *J. I. A.*, xxxii, 391). It will be seen that all of these formulas require values of $l$ for ages both greater and less than $x$; and, if $x$ be a select life, we can put no interpretation on $l_{[x]-1}$, $l_{[x]-2}$ etc.; so that none of these formulas can be applied to find $\mu_{[x]}$.

Another method of finding $\mu$ is to ascertain the mathematical law according to which $l_x$ varies. If this law be known, the differential coefficient of $l_x$ can be deduced, and thus the exact value of $\mu_x$ obtained. This is the method adopted in the table given in the *Text-Book*; but, as few mortality tables follow any known mathematical law, it is in general not applicable, and in particular it cannot be applied to the *Select Life Tables* published by the Institute of Actuaries.

Again, if a very large mortality experience were available, and the duration of each risk were stated to the nearest day, and a mortality table were formed showing the number living at the end of each day instead of at the end of each year, we should be able to get a very close approximation to the force of mortality. For, as one day is a very small interval in life, it may be taken as a rough representation of $dx$, and the formula for $\mu_x$ becomes

$$\mu_x \text{ approximately} = \frac{l_x - l_{x+\frac{1}{365}}}{l_x \cdot \frac{1}{365}} = 365\left(\frac{l_x - l_{x+\frac{1}{365}}}{l_x}\right),$$

or 365 times the probability of dying in one day. The data from which the above-mentioned Select Tables were constructed do not, however, admit of this method being employed.

The problem is to determine the differential coefficient of a function whose mathematical law is unknown, and whose values are only known for integral values of the variable; that is to say, the initial value and the differences of the function being known. Now the differential coefficient of a function can be expressed in terms of its differences by the following formula (*Text-Book*, part 2, page 453):—

$$\frac{du_x}{dx} = \delta u_x - \frac{\delta^2 u_x}{2} + \frac{\delta^3 u_x}{3} - \frac{\delta^4 u_x}{4} + \frac{\delta^5 u_x}{5} \quad . \quad . \quad . \quad \text{(A)}$$

This formula may be applied to find $\mu$ as follows:—

$$\mu_x = -\frac{1}{l_x}\frac{dl_x}{dx} = -\frac{d}{dx}\left(\log_e l_x\right) = -\frac{1}{M}\frac{d}{dx}\left(\log_{10} l_x\right),$$

where $M$ is the modulus of the common system of logarithms, and is equal to ·43429 . . . Now $\log_{10} l_x$ is tabulated for the Select Tables, and, by differencing it, we obtain successively $\delta(\log_{10} l_x)$, $\delta^2(\log_{10} l_x)$, $\delta^3(\log_{10} l_x)$, etc. If these be divided by 1, 2, 3, etc., respectively, the odd terms added together and subtracted from the sum of the even terms, the result is $-\frac{d}{dx}\left(\log_{10} l_x\right)$; and, if this be now divided by $M$, the final result is $\mu_x$.

If the table be written in reverse order, and then the above process be gone through, the result will be $-\mu$; hence the method can be applied to the end as readily as to the beginning of a table, and by this means the value of $\mu$ at the last age in the table can be found. The progress of differencing may of course be carried to as many orders of differences as is thought desirable; and, if $\log l_x$ followed a regular law, the differences of each order would tend to be smaller than those of the preceding order, and the approximation would increase in accuracy with each successive order taken into account in the calculation.

As a first trial of the method, I applied it to the $H^{M(5)}$ Table, proceeding as far as fifth differences, and calculated a complete table of $\mu_x$ on this basis. For comparison I calculated a second table of $\mu_x$ by the formula

$$\mu_x = \frac{l_{x-1} - l_{x+1}}{2\, l_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(B)}$$

The results were in most cases close enough for practical purposes; but those by formula A applied as far as fifth differences showed some irregularities, owing to the fact that the $H^{M(5)}$ Table does not follow a perfectly regular law, its graduation being in fact not very smooth. The effect of this is that the successive orders of differences do not diminish continually; but in general only do so for a few orders, and then begin to increase, and tend to become not only excessively irregular but also very large. It is clear, therefore, that, instead of proceeding in all cases to fifth differences, it will be more accurate to proceed only so far as the successive orders diminish and remain fairly regular, and to neglect the large and irregular differences arising from the irregularities in the original table. I therefore calculated a third table of $\mu$, similar to the first, but proceeding in each case with the differencing only so far as the successive orders continued to diminish, and the results thus brought out are considerably better than those in either of the two former tables. The method I adopted to test the accuracy of the results was to evaluate the integral

$$\int_0^\infty l_{x+t}\mu_{x+t}dt$$

by means of Hardy's formula above mentioned, in the cases where $x = 20$, 30, 40, 50, 60, and 70. The exact value of the integral is of course $l_x$, being the sum of all the deaths after age $x$, and the following table shows the exact values for the ages in question, and the approximations deduced by means of Hardy's formula from the three tables of $\mu$:—

TABLE 1.

| Age. $x$. | Tabular value of $l_x$. | Approximation No. 1. Formula A applied as far as 5th differences. | Approximation No. 2. Formula B. | Approximation No. 3. Formula A applied as far as minimum differences only. | Error by Approximation No. 1. | 2. | 3. |
|---|---|---|---|---|---|---|---|
| 20 | 9607 | 9399 | 9561 | 9505 | −208 | − 46 | −102 |
| 30 | 8693 | 8692 | 8945 | 8807 | − 1 | +252 | +114 |
| 40 | 7865 | 7835 | 7811 | 7828 | − 30 | − 54 | − 37 |
| 50 | 6894 | 6845 | 6850 | 6856 | − 49 | − 44 | − 38 |
| 60 | 5516 | 5525 | 5607 | 5573 | + 9 | + 91 | + 57 |
| 70 | 3533 | 3557 | 3598 | 3584 | + 24 | + 65 | + 51 |
| Sum of the errors, . . | | | | | −255 | +264 | + 45 |

It will be seen that the first approximation gives in most cases better results than the second, and the maximum error by it is less; but the sum of the errors is nearly as large, though of different sign. The third approximation gives in most cases results between those of the other two; and the maximum error by it is 114, compared with 208 and 252 in the other two, and the sum of the errors by it is small, being only 45. These figures seem to indicate that the third approximation gives, on the whole, the best results; and I therefore adopted it in preference to the other two, and proceeded to apply the same method to the Select Tables.

In this manner I calculated $\mu$ for all ages at entry from 15 to 75, and all durations of insurance (see the appended Tables 3 and 5). These results I tested in the same way as those derived from the $H^{M(5)}$ Table, the following being the figures:—

TABLE 2.

| Age. $x$. | Tabular value of $l_{[x]}$. | Approximation, using tabular value of $\mu_{[x]}$. | Approximation, assuming $\mu_{[x]}=0$. | Error, if tabular $\mu_{[x]}$ is used. | Error, if $\mu_{[x]}=0$. |
|---|---|---|---|---|---|
| 20 | 9564 | 9362 | 9288 | −202 | −276 |
| 30 | 8628 | 8661 | 8602 | + 33 | − 26 |
| 40 | 7780 | 7686 | 7628 | − 94 | −152 |
| 50 | 6749 | 6720 | 6662 | − 29 | − 87 |
| 60 | 5350 | 5413 | 5340 | + 63 | − 10 |
| 70 | 3354 | 3402 | 3337 | + 48 | − 17 |
| Sum of the errors, . . | | | | −181 | −568 |

These figures do not show as much regularity as might be desired; but, seeing that the maximum error, namely, at age 20, is little more than 2 per cent of the true value, the approximations to $\mu$ may perhaps be taken as sufficiently close for practical purposes. It will be noticed that I have calculated approximate values of $l_{[x]}$ on the assumption that $\mu_{[x]} = 0$, in addition to the approximations based on the values of $\mu_{[x]}$ given in the appended table, for reasons which I will now proceed to explain.

So far as I am aware, the general opinion among actuaries is that at the moment of selection the force of mortality is zero; and the reasons for this view are stated in a paper contributed by Dr. Sprague to the *Journal of the Institute of Actuaries* (xx, 100, 101), as follows:—"The effect of the medical examination is to "reject all those persons who are suffering from diseases tending "to shorten life, which can be detected by the examiner. If "we had the means of making accurate observations upon a large "number of persons, say the residents in a particular town or the "members of a particular profession, we should find that a certain "number of these, perhaps 5 per cent of the whole, are at any "particular time suffering under complaints that will, either "probably or certainly, prove fatal after a longer or shorter time; "and if, after the lapse of, say, a year, we could examine carefully "the causes of death of those persons among them who had died, "we should find that much more than 5 per cent, probably even "more than 50 per cent, of the total deaths would occur among "those persons whom we had previously noted as sickly. A "strict medical examination, then, greatly reduces the mortality "in the years which immediately follow it, that is to say, in the "early years of the insurance. The fewer the number of years "which have elapsed since the medical examination, the greater "will be its effect on the mortality; so that, in the first year "subsequent to examination, the mortality is very light; in the "first six months after examination, it is still lighter in propor-"tion; while it is an extremely rare thing for a life assured to "die within three months of the medical examination. But in "the application of Mr. Woolhouse's formula, $\mu$ is the value of "the force of mortality at the instant the policy is effected; and "we have therefore to consider the effect of a medical examination "on the mortality, not during the three months, nor even during "the month or the week following the examination, but during "the day or even the hour thereafter. We thus see that, assum-"ing the insurance to be effected immediately after the life is "passed by the doctor, all persons labouring under diseases from "which it is conceivable that they should die within a day, will "be eliminated; and we may almost say that there is only left "the chance of the life being knocked down, run over, and

" killed, when leaving the office of the company. The chance of " this or any other similar cause of death, is so extremely minute, " that we arrive at the conclusion that, at the instant a policy is " effected, $\mu = 0$."

In the discussion following the reading of that paper, objection was taken to this view, and the author expressed the opinion that the point was well deserving further consideration, and would no doubt receive it; but, so far as I am aware, the subject has hitherto not been pursued further. It will be observed that the view that $\mu_{[x]} = 0$ is based upon several assumptions—namely, that the insurance is effected immediately the life is passed by the doctor, that the medical examination is successful in eliminating all persons labouring under diseases from which it is conceivable that they should die within, say, a day, and that the chance of death by violence or accident is so extremely minute that it may be left out of account. The first of these assumptions is not correct, at all events in this country; though I have heard that in some thinly-populated parts of Australia the insurance agent travels about with a doctor, and, on coming to a house, canvasses the occupants, and, if successful, gets them examined by the doctor at once, and completes the matter on the spot. The Institute Experience was, however, compiled from data furnished by British companies, whose procedure is different. Here the doctor's report has to be sent to the head office of the company, the proposal submitted to the directors (or officials), and an acceptance issued, before the first premium can be paid. It not infrequently happens that the proposal papers are incomplete when the medical examination is made, and referees' or agents' reports have to be obtained, causing further delays. It must also be remembered that proposers do not always pay the first premium immediately they receive the acceptance, and (especially in cases connected with loan transactions) a considerable interval is often allowed to elapse before the case is finally completed. It is true that offices usually reserve the right to require a fresh medical examination if the premium be not paid within, say, 14 days of the date of acceptance; but this is rarely insisted upon, and it is probably safe to say that, on the average, at least a week elapses between the medical examination and the commencement of the insurance. As to the second assumption, I am not qualified to express an opinion further than to remark that it seems to infer the infallibility of all medical examiners, and that it is conceivable that there may be some diseases so insidious that even the most expert doctor might fail to discover them. The third assumption is of a different nature, and there are plenty of data available to test its accuracy. From the Registrar-

General's returns for England and Wales I have extracted the following figures:—

| *Year.* | 1885 | 1886 | 1887 | 1888 | 1889 | 1890 |
|---|---|---|---|---|---|---|
| Total deaths, . | 522,750 | 537,276 | 530,758 | 510,971 | 518,353 | 562,248 |
| Deaths by accident or violence, . | 15,248 | 15,171 | 15,870 | 15,209 | 15,315 | 16,549 |

The ratio of the deaths by accident or violence to the total deaths is very nearly the same for each of these years, being about 3 per cent, and, as the probability among the whole population of dying in a year is about ·02, it follows that the probability of dying by accident or violence is about ·0006. Since the medical examination cannot eliminate the risk of death by accident or violence, it is certain that the force of mortality among newly selected lives cannot be absolutely zero; but it remains to be seen whether it is so small that, for practical purposes, it can be disregarded. To test this, I evaluated the integral $\int_0^\infty l_{[x]+t}\mu_{[x]+t}dt$ for ages 20, 30, 40, 50, 60, and 70, on the assumption that $\mu_{[x]}=0$; and the results are given in the foregoing table (2). It will be seen that the resulting errors are in three cases greater, and in three cases less, than those resulting from the calculated values of $\mu$; but the maximum error when $\mu_{[x]}=0$, is considerably greater than when the tabular value of $\mu_{[x]}$ is used, and every error is of the same sign, namely, negative, so that the sum of the errors is very large. The fact that this assumption produces in each case too small a result, tends to prove that it is an appreciable, though slight, understatement of the risk—in other words, that $\mu_{[x]}$ must have some finite but small value; and I think that, on the whole, the calculated values of $\mu_{[x]}$ gives better results than are obtained on the assumption that the value is zero. The only means of definitely settling the question, is to investigate the experience of insurance companies; and I have obtained particulars in regard to one office, of all the deaths that occurred within a year of the commencement of the insurance. The total number of such deaths (excluding cases of children's endowments with returnable premiums) was 204, of which 5 occurred in the first month, 14 in the second, 18 in the third, 9 in the fourth, 10 in the fifth, 19 in the sixth, 17 in the seventh, 22 in the eighth, 19 in the ninth, 19 in the tenth, 25 in the eleventh, and 27 in the twelfth month, the average age at entry of those who died being 39·3 years. The earliest claim was on the 13th day after acceptance, two deaths occurred on the 22nd day, one on the 25th, and one on the 31st. There were two other deaths within the first month, which I have not taken into account, as in one

case the policy was obtained by fraud, and in the other there was good reason to believe there had been foul play, and neither claim was admitted. In considering these figures, it must be remembered that the commencement of the insurance is in every case taken as the date on which the acceptance was issued, and that on the average the first premium was not paid till at least a week later, so that the 5 deaths stated as occurring in the first month really took place within less than three weeks from the actual commencement of the risk. The total number of entrants was about 40,000, so that the mortality during the first insurance year was at the rate of about $\frac{1}{2}$ per cent, which is, as we might have expected, the rate shown by the Select Life Tables for age 35—that being probably very near the average age at entry. If the mortality during the first month of insurance were correctly represented by the calculated value of $\mu_{[35]}$ (·00299), the number of deaths would have been $40{,}000 \times \cdot 00299 \div 12 = 10$. A deduction of at least one-fourth must be made from this, in order to allow for the time between the date of acceptance and the commencement of the risk; so that the expected number of deaths on this basis is about $7\frac{1}{2}$, while the actual number of deaths was only 5. It is to be observed also that the rate of mortality during the first month is greater than the force of mortality at the moment of entry, and for both these reasons it would seem that the calculated value of $\mu_{[35]}$ is too large. The total number of deaths is not large enough to make a very satisfactory graduation, and thus ascertain their distribution throughout the year; but it may be pointed out that during the first four months there were 46 deaths, in the second four months there were 68, and in the third four months there were 90, and that these numbers increase in arithmetical progression.

Therefore, if we assume that throughout the first year of insurance the number of deaths in the successive months increases in arithmetical progression, we can easily construct a graduated table which will reproduce exactly the total number of deaths, and also the number occurring in each group of four months. Let the number of deaths in the first month be $A+B$, in the second $A+2B$, and so on, the number in the $n$th month being $A+nB$. Summing these, we see that the number of deaths in the first four months is $4A+10B$, and in the second four months $4A+26B$; but the actual number in the first four months was 46, and in the second four months it was 68, so that we have two equations to find $A$ and $B$. Solving these, we find $A=8{\cdot}0625$, $B=1{\cdot}375$. The adjusted number of deaths in the first month (being equal to $A+B$) is 9·4375, and the continued addition of $B$ gives the following adjusted table:—

| Month. | Adjusted number of deaths. | Total in each group of 4 months. | Unadjusted number of deaths. |
|---|---|---|---|
| 1. | $A+B$ 9·4375 | 46 | 5 |
| 2. | $A+2B$ 10·8125 | | 14 |
| 3. | $A+3B$ 12·1875 | | 18 |
| 4. | $A+4B$ 13·5625 | | 9 |
| 5. | $A+5B$ 14·9375 | 68 | 10 |
| 6. | $A+6B$ 16·3125 | | 19 |
| 7. | $A+7B$ 17·6875 | | 17 |
| 8. | $A+8B$ 19·0625 | | 22 |
| 9. | $A+9B$ 20·4375 | 90 | 19 |
| 10. | $A+10B$ 21·8125 | | 19 |
| 11. | $A+11B$ 23·1875 | | 25 |
| 12. | $A+12B$ 24·5625 | | 27 |
| | Total, 204· | 204 | 204 |

Having obtained a general expression for the number of deaths, it would be easy to deduce an expression for the number living at any time, and by direct differentiation to find the value of $\mu$; but the following method (due to my father) is simpler.

By summation, the number of deaths in the first $n$ months is seen to be equal to $nA+\frac{n(n+1)}{2}B$; therefore (taking 40,000 as the number of entrants) the probability of dying in the first $n$ months is $\frac{nA+\frac{n(n+1)}{2}B}{40,000}$. This is the rate of mortality *per n months*, and, if this rate were to remain unaltered for a complete year, it would follow by simple proportion that the rate *per annum* would be

$$\frac{12}{n}\times\frac{nA+\frac{n(n+1)}{2}B}{40,000}=12\times\frac{A+\frac{n+1}{2}B}{40,000}.$$

But, by definition, the force of mortality at the moment of entry is the limit of this expression when $n=0$, namely, $12\times\frac{A+\frac{B}{2}}{40,000}$. Substituting for $A$ and $B$ their values, we see that at the moment of entry $\mu=\cdot00263$. If the above graduation be accepted as satisfactory—and, after many attempts by myself and some of my friends, I am satisfied that it is about as good an adjustment as can be made—and if 35 be taken as the average age at entry,

it follows that the tabular value of $\mu_{[36]}$ is about ·00036 too large.

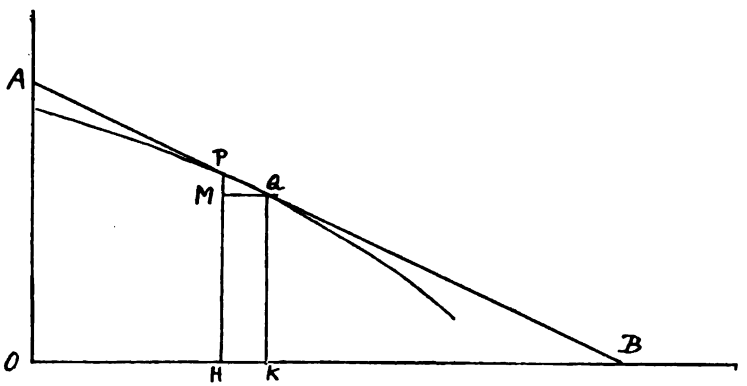
These considerations, as well as the figures given above relating to the risk of death by accident or violence, and the fact that accident insurance companies can grant policies insuring against the risk of death by accident, at annual premiums of about 25s. per £1000 (or ·00125 per unit), seem to indicate that the calculated values of $\mu$ at the moment of selection are too large, and that the true values are intermediate between them and zero. They are, however, the results of the application of a method which ought theoretically to give good approximations to the true values; and I therefore submit them for what they are worth, in the belief that they at all events deserve consideration, and in the hope that their publication may lead others to pursue the subject further. The tests I have applied to them show that, even if they are too large, the error resulting in their application to practical calculations is small; and as the values of $\mu$ after the first year of insurance are certainly very fair approximations, I trust that the complete tables may prove of some use in the calculation of complicated benefits. In order to facilitate their use for such purposes, I have calculated, in addition to tables of $\mu$, complete tables of $\log l_x\mu_x$, this being the function most suitable for calculation by means of a formula of approximate summation. (See Tables 4 and 5 appended.) The radix in both these tables is the same as in the *Select Life Tables* published by the Institute of Actuaries, namely, 1,000,000 at age 10, or 100 times the radix of the $H^{M(5)}$ Table.

It occurred to me as possible that the method of compilation of the Institute Experience, by calendar years, might have affected the rate of mortality in the first insurance year, and that this might be the reason why the values of $\mu_{[x]}$ are larger than would have been expected; and I therefore examined a few figures in the New Experience, which has been compiled by policy years, with a view to ascertaining whether it would give different results. Taking the figures for ages at entry from 28 to 32 in the whole-life class with and without profits, and grouping them together for each insurance year in order to get a larger body of facts and thus secure a smoother progression, I calculated the unadjusted rates of mortality for the central age 30, during each of the first five years of insurance, and from them deduced the value of $\mu_{[30]}$ in the manner explained above, with the remarkable result that, although the rates of mortality are considerably less than the rates shown by the Select Tables, yet the force of mortality at entry was identical, namely, ·00272. Taking the ages 33 to 37 and grouping them similarly, the value of $\mu_{[35]}$ came out as ·00214, compared with ·00299 in the Select Tables.

These last two results are irregular, owing of course to the fact that the data were not in any way graduated, but they tend to confirm my results, and to show that any error which might have arisen from the use of the calendar year method in the Institute Experience, has been kept within small limits by the method of construction of the Select Tables.

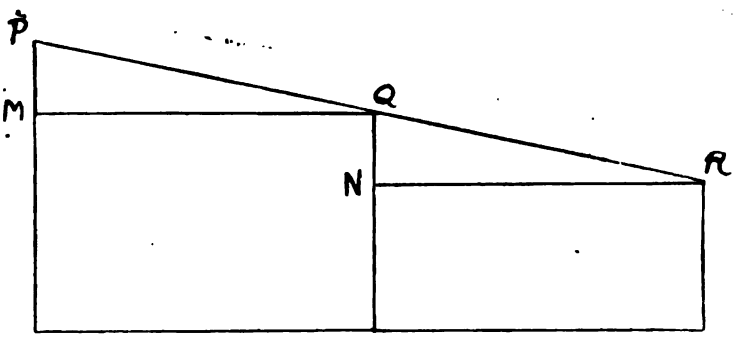
It may be useful to give a geometrical interpretation of my method of calculation. If we plot down a curve in which the abscissas are the ages attained, and the ordinates are the logarithms to base $e$ of the numbers living, the force of mortality at any age $x$ will be numerically equal to the ratio of the segments cut off the axes by the tangent to the curve at the point whose abscissa is $x$. This proposition can be easily demonstrated, as follows:—

Let $P$ and $Q$ be two points on the curve, close together, $PH$ and $QK$ the corresponding ordinates, $A$ and $B$ the points where the line $PQ$ cuts the axes, and $QM$ be drawn parallel to the $x$-axis meeting $PH$ in $M$. Then $HK$ represents a small increment in $x$, and $PM$ the corresponding decrement in $\log_e l_x$, also $QM = HK$.



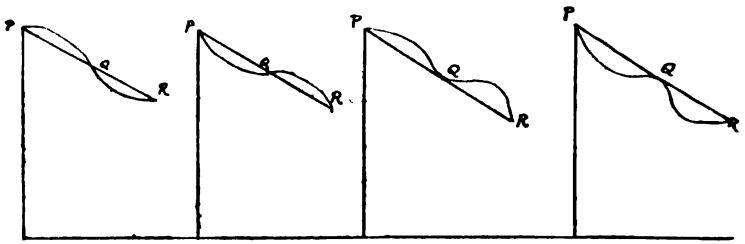

If now the two points $P$ and $Q$ are made to approach and become indefinitely close together, $HK$ will become indefinitely small and ultimately become equal to $dx$, similarly $PM$ will become ultimately numerically equal to $d(\log_e l_x)$, so that the ratio $\frac{PM}{HK}$ or $\frac{PM}{QM}$ will be numerically equal to $\frac{d(\log_e l_x)}{dx}$, that is to $\mu_x$; also the line $PQ$ will become the tangent to the curve at the point $P$, and the lines $OA$, $OB$ will become the segments cut off the axes by the tangent. Now the triangles $AOB$ and $PMQ$ are

similar, therefore the ratio $\frac{AO}{BO}$ is equal to the ratio $\frac{PM}{QM}$, so that $\mu_x$ is numerically equal to the ratio of the segments. Hence, if we could construct such a curve and draw the tangent to it, we could at once find the value of $\mu$ by reading off the lengths of the segments cut off the axes and finding their ratio. The difficulty in constructing the curve lies in the fact that we only know the values of $\log_e l_x$ for integral values of $x$, and the problem is therefore to construct a curve having given only a finite number of points on it. That is to say, all the intermediate points must be interpolated, and this can only be done approximately. The first approximation is to connect all the points by straight lines, and this is the equivalent of interpolation by first differences. If at any age $x$ we find that $\delta^2(\log_e l_x)=0$, it follows that the points on the curve corresponding to the ages $x$, $x+1$, and $x+2$ are in a straight line. For, let $P$, $Q$, $R$, be the three points. Then $PM$ represents $\delta(\log_e l_x)$, and $QN$ represents $\delta(\log_e l_{x+1})$; and, because the second difference is zero, the first differences are equal, therefore $PM=QN$, and, as $MQ=NR$, the triangles $PMQ$ and $QNR$ are similar, and hence $PQR$ is a straight line.




In this case it is clear that, no matter what may be the positions of the points beyond $R$, the only satisfactory curve which can be interpolated between $P$ and $R$ is the straight line $PQR$, the only other possible cases being represented by the following diagrams, which obviously introduce undue fluctuations in the curve and therefore also in the value of $\mu$. That function must, by the nature of the case, vary more or less steadily, and certainly it cannot, during the first year of insurance, increase for part of the

time and then begin to diminish again, or begin by diminishing and then increase again.




It thus appears that at any age where $\delta^2(\log_e l_x)=0$ it would be incorrect, in finding the value of $\mu$, to take into account the higher orders of differences, even though they do not vanish; and therefore at such a point $\mu_x$ must either equal or closely approximate to $-\delta(\log_e l_x)$.

Again, if at any age $x$, $\delta^3(\log_e l_x)=0$, it may be shown that the four points whose abscissas are $x$, $x+1$, $x+2$, and $x+3$, lie on a parabola whose axis is parallel to the $y$-axis; and the interpolated curve must either coincide with this parabola or lie very close to it, no matter what may be the position of subsequent points on the curve, otherwise undue fluctuations would be introduced into the curve. In such a case the differences of higher order than the third must be left out of account, and the formula for $\mu$ becomes $-\delta(\log_e l_x)+\delta^2(\log_e l_x)$. Similarly, if $\delta^4(\log_e l_x)=0$, the curve must very closely approximate to a cubic parabola, and the formula for $\mu_x$ must involve three terms, and so on. These considerations indicate that generally in an interpolation the best results will not be got by proceeding to the same order of differences throughout; but by studying the character of the differences, and stopping in each case at the point where they decrease to a minimum.

## TABLE 3.—$\mu_{[x-t]+t}$: Dr. Sprague's Select and $H^{M(5)}$ Tables: Ages 15-76.

| Age. | Years elapsed since Date of Assurance. | | | | | | Age. |
|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 or more. | |
| $x$ | $\mu_{[x]}$ | $\mu_{[x-1]+1}$ | $\mu_{[x-2]+2}$ | $\mu_{[x-3]+3}$ | $\mu_{[x-4]+4}$ | $\mu_x$ | $x$ |
| 15 | ·00256 | .... | .... | .... | .... | ·00304 | 15 |
| 16 | ·00350 | ·00344 | .... | .... | .... | ·00304 | 16 |
| 17 | ·00378 | ·00398 | ·00398 | .... | .... | ·00398 | 17 |
| 18 | ·00378 | ·00493 | ·00493 | ·00493 | .... | ·00493 | 18 |
| 19 | ·00336 | ·00569 | ·00565 | ·00565 | ·00565 | ·00565 | 19 |
| 20 | ·00253 | ·00622 | ·00735 | ·00735 | ·00735 | ·00735 | 20 |
| 21 | ·00265 | ·00702 | ·00853 | ·00853 | ·00853 | ·00853 | 21 |
| 22 | ·00267 | ·00677 | ·01004 | ·01004 | ·01004 | ·01004 | 22 |
| 23 | ·00256 | ·00663 | ·00956 | ·01067 | ·01067 | ·01067 | 23 |
| 24 | ·00260 | ·00652 | ·00951 | ·01078 | ·01191 | ·01191 | 24 |
| 25 | ·00267 | ·00629 | ·00870 | ·01066 | ·01091 | ·01091 | 25 |
| 26 | ·00269 | ·00610 | ·00840 | ·01034 | ·01004 | ·01034 | 26 |
| 27 | ·00260 | ·00601 | ·00838 | ·00997 | ·01013 | ·01013 | 27 |
| 28 | ·00265 | ·00603 | ·00840 | ·00965 | ·00986 | ·00986 | 28 |
| 29 | ·00267 | ·00596 | ·00881 | ·00956 | ·00974 | ·00974 | 29 |
| 30 | ·00272 | ·00603 | ·00817 | ·00933 | ·00933 | ·00933 | 30 |
| 31 | ·00260 | ·00615 | ·00818 | ·00916 | ·00912 | ·00912 | 31 |
| 32 | ·00288 | ·00622 | ·00815 | ·00919 | ·00942 | ·00942 | 32 |
| 33 | ·00269 | ·00612 | ·00811 | ·00926 | ·00926 | ·00926 | 33 |
| 34 | ·00274 | ·00645 | ·00822 | ·00907 | ·00907 | ·00907 | 34 |
| 35 | ·00299 | ·00661 | ·00824 | ·00923 | ·00926 | ·00926 | 35 |
| 36 | ·00302 | ·00677 | ·00822 | ·00939 | ·00988 | ·01025 | 36 |
| 37 | ·00313 | ·00693 | ·00827 | ·00966 | ·00988 | ·01045 | 37 |
| 38 | ·00313 | ·00702 | ·00863 | ·00979 | ·01016 | ·01105 | 38 |
| 39 | ·00322 | ·00716 | ·00875 | ·00981 | ·01066 | ·01117 | 39 |
| 40 | ·00336 | ·00730 | ·00896 | ·01039 | ·01121 | ·01135 | 40 |
| 41 | ·00338 | ·00789 | ·00914 | ·01004 | ·01124 | ·01124 | 41 |
| 42 | ·00352 | ·00767 | ·00987 | ·01057 | ·01151 | ·01172 | 42 |
| 43 | ·00375 | ·00774 | ·00958 | ·01087 | ·01184 | ·01186 | 43 |
| 44 | ·00391 | ·00817 | ·00992 | ·01126 | ·01186 | ·01197 | 44 |
| 45 | ·00394 | ·00845 | ·01016 | ·01158 | ·01227 | ·01271 | 45 |
| 46 | ·00419 | ·00877 | ·01052 | ·01207 | ·01292 | ·01345 | 46 |
| 47 | ·00437 | ·00908 | ·01089 | ·01257 | ·01354 | ·01407 | 47 |
| 48 | ·00461 | ·00939 | ·01140 | ·01313 | ·01423 | ·01492 | 48 |
| 49 | ·00486 | ·00981 | ·01186 | ·01370 | ·01483 | ·01596 | 49 |
| 50 | ·00511 | ·01022 | ·01248 | ·01412 | ·01570 | ·01681 | 50 |
| 51 | ·00516 | ·01059 | ·01315 | ·01515 | ·01669 | ·01739 | 51 |
| 52 | ·00560 | ·01112 | ·01395 | ·01598 | ·01739 | ·01840 | 52 |
| 53 | ·00615 | ·01174 | ·01481 | ·01688 | ·01821 | ·01946 | 53 |
| 54 | ·00668 | ·01216 | ·01568 | ·01794 | ·01948 | ·02056 | 54 |
| 55 | ·00702 | ·01288 | ·01688 | ·01927 | ·02063 | ·02185 | 55 |
| 56 | ·00744 | ·01370 | ·01817 | ·02047 | ·02185 | ·02296 | 56 |
| 57 | ·00804 | ·01460 | ·01927 | ·02194 | ·02349 | ·02459 | 57 |
| 58 | ·00859 | ·01559 | ·02079 | ·02386 | ·02547 | ·02637 | 58 |
| 59 | ·00898 | ·01669 | ·02206 | ·02577 | ·02724 | ·02759 | 59 |
| 60 | ·00963 | ·01759 | ·02406 | ·02828 | ·02975 | ·03012 | 60 |
| 61 | ·01029 | ·01815 | ·02549 | ·03092 | ·03212 | ·03212 | 61 |
| 62 | ·01108 | ·01898 | ·02894 | ·03366 | ·03486 | ·03486 | 62 |
| 63 | ·01190 | ·01900 | ·03228 | ·03606 | ·03751 | ·03751 | 63 |
| 64 | ·01333 | ·02029 | ·03647 | ·03882 | ·04119 | ·04119 | 64 |
| 65 | ·01384 | ·02049 | ·03928 | ·04184 | ·04391 | ·04391 | 65 |
| 66 | ·01425 | ·02303 | ·04757 | ·04757 | ·04757 | ·04757 | 66 |
| 67 | ·01471 | ·02577 | ·05052 | ·05052 | ·05052 | ·05052 | 67 |
| 68 | ·01536 | ·02901 | ·05441 | ·05441 | ·05441 | ·05441 | 68 |
| 69 | ·01628 | ·03198 | ·05786 | ·05786 | ·05786 | ·05786 | 69 |
| 70 | ·01745 | ·03468 | ·06245 | ·06245 | ·06245 | ·06245 | 70 |
| 71 | ·01886 | ·03682 | ·06761 | ·06761 | ·06761 | ·06761 | 71 |
| 72 | ·02086 | ·03859 | ·07458 | ·07458 | ·07458 | ·07458 | 72 |
| 73 | ·02114 | ·03956 | ·08188 | ·08188 | ·08188 | ·08188 | 73 |
| 74 | ·02211 | ·04857 | ·09230 | ·09230 | ·09230 | ·09230 | 74 |
| 75 | ·02245 | ·04626 | ·09947 | ·09947 | ·09947 | ·09947 | 75 |
| 76 | .... | ·05204 | ·11126 | ·11126 | ·11126 | ·11126 | 76 |

## TABLE 4.—LOG $l_{[x-t]+t}\ \mu_{[x-t]+t}$: DR. SPRAGUE'S SELECT AND $H^{M\ (5)}$ TABLES: AGES 15-76.

| Age. | Years elapsed since date of Assurance. | | | | | | Age. |
|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 or more. | |
| $x$ | $\lambda(l\mu)_{[x]}$ | $\lambda(l\mu)_{[x-1]+1}$ | $\lambda(l\mu)_{[x-2]+2}$ | $\lambda(l\mu)_{[x-3]+3}$ | $\lambda(l\mu)_{[x-4]+4}$ | $\lambda(l\mu)_x$ | $x$ |
| 15 | 3·40029 | .... | .... | .... | .... | 3·47865 | 15 |
| 16 | ·53548 | 3·55233 | .... | .... | .... | ·55233 | 16 |
| 17 | ·56662 | ·59005 | 3·59005 | .... | .... | ·59005 | 17 |
| 18 | ·56457 | ·58052 | ·58052 | 3·58052 | .... | ·58052 | 18 |
| 19 | ·51110 | ·74038 | ·75257 | ·75257 | 3·75257 | ·75257 | 19 |
| 20 | ·38426 | ·77594 | ·84860 | ·84860 | ·84860 | ·84860 | 20 |
| 21 | ·39849 | ·82499 | ·94864 | ·94864 | ·94864 | ·94864 | 21 |
| 22 | ·39728 | ·80402 | ·97645 | ·97645 | ·97645 | ·97645 | 22 |
| 23 | ·37334 | ·79014 | ·95013 | ·99428 | ·99428 | ·99428 | 23 |
| 24 | ·37643 | ·77749 | ·94323 | ·99609 | 4·00723 | 4·00723 | 24 |
| 25 | ·38327 | ·75753 | ·90011 | ·98860 | 3·99885 | 3·99885 | 25 |
| 26 | ·38264 | ·74010 | ·88037 | ·97074 | ·97074 | ·97074 | 26 |
| 27 | ·36333 | ·72914 | ·87474 | ·95054 | ·95754 | ·95754 | 27 |
| 28 | ·36687 | ·72662 | ·87163 | ·93191 | ·94119 | ·94119 | 28 |
| 29 | ·36657 | ·71754 | ·86270 | ·92353 | ·93186 | ·93186 | 29 |
| 30 | ·37002 | ·71847 | ·85136 | ·90885 | ·90885 | ·90885 | 30 |
| 31 | ·34725 | ·72269 | ·84484 | ·89725 | ·89507 | ·89507 | 31 |
| 32 | ·38676 | ·72355 | ·84207 | ·89430 | ·90509 | ·90509 | 32 |
| 33 | ·35371 | ·71271 | ·83560 | ·89356 | ·89356 | ·89356 | 33 |
| 34 | ·35658 | ·73063 | ·83735 | ·88081 | ·88081 | ·88081 | 34 |
| 35 | ·39047 | ·73682 | ·83414 | ·88403 | ·91263 | ·91263 | 35 |
| 36 | ·38923 | ·74274 | ·82836 | ·88709 | ·90920 | ·92519 | 36 |
| 37 | ·40089 | ·74835 | ·82618 | ·89505 | ·90465 | ·92957 | 37 |
| 38 | ·39623 | ·74942 | ·84044 | ·89553 | ·91189 | ·94888 | 38 |
| 39 | ·40411 | ·75318 | ·84147 | ·89180 | ·92827 | ·94885 | 39 |
| 40 | ·41755 | ·75670 | ·84686 | ·91177 | ·94539 | ·95148 | 40 |
| 41 | ·41555 | ·75731 | ·85085 | ·89221 | ·94140 | ·94140 | 41 |
| 42 | ·42776 | ·76821 | ·85672 | ·90958 | ·94699 | ·95475 | 42 |
| 43 | ·44989 | ·76688 | ·86109 | ·91666 | ·95392 | ·95479 | 43 |
| 44 | ·46246 | ·78532 | ·87115 | ·92678 | ·94959 | ·95330 | 44 |
| 45 | ·45895 | ·79398 | ·87554 | ·93356 | ·95908 | ·97430 | 45 |
| 46 | ·47968 | ·80410 | ·88511 | ·94560 | ·97563 | ·99330 | 46 |
| 47 | ·49173 | ·81001 | ·89375 | ·95737 | ·99008 | 4·00656 | 47 |
| 48 | ·50716 | ·82063 | ·90689 | ·96955 | 4·00534 | ·02610 | 48 |
| 49 | ·52322 | ·83240 | ·91715 | ·98135 | ·01646 | ·04854 | 49 |
| 50 | ·53782 | ·84303 | ·93215 | ·98710 | ·03421 | ·06401 | 50 |
| 51 | ·53391 | ·85079 | ·94721 | 4·01038 | ·05326 | ·05361 | 51 |
| 52 | ·56109 | ·86399 | ·96516 | ·02435 | ·06302 | ·06785 | 52 |
| 53 | ·59336 | ·87923 | ·98261 | ·04108 | ·07495 | ·10853 | 53 |
| 54 | ·61727 | ·88544 | ·99883 | ·05881 | ·09544 | ·11913 | 54 |
| 55 | ·63278 | ·89946 | 4·02157 | ·08082 | ·11118 | ·13641 | 55 |
| 56 | ·64767 | ·91846 | ·04393 | ·09724 | ·12641 | ·14810 | 56 |
| 57 | ·67064 | ·93573 | ·05946 | ·11721 | ·14738 | ·16764 | 57 |
| 58 | ·68806 | ·95325 | ·08160 | ·14270 | ·17162 | ·18655 | 58 |
| 59 | ·69502 | ·97119 | ·09575 | ·16460 | ·18923 | ·19480 | 59 |
| 60 | ·71172 | ·98117 | ·12106 | ·19261 | ·21502 | ·22045 | 60 |
| 61 | ·72629 | ·98077 | ·13263 | ·21812 | ·23490 | ·23490 | 61 |
| 62 | ·74260 | ·98410 | ·17322 | ·24047 | ·25593 | ·25593 | 62 |
| 63 | ·75785 | ·96965 | ·20477 | ·25451 | ·27198 | ·27198 | 63 |
| 64 | ·79001 | ·98155 | ·24082 | ·26936 | ·29562 | ·29562 | 64 |
| 65 | ·78676 | ·96835 | ·25539 | ·28337 | ·30486 | ·30486 | 65 |
| 66 | ·77888 | ·99890 | ·31982 | ·31982 | ·31982 | ·31982 | 66 |
| 67 | ·77057 | 4·02635 | ·32464 | ·32464 | ·32464 | ·32464 | 67 |
| 68 | ·76557 | ·05504 | ·33414 | ·33414 | ·33414 | ·33414 | 68 |
| 69 | ·76526 | ·07296 | ·33500 | ·33500 | ·33500 | ·33500 | 69 |
| 70 | ·76745 | ·08165 | ·34365 | ·34365 | ·34365 | ·34365 | 70 |
| 71 | ·76996 | ·07872 | ·34994 | ·34994 | ·34994 | ·34994 | 71 |
| 72 | ·76853 | ·06708 | ·36174 | ·36174 | ·36174 | ·36174 | 72 |
| 73 | ·74641 | ·04221 | ·36818 | ·36818 | ·36818 | ·36818 | 73 |
| 74 | ·72411 | ·04451 | ·38703 | ·38703 | ·38703 | ·38703 | 74 |
| 75 | ·68521 | ·02769 | ·37282 | ·37282 | ·37282 | ·37282 | 75 |
| 76 | .... | ·03196 | ·37595 | ·37595 | ·37595 | ·37595 | 76 |

TABLE 5.—$\mu_x$ AND LOG $l_x\mu_x$, $H^{M\,(S)}$: AGES 10-14 AND 77-96.

| Age. $x$ | $\mu_x$ | $\lambda l_x\mu_x$ | Age. $x$ | $\mu_x$ | $\lambda l_x\mu_x$ |
|---|---|---|---|---|---|
| 10 | ·00435 | 3·63868 | 84 | ·21451 | 4·12244 |
| 11 | ·00372 | ·57000 | 85 | ·23040 | ·05709 |
| 12 | ·00315 | ·49571 | 86 | ·23931 | 3·97114 |
| 13 | ·00290 | ·45805 | 87 | ·26003 | ·89933 |
| 14 | ·00311 | ·48674 | 88 | ·27134 | ·80273 |
| [For | ages 15-76, see | Tables 3 and 4] | 89 | ·29595 | ·71919 |
| 77 | ·11812 | 4·35224 | 90 | ·30643 | ·60643 |
| 78 | ·12749 | ·33209 | 91 | ·38921 | ·56331 |
| 79 | ·14009 | ·31520 | 92 | ·42308 | ·43260 |
| 80 | ·15061 | ·28371 | 93 | ·47855 | ·29271 |
| 81 | ·16477 | ·25432 | 94 | ·68424 | ·19694 |
| 82 | ·18092 | ·21985 | 95 | ·82632 | 2·95854 |
| 83 | ·19621 | ·17295 | 96 | 1·20860 | ·68436 |

## DISCUSSION.

THE PRESIDENT (Mr. G. M. Low) said that Mr. Sprague had presented their old friend $\mu_x$ in an interesting light, and they would now know more about it than before. When the paper was in print they would be able to study it more at leisure, with the geometrical illustrations. The subject was one on which he would not venture to offer any criticism. It was one of the new methods that had come into use since he was a student, and his acquaintance with the higher mathematics had not enabled him to keep abreast of all these developments which had proved so important and useful.

Mr. V. MARR thought the author of the paper deserved great credit for the labour and time expended in the calculation of the tables relating to the force of mortality. Among these were included the values of log $l_x \mu_x$, which was a very convenient function to tabulate, as it often shortened the work; and he hoped it would be published in connection with the New Experience. With regard to the terms to be used in applying an approximate summation formula, this was a difficulty always encountered by students. He did not know that any rules had been laid down elsewhere, and considered that the author's remarks on this point would be appreciated. Mr. Sprague had given two different expressions for a reversionary annuity to (20) after (80); but of course one could be got from the other by changing the order of integration, and making the necessary alteration in the limits. Mr. Sprague's formulas for getting $\mu_{[x]}$ were very simple in their application, and he thought they were about the only ones that could be employed, owing to the way in which the data were arranged, according to integral years. To find the force of mortality at the date of medical selection, it must be remembered that the rate of mortality increased very rapidly in the early months; and he did not think that yearly units of time would give good enough results. He had had access lately to an experience where the numbers living were tabulated according to quarter-years of age, and had calculated the force of mortality at age 0 (where the curve for $p_x$ increases rapidly and must be of a somewhat similar nature to that for $q_{[x]+t}$). Using the numbers at integral ages, the force of mortality came out ·1111; taking half-years of age, ·1376; and taking quarter-years, ·1773; so that there was a considerable difference, and most probably the quarter-years gave the most correct result. Mr. G. F. Hardy had applied Makeham's law to Dr. Sprague's Select Tables, and from the resulting net premiums it would seem that the values which he had assigned to Makeham's constants must reproduce the various values of $l_x$ very accurately. He (Mr. Marr) did not know of these constants ever having been published; and it would be interesting to know how the force of mortality deduced from them would compare with that brought out by Mr. Sprague. At the date of selection, with a mathematical formula like that applied by Mr. G. F. Hardy, the force of mortality was, of course, what is represented by the constant in Makeham's general expression. Mr. Sprague's graduation of the deaths in the first insurance year was ingenious; but he thought

that the arithmetical progression which they showed when split up into periods of four months, was just a coincidence, and that Mr. Sprague's A was rather too large for the initial value. He had tried a graduation by taking account of the probable duration of the illness, by means of the table (page viii of Dr. Farr's letter appended to the Registrar-General's Twelfth Report) which had been mentioned by Dr. Sprague and Mr. King in connection with the Analysed Mortality Tables of the latter, and which showed the duration of the illness in the case of deaths. He had, however, not been able to get hold of a mathematical law from which could be deduced the initial value corresponding to Mr. Sprague's $\frac{12}{10000}$ (A + ½B). He would like to hear the views of members upon the suitability of approximate summation formulas when applied to select tables. All these formulas assumed that the function to be integrated followed a regular law, and could be traced by a curve moving evenly and not by sudden jumps; whereas in select tables the function went up pretty quickly at first and more gradually afterwards; and he did not think an approximate formula of summation, such as the one applied, accurate enough. It seemed to him that the best course would be to use the tables in the Institute *Text-Book*, Part II, taking equivalent ages. These tables proceed with regularity, and would probably give a better approximation to the risk.

Mr. W. Young thought that, while the first part of the paper would be useful to students as a résumé of the subject of approximate summation, the second part, dealing with the force of mortality in select tables, was an important contribution to the science of life contingencies. Mr. Sprague had calculated $\mu_{[x]}$ by approximating to the differential coefficient of $l_{[x]}$ by means of the formula A, so far as the successive orders of differences diminished and remained fairly regular, and had said that this should give theoretically a good approximation to the true value. No reasons, however, had been given for this opinion. It appeared to him (Mr. Young) that there was no evidence that the use of this approximation would give a close approach to the value of $\mu_{[x]}$. He thought the formula was only applicable to find an approximate differential coefficient, where the successive orders of differences were regular and rapidly convergent; and the fact that the differences were irregular was an objection to the use of the formula. There was, however, a more serious objection—namely, that in finding $\mu_{[x]}$ from $l_{[x]}$ and the values of $l$ at succeeding *integral* ages, it was tacitly assumed that the law of mortality at the moment of selection would be the same as that represented by the values of $l$ used. Since selection eliminated those lives who were suffering from disease, it was more likely that the law of mortality at the moment of selection was not the same as the law represented by the figures giving the numbers surviving one, two, and three years after selection. He therefore did not think they could have confidence that values so derived gave a trustworthy estimate of the force of mortality at the moment of selection; indeed, it was evident they would be much too large. To test how near Mr. Sprague's figures came to the true values, they would require statistics showing the distribution of the deaths that take place soon after selection. It would be necessary to have a much larger body of facts than Mr. Sprague had given, from which to deduce a close approximation to $\mu_{[x]}$. There were, however, two noteworthy facts in his figures—namely, that the deaths in the first month were about one-third of those in any succeeding month, and that, excluding the first month, the figures were fairly regular and increased slowly. These two characteristics had an important bearing on the value of $\mu_{[x]}$; but Mr. Sprague had graduated the figures as an arithmetical progression, and had lost their chief characteristics by making the deaths in the first month (this being the value which most affects $\mu_{[x]}$) nearly double the original number. He did

not think anything was to be gained by graduating small numbers such as these. Taking the figures as they stood, it appeared that the deaths in the first month were $\frac{5}{204}$ths of the whole year's deaths; and therefore, if the numbers were considered sufficient to give a good result, $\mu_{[x]}$ would be less than $\frac{60}{204}$ths of $q_{[x]}$, or less than $\cdot 3q_{[x]}$. He found the values of $\mu$ in Mr. Sprague's table were throughout nearly twice this value, which itself appeared to him to be an overestimate—especially if they examined the number of days after acceptance on which the five deaths occurred. It seemed, therefore, not unlikely that the value 0 was nearer the true value of $\mu_{[x]}$ than the values tabulated by Mr. Sprague. The question of the actual value of $\mu_{[x]}$ was purely theoretical, as it was not very material in practical calculations whether they used the value 0, or allowed a small value, or as a maximum the values of Mr. Sprague. It was true that premiums should be calculated on a select basis, but $\mu_{[x]}$ was required only in those cases where the benefit was so complex that a summation formula was necessary, and it affected only one of the four to six values used. The premiums for complex and unusual assurances were often very small, and in all cases there ought to be a substantial loading. In the case of the single premium for a contingent assurance on a young life against an old one, he estimated the difference between using 0 and Mr. Sprague's value, as less than 5 per cent of the premium. For a complex benefit a difference of 5 per cent would not be important, in view of the large loading. Mr. Sprague's figures, however, recommended themselves, inasmuch as they gave higher premiums, and the error was on the safe side. He thought they were indebted to Mr. Sprague for supplying tables which would enable them for the first time to calculate on a select basis the premium for a complex benefit.

Mr. J. Chatham remarked that the force of mortality must always be of importance to actuaries, because it now entered so much into their calculations; but its value at the date of selection was more of a theoretical than a practical nature. The New Experience would likely soon supersede Dr. Sprague's Select Tables, and would be founded upon a law of mortality calculated upon a mathematical formula where the value of $\mu$ would be definitely known or could be easily ascertained. He thought the paper a valuable one for students, because he did not remember any previous paper in which the force of mortality had been so well described and the explanations of its use so well given. He had made considerable use of the *Text-Book* table in complex calculations; and it gave, as a rule, slightly higher premiums at the older ages, so that it erred on the safe side. With regard to Mr. Sprague's estimate of ·0006 as the chance of death by accident or violence in the general population, he had found from the experience of the accident department of his own office, that the risk of death from accident was about ·0005, so that the author's approximation was very close. It would naturally be greater than that derived from the lives selected by an office; and again they could not be quite sure what was included by the Registrar-General under the term 'violence'.

Mr. M. M. Lees said that the practical hints given by Mr. Sprague in connection with the calculation of continuous benefits would be very useful. As regards his calculated values of $\mu_{[x]}$, the author had admitted that at the moment a policy was effected, they were larger than they should be. It was essential, in the calculation of this function, that they should have a fixed idea of the moment when a policy was effected. There were two dates over which some confusion might arise—namely, the date when the acceptance notice was issued, and the date when the first premium was paid. In order that an assurance might be put in force, the life must be alive and in good health when the first premium was paid; and as, according to the acceptance notice, no claim could arise until the premium was

paid, the moment it was paid was the time the policy was effected. But, according to their methods, the date of entry was taken to be the date of the acceptance notice, and no account was taken of the deaths between the two dates. He submitted that they could not calculate directly, from Dr. Sprague's Select Tables, the value of $\mu$ at the moment the acceptance notice was issued. To borrow the example used by Mr. Sprague in the first part of his paper, they might take the case of a railway train travelling towards a station, stopping there, and the steam being let off; it cannot start again until the steam reaches the cylinders. The waiting period was much like the time which elapsed between the medical examination and the payment of the first premium; and the problem before them was, for want of data, very similar to calculating the rate at which the train was travelling while standing still in the station. Again, supposing they had a body of lives large enough to enable them to tabulate the deaths from day to day, commencing with the day the acceptance notice was issued, it was clear that, as these notices were sent to the parties through the post, the first premium was seldom or never paid on that day; and consequently there were no deaths. They would therefore find that they could not even apply the formula $365\frac{(l_x - l_{x+\frac{1}{52}})}{l_x}$, to get an approximation to the force of mortality at the moment of acceptance. They had been told that, on the average, a week elapsed between the acceptance and the payment of the premium; it appeared to him therefore that the values brought out were not the values of $\mu_{[x]}$, but approximately the values of $\mu_{[x]+\frac{1}{52}}$. In order to show that $\mu_{[x]}$ is not equal to 0, Mr. Sprague had tried to reproduce the tabular values of $l_{[x]}$ for various ages by means of a formula for approximate summation, taking $\mu_{[x]}$ as 0, but had found that the errors were all in one direction and negative. This he (Mr. Lees) thought was not due to the assumption that $\mu_{[x]}$ is 0, but might have something to do with the methods which had to be resorted to in order to ascertain the numbers exposed to risk and the corresponding deaths in the first year of assurance.

Mr. A. Hewat recommended the paper not only to students of the Faculty, but also to the examiners, as he considered it would be useful for their purposes. The author had dealt in an interesting way with his subject. He was a bold actuary who would call in question anything that Dr. Sprague had said, or any conclusion brought out by him. He thought, however, there was a good deal in the reasons given in the paper for differing from Dr. Sprague. He liked the graduation formula; but, as had been already pointed out, the arithmetical progression might be only an accident.

Mr. J. J. McLauchlan concurred with other members in thinking that the excellent description of the force of mortality, the illustrations used, and the various explanations of its relations to other functions, were very useful. Indeed, they reminded him of passages in Dr. Sprague's own papers, which show how important he considered it that the readers of them should fully understand the nature of the functions dealt with; and he was glad to have his own ideas refreshed by hearing that part of the paper. Those who had to calculate complicated benefits by the use of summation formulas, would find the remarks on these formulas exceedingly useful.

Mr. H. W. Brown remarked that he would be glad if Mr. Sprague would explain why he did not extend the differential coefficient of $l_x$ in terms of its finite differences, but passed on instead to $\log_e l_x$. It had occurred to him to investigate whether the errors exhibited in Table 1 as the result of approximation No. 3, were for the most part due to the mere approximativeness of the summation formula or to the inexactness of the $\mu$'s them-

selves. He had accordingly used Mr. G. F. Hardy's formula for the approximate evaluation of $\int_0^{2n} l_x\mu_x dx$ for ages 20, 30, 50, and 70, and of $\int_0^{18n} l_x\mu_x dx$ for ages 40 and 60, giving to $n$ the values 1, 2, or 3, as the case might be. The errors brought out were :—

| | | | | | | |
|---|---|---|---|---|---|---|
| Age . . | 20 | 30 | 40 | 50 | 60 | 70 |
| Error . . | −58 | +17 | +20 | +25 | +8 | −19 |

which summed to −7, in place of Mr. Sprague's 45. Thus the error arose in great measure from lack of precision in the summation formula, and not from want of accuracy in the $\mu_x$ values. Moreover, an extended summation formula like this seemed especially desirable when they came to deal with select values, since it did not involve a standing jump, as it were, from the select to the non-select, but allowed the one to merge gradually into the other. Coming to the appraisement of $\mu_{[x]}$, he thought Mr. Sprague had amply proved his contention that it has a definite value; but when he proceeded to ascertain this value by means of the numbers living at the end of the next few years after entry, he (Mr. Brown) was reckless enough to differ from him. Mr. Sprague's method seemed to be an admirable one for finding the value of $\mu$ for every year after the first; but he could not bring himself to believe that the mortality of the second and subsequent years had any very definite connection with the value of $\mu$ at the moment of entry—just the very time when the mortality was changing most rapidly. Of course, the numbers living at the end of each year depended to some extent upon the force of mortality at the moment of entry and at every other moment throughout the years; but that did not imply that $\mu_{[x]}$ could be expressed within the boundaries of a hard and fast mathematical formula, in terms of the differences of $\log_e l_x$. The force of mortality at entry was unique, and its value must be determined uniquely. As an adaptation of Mr. Sprague's process, he would point out that the evaluation of the integral $\int_0^\infty l_{[x]}\mu_{[x]}dx$ by Mr. G. F. Hardy's summation formula already referred to, would produce an approximation to the value of $l_{[x]}$, and the calculations in connection with Table 1 afforded some indication of the probable variation from the true value. He had made allowance for the same errors reduced in the ratio of $l_{[x]}$ to $l_x$. They had now an equation, the numerical value of each of whose factors was known, with the exception of the initial value of $\mu$; and by solving that equation the value of $\mu_{[x]}$ was at once determined. At age 20 the value so brought out was ·00208, as against Mr. Sprague's ·00253; and for ages 50 and 60, ·00456 and ·00925 respectively, in opposition to the ·00511 and ·00963 of Mr. Sprague. The values were somewhat less than Mr. Sprague's in each case; which was interesting in view of the author's persuasion that his values were in general rather high. Of course, these figures ought to be independent of the extent of mortality table involved; and as a test he had worked out the integral $\int_0^6 l_{[x]}\mu_{[x]}dx$, by Mr. Hardy's formula of approximate summation, using the initial values of $\mu$ just brought out, and the results varied from $l_{[x]} - l_{[x]+6}$ by less than one per cent. Perhaps at some other ages the results of this method might not be so satisfactory; but this was probably due to the formation of the Select Tables, which were scarcely suited for such delicate work; and when the New Experience Tables were completed, more trustworthy results might be obtained.

Mr. W. A. Robertson, having referred first to the question of the suitability of the data for Mr. Sprague's purpose, in view of the difficulty

of obtaining accurate results from experiences arranged according to calendar years, proceeded to give a comparison he had made of the force of mortality obtained from Mr. Chatham's *Gotha* experience with that given in the paper. The former values had been calculated by means of central-difference formulas, $\mu_{[x]}$ being obtained by a special method. It might perhaps be argued that the comparison lacked importance by reason that the same formulas were not applied in both cases; but it did not necessarily follow that, because a central-difference formula was inapplicable to the early portion of a select table, it was inapplicable all through. The difficulty referred to by Mr. Sprague in applying such a formula to a select table, was equally evident to Mr. G. King in forming the *Text-Book* force of mortality below age 10. Comparing the *Gotha* figures with Mr. Sprague's, he pointed out that the former increased regularly throughout for the ages at entry he had tabulated, namely, 25, 35, 45, and 55. Mr. Sprague's figures for age 25 at entry formed what might be described as a wavy line. Was it possible that the force of mortality should increase and decrease alternately with the increase in the duration? He did not overlook the fact that between ages 25 and 35 there was a tendency for the rates of mortality to decrease and then increase; but that could not, he thought, be the whole explanation. Then was it likely that $\mu_{[35]+6}$ should be less than $\mu_{[35]+5}$? Speaking generally, the *Gotha* $\mu$'s were less—usually considerably less—than Mr. Sprague's for the principal assuring ages; $\mu_{[x]}$ was especially interesting; $\mu_{[25]}$ was much less than Mr. Sprague's; but the other values were higher. In view of the undoubted irregularity of Woolhouse's graduation of the $H^{M(5)}$ experience, he had worked out, by the central-difference formula, a table of $\mu$ from Dr. Sprague's graduation of the same experience, and had compared the values with Mr. Sprague's. The differences were not inconsiderable, more especially at the older ages. Below age 45, Mr. Sprague's values did not proceed at all smoothly. The values deduced from Dr. Sprague's graduation decreased from age 26 to age 32, and thereafter increased regularly throughout; and at ages above 80 were always considerably higher. At that age he had checked the values by evaluating the integral $\int_0^\infty l_{x+t}\mu_{x+t}dt$, getting the result 13,265, the correct value being 13,268. Turning to the data, and comparing the expected deaths by Dr. Sprague's graduation with the actual, it would be found that the graduated rates of mortality at ages over 80 were undoubtedly too high. Dr. Sprague had told them that it was impossible to follow the facts more closely without sacrificing the regular increase in the rate of mortality above 70. It had, however, occurred to him that the difference between the actual and the adjusted mortality might be to some extent due to the nature of the experience, to which Dr. Sprague did not appear to have given sufficient weight. The advantage of selection diminished much less rapidly at the older than at the younger ages, where it is less effective. Comparing the *Gotha* $^{M(6)}$ with the Select values, even after ten years the latter did not approach the former; and this light mortality would tend to prevent a regular increase in the mortality. To justify select tables formed on the principle of joining the select values on to the aggregate after a term of years, it would seem to be almost essential to adopt a term varying with the age at entry, somewhat after the method adopted by Dr. Sprague in forming a marriage table for widowers. Mr. Sprague's values at the latter portion of his table seemed to be low—in fact, they were sometimes actually less than the $H^M$, whether those of the *Text-Book* or those deduced by the central-difference formula. Mr. Sprague had given the general central-difference formula for $\mu$. He (Mr. Robertson) thought it might be interesting to give a general finite-difference formula that he had deduced.

They had

$$\mu = -\frac{1}{l}\frac{dl}{dx} = -\frac{1}{l}(\Delta l - \tfrac{1}{2}\Delta^2 l + \tfrac{1}{3}\Delta^3 l - \ldots).$$

Now, stopping successively at the several differences, they had

$$\begin{array}{ll}
\mu = q & (1) \\
\mu = q + \tfrac{1}{2}(q - {}_1|q) & (2) \\
\mu = q + \tfrac{1}{2}(q - {}_1|q) + \tfrac{1}{3}(q - 2_1|q + {}_2|q) & (3) \\
\mu = q + \tfrac{1}{2}(q - {}_1|q) + \tfrac{1}{3}(q - 2_1|q + {}_2|q) + \tfrac{1}{4}(q - 3_1|q + 3_2|q + {}_3|q) & (4) \\
\mu = (4) + \tfrac{1}{5}(q - 4_1|q + 6_2|q - 4_3|q + {}_4|q) & (5)
\end{array}$$

and so on.

They thus obtained the following formula :—

$$\mu = q(1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \tfrac{1}{5} + \ldots) - {}_1|q(\tfrac{1}{2} + \tfrac{2}{3} + \tfrac{3}{4} + \tfrac{4}{5} + \ldots)$$
$$+ {}_2|q(\tfrac{1}{3} + \tfrac{3}{4} + \tfrac{6}{5} + \ldots) - {}_3|q(\tfrac{1}{4} + \tfrac{4}{5} + \ldots) + \ldots$$

Mr. Sprague had told them that the chief use of $\mu$ was in connection with continuous benefits. The quest after the law of mortality had revealed the function $c$, which was now so much employed to obtain an average valuation age in connection with the grouping of endowment-assurance and limited-payment policies according to the term to run. He must confess that the necessity for calculating the function $Z^M$ had never appeared to him as obvious, when exactly similar results might have been obtained, and with no more labour, by scheduling the *Text-Book* force of mortality. He had doubted whether a summation formula such as Woolhouse's could be applied to the calculation of a select benefit; but, on working out $a_{[30]}$ and $a_{[50]}$ at 3 per cent, which involved the calculation of but three terms (the first being the antilogarithm of the coefficient), the errors were only ·009 and ·003 respectively.

Mr. S. F. M. Cumming asked whether the 204 deaths were arrived at after the whole 40,000 lives had been exposed for twelve months, or only until the first renewal date?

Mr. J. Nicoll had never studied formulas of the differential calculus with so much interest as he had done those in this paper, and hoped Mr. Sprague would take up other formulas of the *Text-Book* in a similar way. Mr. Sprague had said that the force of mortality for age $[x] - 1$ had no meaning; and it might be pointed out that the same difficulty occurred in connection with the formula for contingent assurances, which had to be altered when select tables were used. As to the risk of death from accident being equal to zero at the outset, he was inclined to agree with Dr. Sprague that it was so. Accidents did not usually happen to people newly selected for assurance, or to others physically capable, but more generally, perhaps, to the blind, the deaf, and the lame, the very weaklings of the general population (from which source Mr. Sprague's figure was drawn), who could have no chance of being assured; so that the risk in the case of newly assured lives must be extremely small, if appreciable at all. The 204 deaths of the first year were only the experience of one company, and it would have been well if some other experience could have been adduced to confirm the conclusions drawn in the paper.

Mr. A. G. Donald said that as regards $q_{[x]}$, objection had been taken to mixing up the mortality of the first year with that of any other, and it appeared to him that this applied with no less force to the determination of the still more exceptional quantity $\mu_{[x]}$. If Mr. Sprague's formula were applied to a carefully adjusted select curve which merged smoothly into the ultimate, and if it were carried to a sufficient number of terms, the results might be good; but when used to a varying number of terms, depending on the irregularity of the data, he doubted what authority to assign to the

method. From the nature of the case, the curve varied rapidly near its beginning, and the data of subsequent years were not very helpful in indicating its course at first. Moreover, when $\mu_{[x]}$ was deduced from the materials used, one felt that even the slight arbitrary adjustments and the necessary defects of the Select Tables might give results somewhat astray; for delicate work could not be expected from these tables. The heart of Mr. Sprague's paper lay in his application of formula A. But that formula was not exact; it was only approximate, and they had no indication of the degree of its approximation at the initial point of a curve not represented by a mathematical equation—a curve, moreover, which changed its direction rapidly near the beginning. With people dying off with mathematical care or scientific enthusiasm, its result might be good; but when use was made of the tables under consideration, the answer contained the resultant of several errors for which they could not make a correction. All that they had given was a series of points; and they were endeavouring to determine the direction of the curve at the first point. Now, there was a possible infinity of curves connecting the first point to the second, with a corresponding number of directions of the initial tangent. If they had the figures of the office whose experience Mr. Sprague had investigated, they would have $l_{[x]}$ and then $l_{[x]+1}$; and the $\mu_{[x]}$ deduced by the formula would have been precisely the same whether the deaths had occurred as they did in fact, or if they had happened according to Mr. Sprague's redistribution—two series which suggested values of $\mu_{[x]}$ considerably different. In fact, the formula implicitly assumed that only the number of deaths in the first year mattered, and not their distribution; and Mr. Sprague had not offered any proof of this fundamental hypothesis. Further investigation might show that the curve under discussion was fairly regular, and not subject to rapid change at the beginning; but the method employed seemed to start from this assumption—a position not yet clearly made out. On the face of it, however, there seemed nothing improbable in the view that $\mu_{[x]}$ was a constant; that at the instant there was no deterioration of the life; but that death was the consequence of one force, which was chance; such a value of $\mu_{[x]}$ would be quite satisfactory in approximating to the value of a continuous benefit. But if this were thought to be merely theory, one would more naturally seek for an *à posteriori* value of $\mu_{[x]}$ by some such method as Mr. Brown's. If the $H^{M(5)}$ $\mu$'s were satisfactory, it would seem possible from the $H^{M(5)}$ results to deduce $\mu_{[x]}$ without making use of the mortality of the second and subsequent years; for the figures in the summations for $l_x$ and $l_{[x]}$ differed only as to the first term, and a linear equation could be easily deduced in which the select $\mu$ was the only unknown, and from which it could be determined. As the function which Mr. Sprague had used to test the accuracy of his $\mu_{[x]}$ values was largely built up of the corresponding function deduced from the $H^{M(5)}$ table, the result got for comparative purposes was really disturbed by the sum of two errors due respectively to the Select and the $H^{M(5)}$ $\mu$. Moreover, it somewhat discredited the accuracy of the results to note that the number of deaths introduced by the first term in the summation formula was small; and if they considered what different values of $\mu_{[x]}$ were possible without increasing the numerical error of the result as compared with the true value of $l_{[x]}$, they would, at the ages taken by Mr. Sprague, get values as low as 12 per cent of $\mu_{[x]}$, and as high as 646 per cent.

The President said it was significant of the tendency of modern actuarial science towards scientific accuracy, that a paper of this technical nature should have excited so much interest and produced so good a discussion. He proposed a vote of thanks to the author, which was cordially given.

Mr. Sprague, in reply, said that his object in writing the paper had been

a practical one—namely, to find a means by which an approximation could be made to the value of a complicated benefit by select tables, as he was strongly of opinion that these were the proper tables to use for the calculation of premiums. The question of finding the force of mortality at entry, though of great interest, was merely incidental and of comparatively little practical importance. The first month (during which five deaths occurred) was measured from the date of acceptance, not from the commencement of the risk. He had stated in the paper that the interval between acceptance and commencement of the risk might be taken as at least a week; but he had little doubt that this was an underestimate, and that the actual interval was more nearly a fortnight. The five deaths therefore occurred among lives who had on the average been exposed to risk for little more than a fortnight, and the number of deaths in the first month's insurance would therefore be nearly double. His graduation of the deaths therefore produced a result in reasonable accordance with the facts regarding the first month's insurance. No doubt it was a mere accident that the deaths, taken in groups of four months, were in exact arithmetical progression; but the general tendency of the figures was undoubtedly to increase in some such progression, and he thought the adjustment represented the facts fairly well. He did not agree with the idea that no attempt should be made to graduate small figures. With regard to Mr. Brown's question—he had employed $\log l_x$ instead of $l_x$ in the formula, because the former possessed a slight mechanical advantage. Up to the final operation the work was the same in both cases; but, if $\log l_x$ were employed, this operation was to divide by M, while, if $l_x$ had been taken, the final step would have been to divide by $l_x$. It was easier to divide by a constant quantity than by a variable, as the logarithm of the divisor could be written on a slip of paper and subtracted rapidly from the logarithms of the various dividends. With regard to Mr. Cumming's question, the 40,000 was a rough estimate of the number of entrants; and the 204 deaths were those which occurred within one year from the date of acceptance.

No. 4. Vol. I.

# TRANSACTIONS

OF THE

# FACULTY OF ACTUARIES.

CONSTITUTED 1856.

INCORPORATED 1868.

## Some Practical Notes on Reversions, with Results of an Investigation into the Mortality among Life-Tenants—By Neil Campbell, F.F.A.

With Abstract of Discussion thereon.

LONDON:
CHARLES & EDWIN LAYTON,
FARRINGDON STREET.
1902.

*Price* 1s. 6d.

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TRANSACTIONS

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FACULTY OF ACTUARIES.

# *Some Practical Notes on Reversions, with Results of an Investigation into the Mortality among Life-Tenants.* By NEIL CAMPBELL, F.F.A., *of the Scottish Equitable Life Assurance Society.*

[Read before the Faculty, 3 February 1902.]

FROM the published list of the papers read to the late Actuarial Society, it would appear that, notwithstanding its interest and importance, little or no attention was paid to the subject of reversions and other allied securities. This is probably accounted for by the fact that numerous able papers have been read from time to time to the Institute of Actuaries, and the field may have been considered as pretty well exhausted. The Institute papers, however, for the most part confine themselves to particular branches of the subject, many of them almost entirely to questions of formula; and to a person beginning the subject, the variety and detail of such papers must be perplexing, not to say alarming. Especially will this be the case if the student is seeking information as to the actual business handling of reversionary transactions, for the theoretical is more frequently in evidence than the practical. It seemed, therefore, that it might not be useless to have a paper of the nature of an introduction to the practical working of reversions—a paper which might form a base for attacking the subject in its more complex developments—and the following notes have been written on these lines. I do not know whether anything of the nature of an apology is owing in respect of a paper which naturally embodies much that will appear common-

place to those familiar with the subject; but I take it that the aim of the Faculty is not only to stimulate original research, but also to encourage any work which is likely to be useful to junior members.

I will seldom refer to any of the existing papers as I go along, as it has been suggested that a complete list of such papers, with an indication of their contents would be handy for reference; and as this is evidently preferable to a number of scattered references, I have compiled a bibliography of this kind, and added it as an appendix.

It is important at the outset to have some clear idea of the nature of a reversion. Let us take a simple case. Suppose a marriage settlement where the fund is settled on the wife for life, to go thereafter to the husband for life, and subsequently to be divided among the children. The wife would have a life interest in the fund, the husband would have a reversionary life interest to run from the death of his wife, and the children would have reversions expectant on the death of the parents. If the fund was to be divided among all the children who attained twenty-one, whether they survived to the actual date of division or not, such children would become entitled to absolute reversions—in other words, on attaining majority their interests would become vested, and their shares would be payable certainly when the reversions fell in, either to themselves, or to their heirs or assigns. It is not uncommon to make daughters' shares vest on marriage, if that takes place before twenty-one. The fund, on the other hand, may be left only to those children who survive the parents, and the reversions would then be contingent. If in the event of there being no children who attain a vested interest, the fund, let us say, is to revert to the settlor, he also has a reversion contingent on there being no such issue. It frequently happens that when a reversioner predeceases the life-tenant, his share goes to his children, who will usually take, to use the common technical phrase, *per stirpes*, their interest being in this case limited to the share of the deceased reversioner. If such children were to rank equally with the original reversioners, the division would be *per capita*.

These may be taken as typical contingencies—survivorship and issue—with no complications; but in practice many variations are encountered, for example, where a life-tenant has power to appoint a life interest to a present or future husband. An interesting case frequently met with in connection with land is where the reversioner has to retain, or it may be assume, a certain name and arms in order to succeed. Again, it is not uncommon to have an interest, as next of kin, in property belonging to a person who is a lunatic, and unable consequently to dispose of it by will.

In such a case there is the risk of nearer relatives turning up, also of the lunatic recovering reason, or of having made a will before becoming insane.

The advantages to an office of dealing in reversions are that a higher rate of interest can be obtained than that ruling among most other securities, the market being more restricted, and that life and other insurance business frequently accrues. On the other hand, there are certain risks peculiar to this class of business. Trustees may misappropriate or injudiciously invest funds; mistakes may be made in interpreting the will or other document under which the reversion arises; if a reversion is dependent on survivorship, the reversioner may disappear. In addition, reversionary transactions often entail a great deal of trouble, and a reasonable doubt may be raised whether the life business obtained is of a profitable class.

There are three methods of dealing with this class of securities: —(1) By absolute purchase, either of a whole reversion, or of a reversionary charge secured upon it; (2) by purchase, allowing a power of repurchase for a limited time; and (3) by way of ordinary mortgage. In regard to the last method, however, some special precaution has to be used, because, if the mortgage is called in question, the burden is laid on the lender to show that the transaction has been fair in every way. It has been thought, for example, that if the reversion was absolute, and a life policy made a condition of the transaction, the Court might set the transaction aside, also that intricacy of detail might be assumed beyond the comprehension of the borrower (*J.I.A.*, xxv, 57, 58, Godefroi). But the chief practical objection to lending is that the security does not produce income; and the reversioner being probably impecunious, there is considerable risk of the interest being unpaid, in which case the reversion would have to be realised, a process involving trouble. This objection would not hold if there was a large margin of value, so that interest and premiums might safely accumulate if necessary; or if the position of the borrower made it unlikely that the interest would fall into arrear, in a case, for instance, where the borrower had a good income which he could not charge.

Perhaps, on the whole, then, it is preferable to deal with reversions by way of purchase, either with or without a power of repurchase. It is to be noted, however, in passing, that some doubt has been thrown on the advisability of granting a power of repurchase. Mr. Turnbull, in his address to the Actuarial Society in 1900, cited an opinion that a power to redeem upon repayment of the purchase money with interest, makes the transaction a mortgage. (See *Transactions of the Actuarial Society of Edinburgh*, vol. iv, p. 528.) This is a purely legal point on which we can

hardly venture an opinion; but if this view is correct, it seems singular that the matter has not been brought into Court and decided definitely, as there must have been many cases where a reversion has fallen in shortly after the expiry of the power of repurchase, and where consequently the gain to the vendor to have the transaction considered as a mortgage would be very great. There is certainly some semblance to a mortgage when the conditions of repurchase are the repayment of the purchase money and premiums (if any) accumulated at compound interest; but if this is considered a stumblingblock, it may perhaps be avoided by naming a definite sum for each year or half-year of the term, leaving out any mention of interest. It is, of course, evident that the form of deed must be carefully drawn, so as not to leave the slightest ambiguity on the point.

Suppose we now consider an actual reversionary transaction brought before an office. At the outset it is advisable to have full particulars, at least as far as possible, as imperfect information may give rise to trouble in amending quotations and conditions. The following particulars should be furnished:—

(1) Name, residence, and occupation of the reversioner, with date of birth if the reversion is dependent on survivorship.
(2) Name, etc., of the tenant for life, with date of birth.
(3) Full particulars of fund or property.
(4) Full description of the reversioner's interest.
(5) Names, etc., of the trustees, with their powers of investment.
(6) Rate of duty.
(7) The existing incumbrances (if any) on the reversion.
(8) Name, etc., of vendor if other than the reversioner.

Particulars having been furnished, it may appear on the face of it that the reversion is not a suitable one to be dealt with. The transaction may be too small to be worth entertaining in the case of an office with large funds, or it may be too remote, for example, if the life-tenant is very young. Again, there may be some contingency involved too serious to be got over, such as a too great issue risk. Special contingencies, like the "name and arms" risk before alluded to, can sometimes be got over by effecting a policy with some insurance or guarantee office, but this is not always practicable.

Again, there may be some stumblingblock in connection with the trustees. When there is only one trustee, it is a question depending on circumstances whether it should not be made a condition of the transaction that another trustee be appointed,

in order to lessen the risk of misappropriation or mismanagement of the funds. When the reversioner himself is a trustee, it will be well that he should retire, because any loss sustained through any breach of trust, or other irregularity committed by the trustees, would be payable primarily out of his share. Sometimes trustees have powers of investment much wider than the ordinary statutory powers; but if the trustees are responsible persons, this is not such a serious objection as it might appear. If there is a loss on any investment, the trustees are not necessarily free from personal liability because such investment is within their powers. No matter what their powers are, trustees must exercise what, broadly speaking, may be called ordinary prudence. Frequently the reversioner desires to deal with his reversion without the trustees, or certain of them, being informed of it. This cannot be agreed to with safety, because enquiries, in regard to (among other things) any previous dealings with the reversion, have to be made of the trustees by the solicitors of the office when investigating the title; and moreover, when the transaction has been completed, notice has to be given to them of the purchaser's interest.

If everything is in order, the first consideration is to form an estimate of what the value of the property or fund is likely to be when the reversion falls in, an estimate which may safely be assumed to be, on the average, within the amount which will be actually realised. This is a matter often requiring a good deal of care and judgment. From the nature of the case, no definite rules can be laid down, but we may touch on some of the points to be kept in view in making such an estimate. If the trust estate is not real property, the fund will generally consist of trustee securities, but not necessarily; for, as already stated, trustees have sometimes wider powers than the statutory powers, and frequently, by a special clause in a will, they are empowered to retain investments which formed part of the estate at the testator's death, though these investments would otherwise be beyond their powers. Should the fund include mortgages of real property, it will probably be considered advisable to make a small arbitrary deduction from the amount, to allow for the risk of their being changed for less satisfactory mortgages, or other investments on which there might be a depreciation; and any quotation should be made subject to evidence being furnished that all mortgages are well secured. In regard to this evidence, any recent valuations of the mortgaged property should be produced; and if there are none, full particulars, including statements of rental and outgoings, should be furnished. Special care will have to be taken if a large proportion of the fund consists of a single mortgage. Stock Exchange securities of different kinds bulk largely in trust funds. To get a satisfactory estimate of

these, the prices for some years back should be considered, as well as the price of the day. In the case of high-class debenture or other permanent stocks, these prices, taken with the general tendency of the money market, should enable one to get a tolerably good estimate. In the cases of more fluctuating stocks, say ordinary stocks of railway and other companies, considerable discretion requires to be exercised; and in addition to the prices, the position and prospects of the individual company should be considered, and the estimate should allow a good margin. An important element in the case of many stocks is that of redemption, which may be at a fixed date, or at the option of a company or government after a certain date. If the redemption is likely to take place during the currency of the reversion, the stock cannot be taken higher than the redemption value, no matter what the present or past prices should be. Even if the redemption cannot take place within that time, it may be a strong factor affecting the price when the reversion falls in, and must be allowed for. Shares having a liability, if they form a large proportion of the fund, may render the transaction too speculative.

If the trust estate consists entirely, or almost entirely, of real property, a valuation on behalf of the purchaser should be insisted on, and care must be taken to allow fully for possible future depreciation. The necessity for this is apparent when we consider that the life-tenant has only a temporary interest in the property, and will naturally be loth to put out money in the repairs and other work necessary to keep up the value of the property, but from which he himself will obtain no benefit in the way of increased income. Neglect of this kind to a greater or less extent is therefore probable, and if continued for a number of years may result in a serious depreciation. For a similar reason, the value of timber on an estate should not be counted on, as it is very much at the mercy of the life-tenant, though the parties interested might bring an action against him for excessive cutting of timber (unless he holds the estate without "impeachment of waste", as it is called in England). Dealings with reversions of this class may very properly be carried out by way of reversionary charge. This is more equitable, as, on account of the large margin required for depreciation, a sale of the whole reversion can only be made on terms onerous to the vendor, and which might appear to be hardly creditable to an office. In the same way, it is also more equitable in the case of a reversionary life interest, to purchase a reversionary annuity of a fixed amount secured on the life interest; and this arrangement has the additional advantage of leaving a margin of income to the reversioner, who thus has an inducement to appear from time to time, otherwise the purchasers

might have trouble in proving his existence. It is to be noted that by the Sales of Reversions Act, 1867, no *bonâ fide* purchase, without fraud or unfair dealing, can be set aside merely on the ground of undervalue, but the Courts very properly show a disposition to take a strict view of what constitutes "unfair dealing".

The value of the fund having been estimated, a deduction must be made in respect of any duty payable, also for the expenses of realising the fund when the reversion falls in. These expenses may be found to run, generally speaking, from ½ to 2 per cent, depending partly on the nature and magnitude of the fund; but if the fund is in Court they are likely to be heavier.

Leaving out the question of formula at present, we will assume the calculations have been made, and that terms have been quoted and accepted. If the reversion is contingent on survivorship, the reversioner will have to be examined and passed as a good life, after which solicitors can be instructed to carry out the transaction. If the transaction is English and an absolute purchase, a formal contract should be drawn up and signed before anything further is done; and the sale dates from the signing, so that the purchaser is entitled to the reversion even if it falls in before the assignment deed is completed and the purchase money paid over. It may be interesting to mention that an actual case occurred not so very long ago, where a reversion fell in by the death of the life-tenant about fifteen minutes only after the contract had been signed, but such prompt falling-in is, alas! rare. The contract should give a complete description of the reversion, including all the ages involved (if not already proved), and full details of the trust funds. In Scotland it is not usual to have a contract of this complete nature, but it is thought that a formal letter accepting terms would be binding. This would have to be stamped as an agreement with a sixpenny stamp, if acted on.

On the completion of the transaction, distringases should be placed by the solicitors on any stocks registered in England. The effect of this is that notice is given of the interest of the office, and the office will get notice of any future dealing with the stocks; also, if there is a change of trustees, the new trustees will get notice of the interest of the office. The procedure in placing a distringas is that the company whose stock is in question (or in the case of Consols, India Stock, etc., the bank at which the register of stockholders is kept) is served with copies of a prescribed form of affidavit and notice previously filed in Court. The affidavit and notice run as follows:—

(AFFIDAVIT.)

IN THE HIGH COURT OF JUSTICE,
DIVISION.
IN THE MATTER OF*

* [Here state the nature of the document comprising the stock, and add the date and other particulars so far as known to the deponent sufficiently to identify the document.]

and

IN THE MATTER of the Act of Parliament, 5 Vict. cap 5.

I *A. B.*
of
make Oath and say, that according to the best of my knowledge, information, and belief, *C. D.* of
is beneficially interested in the stock comprised in the [Settlement, Will, etc.] above mentioned, which stock, according to the best of my knowledge and belief, now consists of the stock specified in the notice hereto annexed.

Sworn at
the day of 190 ,
Before me,

A Commissioner for Oaths.

This Affidavit is filed on behalf of *C. D.*,
whose address is [*State address for service*].

(NOTICE.)

To the [*Here add the name of the Company*].

Take Notice that the stock comprised in, and now subject to, the trusts of the [Settlement, Will, etc.] referred to in the affidavit to which this notice is annexed consists of the following (that is to say) [*Here specify the stock*].

This notice is intended to stop the transfer of the stock only, and not the receipt of dividends [*or*, the receipt of the dividends on the stock as well as the transfer of the stock].

(Signed)_____

If there are a large number of stocks, it will usually be sufficient to place distringases on one or two of the principal ones, otherwise the expenses incurred would be unduly heavy. The expense of

placing a distringas, when done by a solicitor, comes to something like £3, 3s.; but if there are several, the expense is not so heavy proportionately. There is no such process in Scotland; but the railway companies, and possibly other companies, will accept notice, without, however, taking any responsibility in the matter. If the fund is in Court in England, a Stop Order, which serves the same purpose as a distringas, will be obtained. The following is an example of a Stop Order:—

IN THE HIGH COURT OF JUSTICE,
CHANCERY DIVISION.

*Monday,* 1901.

In the Matter of the Estate of John Smith deceased

Between Plaintiff
and Defendant.

Upon the application by summons dated
190 , of *A. B.* and *C. D.*, and upon hearing the Solicitors for the applicants, and upon reading an Indenture dated
and made between the said *A. B.* and the said *C. D.* of the other part and the Certificate of the Fund,

It is ordered that no part of the share or interest of the said *A. B.*, of and in the capital of [stocks here described], in Court to the credit of '*Re* John Smith, deceased', be transferred or otherwise dealt with without notice to the said *C. D.*

In regard to other investments, such as mortgages and real property, there are no means by which a purchaser can prevent trustees from realising without his knowledge.

We left out the question of formulas in its natural sequence, and I would gladly leave it out altogether; for not only does it bring us into the stormy waters of controversy, but so much has been written on the subject, that it is difficult to discuss it without indulging in what will perhaps be considered vain repetitions. The matter is, however, both so important and so interesting that we cannot very well avoid touching upon it, even if we do little more than recall the main principles involved. Broadly speaking, the formulas for valuing reversionary interests belong to either of the two groups associated with the distinguished names of Mr. Jellicoe and Dr. Sprague, who, though not, as might be inferred, the originators of all the reversion formulas bearing their names, have been the outstanding exponents and advocates of them. Jellicoe's formulas are based on the principle that reversions should be regarded—to use his own words—'as investments, to be made not subject to any contingency whatever, but as securing to the holder of them, in any case, a given rate of interest so long as he retains them, and as reproducing the capital

invested when such interest shall cease to be paid or to accrue'. Thus his formulas are as follows, confining ourselves for simplicity to the ordinary absolute and contingent reversions:—

Absolute reversion $1-d_{(i)}a_{y(j)}$.
Contingent ,, $1-(P^1_{xy}+d_{(i)})a_{xy(j)}$.

Where $x$=age of reversioner, $y$=age of life-tenant, $P^1_{xy}$=office premium, and 'a' is either the office value of the annuity, or is calculated at a rate, $j$, which will give such value; $i$ is the rate of interest to be realised, and as the risk of loss by mortality fluctuations has been eliminated this may be taken at a comparatively low rate—at a rate, in fact, similar to that ruling among ordinary good investments. Jellicoe did not insist that the above arrangements should be actually carried out, but argued that they should at any rate be allowed for.

Dr. Sprague goes on different principles. His formulas are,

Absolute reversion $1-d_{(i)}a_{y(i)}$, or $A_{y(i)}$.
Contingent ,, $1-(P^1_{xy}+d_{(i)})a_{xy(i)}$.

The difference, it will be seen, is simply that the same rate of interest is used throughout. His arguments against the adoption of Jellicoe's formulas may be condensed into something like the following. To begin with, he objects to the idea of considering 'isolated' reversions, on the ground that in valuing a reversion, even in the case of a single transaction, one must be guided by the market value, and the market value is determined by the prices given by companies which do a sufficiently large business to be able to depend on averages, and which can calculate on the reversions falling in with more or less regularity. Such companies are under no necessity to make each individual reversion, as it were, self-supporting, and as a matter of fact they seldom or never do so by actually purchasing annuities. Jellicoe's formulas do not, therefore, square with the conditions of practice. Also, by his formulas there is a profit on the assumed grant of the annuities, and Dr. Sprague contends that it is much more satisfactory to include the whole profit under one aspect, taking this entirely in the shape of a higher rate of interest.

The difference in values brought out by these formulas will depend naturally on the rates of interest assumed in the calculations, and also to some extent on the ages. Jellicoe gave examples of transactions yielding 5 per cent on the investment, the annuity being taken at $3\frac{1}{2}$ per cent, and the values brought out on this basis are generally less than those obtained by Dr. Sprague's formulas taking 6 per cent, the Carlisle table being used in both cases; but the difference is frequently very slight. (For examples see *J.I.A.*, xxxi, 285.)

On Dr. Sprague's principles there seems to be no theoretical reason why contingent reversions should not be valued at net rates, using simply $A^{1}_{xy}$; but transactions on this basis would hardly be tempting. In the case of absolute reversions the only probable loss is of interest, and this loss is spread over a number of years, and is not likely to cause any real inconvenience, even if the office has only a small number of these transactions on its books. But with contingent reversions taken at net rates it would be different, for not only interest but principal may be lost, and any loss above the average, coming as it would do into the accounts of a single year, might be awkward.

It must be mentioned that Dr. Sprague's formula for a contingent reversion has been called in question, on the ground that it virtually commutes the premiums at the rate at which reversions are purchased, a rate which will always be considerably higher than the ordinary office rate of commutation. Mr. Warner contends (*J.I.A.*, xxxi, 282) that the vendor thus gets an advantage to which he is in no way entitled, and advocates the formula

$$1-d_{(i)}a_{xy(i)}-P^{1}_{xy}a_{xy(j)}$$

where $j$ is the ordinary office rate of commutation. This is certainly the correct formula if it is desired actually to commute the premium, or, what is the same thing, to allow for an office single premium, and at the same time realise interest at rate $i$ on the transaction. Looking to the fact, however, that such commutation is not actually necessary, but is simply a matter of convenience, or even of taste, for these transactions can easily be worked in the books of an office taking annual premiums, the soundness of Dr. Sprague's formula does not seem to be affected, though the advisability of employing it may be a matter of opinion.

The question of how to deal with reversions in the books of an office is of some importance, and though to go into it with any fulness is beyond the scope of this paper, a little consideration may not be out of place. There are the following three recognised methods of dealing with reversions purchased. (See Dr. Sprague's *Life Insurance Accounts*, p. 89.) For convenience we will assume that the reversions in question are absolute.

1. Take no credit for interest, but only for increase in value, either annually or at the periodical investigations; and when a reversion falls in, carry the difference between the ledger balance and the sum received on realisation, to profit and loss on reversions. This method has the advantage of simplicity, and also the advantage that rarely has a loss to be written off. A serious disadvantage, however, is that, no interest being taken credit for, the amount of interest appearing in the revenue account as

received will be smaller than it really is, and the rate of interest earned on the funds will appear correspondingly smaller.

2. Assume that the reversions yield a certain rate of interest, and add interest at that rate to the reversion account each year. The disadvantage of this is that the annual increase in value of reversions is small, and if during a period between the periodical investigations, few reversions fall in, then on the usual revaluation of the reversions at the investigation a sum will have to be written off which might appear unduly large. If the volume of business is large, there is little risk of this; but if small, the first method is preferable; as is also the case when the business is new, because life-tenants are to some extent select lives, and the rate of mortality for the first few years is likely to be light.

3. Provide interest by means of annuities set up in the books of the office. The amount available for the annuity fund will be the difference between the estimated sums in reversion and the purchase moneys. The amount of this annuity fund would be entered as a liability in the annual balance sheet, and would be added to the cost of the reversions on the asset side. This is in exact harmony with the formula $(1-da)$, the 1 forming the sum in reversion and the $da$ the annuity fund to provide interest on it, and this method appears, therefore, to be the more scientific, though the net results from year to year are the same as under 2. This will be apparent if we take a concrete example, and work out a small fund according to each method.

At age 95 the number living according to the Carlisle table is 30. Let us take 30 reversions to 100 each on these lives, the values being calculated to yield 4% by the same table. As $A_{95} =$ ·85868, the total value will be 2576·04; and accumulating interest according to method 2, the fund works out as follows. As A is the value of 1 payable at the end of the year of death, the amount of reversions falling in is deducted at the end of each year.

| Year. (1) | Ledger Balance at beginning of Year. (2) | Year's Interest on (2) at 4%. (3) | Amount of Reversions falling in. (4) | Annual reduction in Ledger Balance, (4) – (3). (5) |
|---|---|---|---|---|
| 1 | 2576·04 | 103·04 | 700 | 596·96 |
| 2 | 1979·08 | 79·16 | 500 | 420·84 |
| 3 | 1558·24 | 62·33 | 400 | 337·67 |
| 4 | 1220·57 | 48·82 | 300 | 251·18 |
| 5 | 969·39 | 38·78 | 200 | 161·22 |
| 6 | 808·17 | 32·33 | 200 | 167·67 |
| 7 | 640·50 | 25·62 | 200 | 174·38 |
| 8 | 466·12 | 18·64 | 200 | 181·36 |
| 9 | 284·76 | 11·39 | 200 | 188·61 |
| 10 | 96·15 | 3·85 | 100 | 96·15 |

Applying the same figures to method 3, we have at the outset as the sum in reversion 3,000, and, the value thereof being 2576·04, the difference 423·96 will be available for the annuity fund.

| Year. | Amount of Annuity Fund. | Interest at 4 % earned by Fund. | Interest at 4 % to be provided by Fund on sums in reversion. | (4)-(3) being amount payable out of fund each year. | Ledger Balance at beginning of year, being sums in reversion. | Amount of reversions falling in. | (6)—(2), net balance to credit of Office, identical with col. (2) above. |
|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 1 | 423·96 | 16·96 | 120 | 103·04 | 3000 | 700 | 2576·04 |
| 2 | 320·92 | 12·84 | 92 | 79·16 | 2300 | 500 | 1979·08 |
| 3 | 241·76 | 9·67 | 72 | 62·33 | 1800 | 400 | 1558·24 |
| 4 | 179·43 | 7·18 | 56 | 48·82 | 1400 | 300 | 1220·57 |
| 5 | 130·61 | 5·22 | 44 | 38·78 | 1100 | 200 | 969·39 |
| 6 | 91·83 | 3·67 | 36 | 32·33 | 900 | 200 | 808·17 |
| 7 | 59·50 | 2·38 | 28 | 25·62 | 700 | 200 | 640·50 |
| 8 | 33·88 | 1·36 | 20 | 18·64 | 500 | 200 | 466·12 |
| 9 | 15·24 | ·61 | 12 | 11·39 | 300 | 200 | 284·76 |
| 10 | 3·85 | ·15 | 4 | 3·85 | 100 | 100 | 96·15 |

## Rate of Mortality among Life-Tenants.

The importance of having some knowledge of the rate of mortality among life-tenants is evident, for, without such knowledge, the most suitable table by which to value reversions can only be a matter of conjecture. The Carlisle table has been largely used in the past, but it is not likely that a table based on general population statistics will show, with any degree of accuracy, what may be expected from a particular class of lives such as life-tenants. Nor are we at liberty to assume that tables based on the experience of a somewhat similar body of lives, such as annuitants, will be reliable, although probably they will give a much closer approximation to the truth. Up to the present, however, no investigation into the rate of mortality among life-tenants appears to have been made. As a beginning in this direction, I am in a position to give the result of a small investigation. The total number of lives was 300, but of these 74 were males who were excluded in the first instance, leaving 226 females, among whom were 113 deaths. The lives are therefore not numerically great, but on the other hand they are confined to the latter part of the mortality table, and they are homogeneous, as they all relate to one class of transactions, namely purchases without a power of repurchase, or where a power of repurchase has expired. The particulars were tabulated on cards, the dates of birth, entry, and exit being given in years and months, and with few exceptions the particulars were com-

plete. The facts being small, the deaths ran irregularly, and in order to get a $q_x$ that could conveniently be graduated, the deaths and exposed to risk were summed up in threes for each year of age. The resulting $q_x$ was graduated by the graphic method, which is peculiarly applicable to a case of this kind. The following table gives the results :—

| Age $x$ | Entrants. | Exposed to Risk. | | Deaths. | Graduated $q_x$ | Deaths expected according to graduated $q_x$ (3)×(5). | Difference between Actual and Expected Deaths (4)−(6). | Accumulated Difference. |
|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | | (4) | (5) | (6) | (7) | (8) |
| | | Years. | Months. | | | | | |
| 54 & under | 33 | 108 | 9 | 0 | ... | ... | ... | ... |
| 55 | 8 | 23 | 2 | 0 | ... | ... | ... | ... |
| 56 | 7 | 31 | 8 | 0 | ... | ... | ... | ... |
| 57 | 8 | 36 | 3 | 0 | ... | ... | ... | ... |
| 58 | 8 | 39 | 4 | 1 | ·016 | ·6 | ·4 | ·4 |
| 59 | 8 | 45 | 9 | 1 | ·017 | ·8 | ·2 | ·6 |
| 60 | 10 | 53 | 2 | 3 | ·018 | 1·0 | 2·0 | 2·6 |
| 61 | 8 | 57 | 1 | 1 | ·019 | 1·1 | − ·1 | 2·5 |
| 62 | 2 | 57 | 7 | 1 | ·021 | 1·2 | − ·2 | 2·3 |
| 63 | 9 | 60 | 9 | 0 | ·023 | 1·4 | −1·4 | ·9 |
| 64 | 9 | 66 | 8 | 1 | ·025 | 1·7 | − ·7 | ·2 |
| 65 | 10 | 70 | 7 | 1 | ·028 | 2·0 | −1·0 | − ·8 |
| 66 | 8 | 75 | 3 | 2 | ·031 | 2·3 | − ·3 | −1·1 |
| 67 | 8 | 82 | 7 | 3 | ·034 | 2·8 | ·2 | − ·9 |
| 68 | 10 | 87 | 0 | 4 | ·037 | 3·2 | ·8 | − ·1 |
| 69 | 9 | 91 | 1 | 3 | ·040 | 3·6 | − ·6 | − ·7 |
| 70 | 6 | 88 | 8 | 5 | ·043 | 3·8 | 1·2 | ·5 |
| 71 | 6 | 85 | 8 | 3 | ·046 | 3·9 | − ·9 | − ·4 |
| 72 | 8 | 84 | 11 | 5 | ·050 | 4·2 | ·8 | ·4 |
| 73 | 6 | 83 | 11 | 4 | ·054 | 4·5 | − ·5 | − ·1 |
| 74 | 6 | 78 | 4 | 6 | ·059 | 4·6 | 1·4 | 1·3 |
| 75 | 6 | 73 | 3 | 7 | ·063 | 4·6 | 2·4 | 3·7 |
| 76 | 10 | 71 | 6 | 2 | ·068 | 4·8 | −2·8 | ·9 |
| 77 | 3 | 71 | 7 | 6 | ·073 | 5·2 | ·8 | 1·7 |
| 78 | 4 | 65 | 5 | 5 | ·078 | 5·1 | − ·1 | 1·6 |
| 79 | 3 | 60 | 3 | 3 | ·083 | 5·0 | −2·0 | − ·4 |
| 80 | 2 | 56 | 5 | 4 | ·088 | 5·0 | −1·0 | −1·4 |
| 81 | 0 | 51 | 5 | 8 | ·093 | 4·8 | 3·2 | 1·8 |
| 82 | 1 | 38 | 8 | 3 | ·100 | 3·9 | − ·9 | ·9 |
| 83 | 1 | 36 | 4 | 3 | ·107 | 3·9 | − ·9 | 0·0 |
| 84 | 3 | 32 | 1 | 6 | ·116 | 3·7 | 2·3 | 2·3 |
| 85 | 1 | 24 | 1 | 0 | ·126 | 3·0 | −3·0 | − ·7 |
| 86 | 0 | 22 | 9 | 4 | ·138 | 3·2 | ·8 | ·1 |
| | | | | 95 | | 94·9 | | |
| 87 | 0 | 18 | 0 | 2 | ... | ... | ... | ... |
| 88 | 2 | 16 | 4 | 4 | ... | ... | ... | ... |
| 89 | 2 | 13 | 5 | 3 | ... | ... | ... | ... |
| 90 | 0 | 11 | 0 | 4 | ... | ... | ... | ... |
| 91 & over | 1 | 26 | 8 | 5 | ... | ... | ... | ... |
| Total, | 226 | 2097 | 4 | 113 | ... | ... | ... | ... |

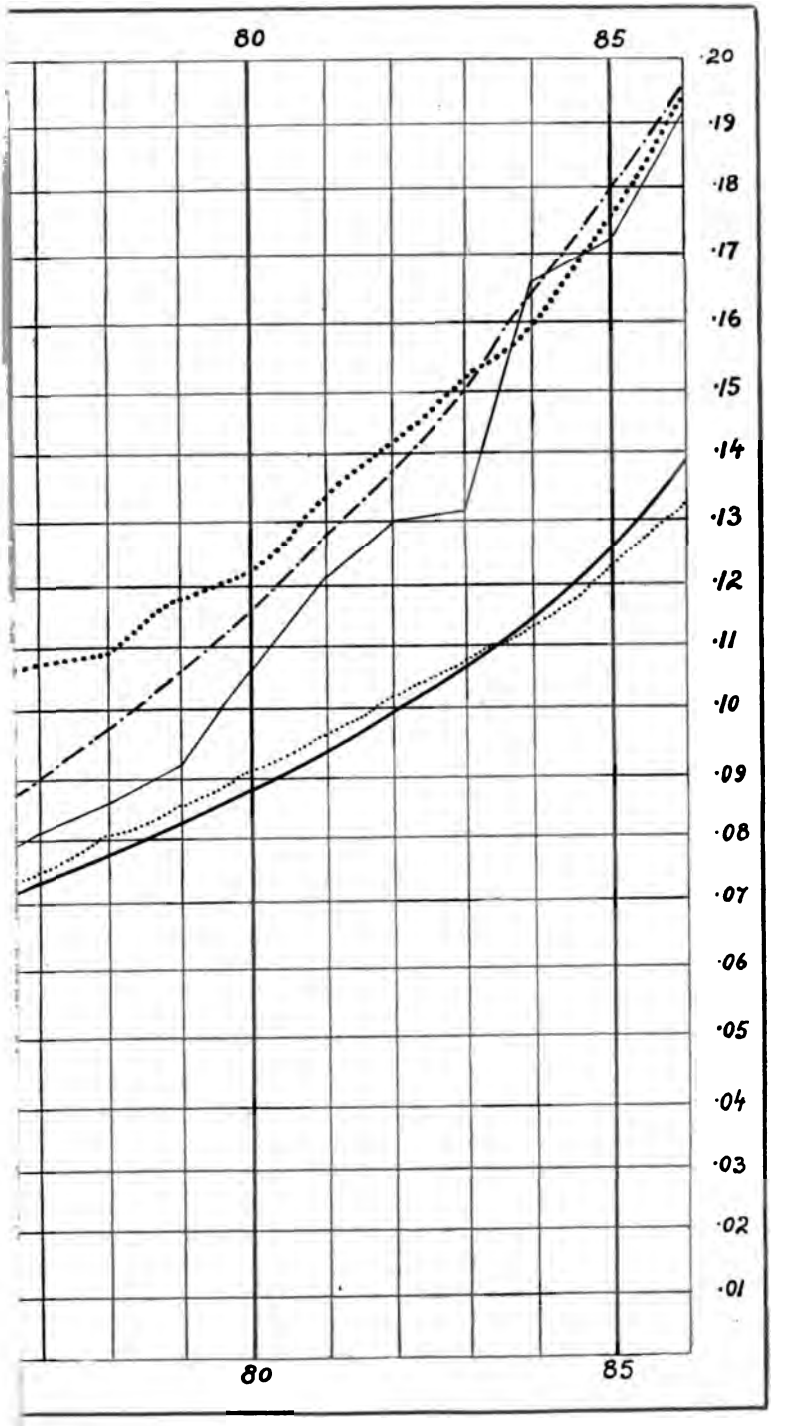
In order to see conveniently how these results compare with other tables, I have plotted down (see annexed diagram) the above rate of mortality, also the rates shown by the following tables:—(1) Carlisle; (2) Female Government Annuitants, 1883; (3) Unadjusted 'New' Female Annuitants, Institute and Faculty Experience; or for shortness, say Office Female Annuitants; and (4) Peerage Females, Dr. Thiele's adjustment (*J.I.A.*, xvi, 43). In regard to (2) and (3) the aggregate tables, taking no account of selection, were used. It will be seen that the rate of mortality of the Life-tenants runs fairly close to that of the Government Annuitants and the Office Annuitants, being less than the former and higher than the latter, from 58 to 71. But at age 71 the Government Annuitants rate increases more rapidly, and the curve representing it begins to diverge, and at age 75 the Office Annuitants rate increases in a similar way. Broadly speaking, then, it appears that the rate of mortality after 75 is distinctly lower among Life-tenants than among Annuitants. The difference is striking, unpleasantly so indeed from the point of view of the purchaser of reversions. Whether a larger body of facts would show different results can only be a matter of surmise for the present; but though a fuller investigation would doubtless bring out a varying degree of divergence, there seems no reason to believe that it would show an entirely different law. To get a really satisfactory comparison, selection should of course have been taken into account, but unfortunately the facts were too small to admit of this. Several attempts were made in this direction by grouping and otherwise, but without any satisfactory result. In fact, the influence of selection could not be said to be apparent, and it is not improbable that in reality it is comparatively slight, as reversioners are, as a rule, more influenced by a pressing impecuniosity than by a consideration of the chances of longevity. The proportion of new entrants, and consequently of select lives, to the exposed to risk in the case of the Life-tenants, is close enough to that of the Office Annuitants to make a comparison of these rates quite sound. As regards the Government Annuitants, the proportion of new entrants is smaller, but I satisfied myself that a similar proportion of new entrants would not materially affect its curve for the purposes of the foregoing comparison.

Some idea of the effects of these varying rates may be got from the following comparison of the actual deaths among the life-tenants, from ages 60 to 89 inclusive, with the expected deaths according to the different tables mentioned, quinquennial groups being taken. The figures speak for themselves.

| Ages. | Actual Deaths among Life-Tenants. | Corresponding Expected Deaths. | | | | |
|---|---|---|---|---|---|---|
| | | Graduated Life-Tenants. | Carlisle. | Government Annuitants. | Peerage Females. | Of Annu |
| 60-64 | 6 | 6 | 11 | 7 | 9 | |
| 65-69 | 13 | 14 | 18 | 14 | 17 | |
| 70-74 | 23 | 21 | 29 | 23 | 23 | 2 |
| 75-79 | 23 | 25 | 36 | 31 | 26 | 2 |
| 80-84 | 24 | 21 | 30 | 29 | 22 | 27 |
| 85 89 | 13 | 15 say | 19 | 20 | 13 | 19 |
| Total, | 102 | 102 | 143 | 124 | 110 | 112 |

Assuming it to be the case that the mortality above age 75 lighter, to a greater or lesser degree, among life-tenants th among annuitants, the question arises whether there is any reas able explanation of this difference. A possible explanation see to lie in the fact that life-tenants, generally speaking, belong to wealthier class than annuitants. At first sight there does n seem much in this, as annuitants are in fairly comfortable circum stances; and certainly the difference will have little, if any, pe ceptible effect in the earlier part of a table such as the abov But at the very old ages, when the hold on life becomes precariou the advantages of affluence in the way of general comfort an command of the best medical skill, combined frequently with th power of changing one's residence so as to get the best climatic conditions, may quite well result in lowering the rate of mortality in a marked degree. This view receives confirmation by the remarkably close agreement at the high ages of the rate of mortality among life-tenants to that of the peerage females. This agreement, I may perhaps remark, did not suggest the above explanation, which was formulated before the agreement was observed. If this similarity is not a mere coincidence—and there seems no reason why it should be—it shows that the low rate of mortality at the high ages is entirely, or almost entirely, a matter of class, and gives weight to the view previously suggested, that selection is not an important factor among life-tenants.

The number of male lives was too small to give any results from them alone, but they were mixed with the female lives, and the rate of mortality taken out. The effect was to increase the rate of mortality throughout, a result which agrees with the experience among annuitants.


80
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In conclusion, the practical result of the investigation is to confirm the doubts already expressed as to the suitability for valuing reversions of any table not based on observations of life-tenants; and the hope may be expressed that a fuller and more exhaustive investigation will be made at an early date to enable such a table to be prepared.

## APPENDIX.

Bibliography of Papers on Reversions, etc., that have appeared in the *Journal of the Institute of Actuaries*, arranged according to volume.

NOTE.—The papers with the titles printed in heavy type are those thought to be most useful to the student.

VOL. PAGE

I. 18* SANG. *On the Contrivances required to render Contingent Reversionary Interests Marketable Securities.*

Mr. Sang was asked to advise on six questions, and his opinions were published under the above heading. Five of the questions related to the contingent reversionary charge or reversionary annuity required for a present payment. The problems are discussed on the principles afterwards advocated by Dr. Sprague, but Mr. Sang's methods have in later years been improved. The sixth question was as to the terms on which a reversionary annuity transaction should be cancelled, but no definite solution is given.

II. 159 JELLICOE. **On the Contrivances required to render Contingent Reversionary Interests Marketable Securities.**

A clear demonstration of the formulas which now go by Jellicoe's name, and of the principles on which they are based. Formulas are given for (1) life annuity, (2) absolute reversion, (3) contingent reversion, and (4) reversionary annuity. A reversionary annuity example is worked out.

V. 239 TUCKER. *On the Methods pursued at the Present Day for estimating the value of Contingent Reversionary Interests.*

Jellicoe's principles are strongly advocated, and examples are worked out of a life interest and of a reversionary life interest. See also a letter by Mr. Tucker, p. 162.

| Vol. | Page. | |
|---|---|---|
| VI. | 61 | JELLICOE. *On the Valuation of Property held for Life and in Reversion; and on the due apportionment of it, when so held on the same life, between the Tenant for Life and the Remainder-man.* |

The question of apportionment takes up practically the whole paper. Rules are given for estimating the respective values when the apportionment is at the desire of (1) both the reversioner and the life-tenant, (2) the reversioner alone, and (3) the life-tenant alone. In the case of (1) the fund is divided proportionately to the market values of the respective interests. As regards (2) and (3) the solutions are on the footing that the life-tenant is provided for by means of a Government annuity in the one case, and the reversioner by means of a paid-up policy in the other.

In the same volume (p. 211) observations on the paper are made by Mr. Wilbraham.

VIII. 12 SPRAGUE. *On the Value of Policies of Assurance in connection with Life Interests.*

A short paper showing how the values of policies may be affected when taken in combination with life interests.

VIII. 310 JELLICOE. *On the Rationale of certain Actuarial Estimates.*

This is an enlargement of the previous paper on apportionment. The values of a life annuity are considered, assuming (1) that it is to be sold, and (2) that it is wanted, the value in the first case being given as the market value, and in the second the sum required to be paid for the annuity to an office or the Government. The cases of an absolute reversion, a contingent reversion, and a reversionary annuity are also dealt with.

VIII. 326 DAY. *On the Purchase of Life Assurance Policies as an Investment.*

A short paper, giving some examples of the value of a policy on Jellicoe's principles. The question of allowing for profits is touched on and examples given.

XIV. 417 **SPRAGUE. On the Valuation of Reversionary Life Interests.**

A classic paper on reversion formulas. The general principles to be observed in calculating the values of reversions are first discussed, the views held by Jellicoe being dissented from, and formulas are demonstrated for absolute and contingent reversions. Formulas for reversionary annuities are then elaborated, and some points in connection with the life policy required are touched on. Tables are appended showing the different values for immediate and reversionary annuities, brought out by the formulas advocated and by Jellicoe's.

VOL. PAGE.

XV. 126 SPRAGUE. *On the value of Reversionary Annuities payable half-yearly, quarterly, etc., according to the conditions which prevail in practice.*

Corrections making a reversionary annuity payable yearly into one payable half-yearly, etc., are dealt with at some length, and an approximate correction is given applicable to any complete annuity. The latter part of the paper is taken up with the demonstration of a reversionary annuity formula on the supposition that the deaths are distributed uniformly over each year of life. (See also vol. xvii, p. 171, Woolhouse.)

XV. 138 DAVIES. *On the Value of Reversionary Life Interests.*

A short note, interesting historically, as it must have been made at least forty years before publication in 1869.

XVI. 269 BADEN. *On the Equitable Apportionment of a Fund between the Life-Tenant and the Reversioner.*

Only one case is taken, namely, where both the life-tenant and the reversioner are desirous of apportionment. A division is advocated on the basis of net functions, taking the rate of interest realised by the fund at the time. The discussion following was very full and interesting. A note by Mr. Porter giving results by seven methods is appended, and further examples are given by Mr. Smyth in a letter (p. 386).

XVII. 229 **SPRAGUE. On Reversionary Life Interests as Securities for Loans.**

The subject is discussed in general terms, details of formula not being entered into. A large part is taken up with the question of the insurance required, and with different devices to reduce the cost of such insurance. An actual transaction is explained where a small policy at the outset was effected, with power to increase during the life-tenant's life. In reply to the discussion, an explanation is given in regard to base fees.

XVIII. 1 **BUNYON. On the Origin and Nature of some of those Limited and Contingent Interests in Property which are commonly submitted to Actuaries for valuation.**

The interests mentioned are those arising under Settlements of either real or personal estate. The different varieties met with under Marriage Settlements in regard to personal property are described, also estates tail in their various aspects. The valuing of reversionary interests is touched on, and some slight modifications are suggested to Jellicoe's formulas (which are otherwise considered satisfactory). The steps taken to secure that a base fee will be enlarged into a fee simple on the death of the tenant for life are discussed, also the doctrine of merger.

| Vol. | Page. | |
|---|---|---|
| XVIII. | 77 | SPRAGUE. *On the Apportionment, or Division by Mutual Consent, of a Fund between the Life-Tenant and the Reversioner.* |

Mr. Baden's views are advocated, and additional arguments given in their favour. Two new cases are discussed—(1) where a tenant for life and a tenant in tail combine to bar the entail; and (2) where two successive tenants for life have a joint power of appointment over an estate or a part thereof. Examples are worked out.

| | | |
|---|---|---|
| XVIII. | 100 | BUNYON. *On some Legal Incidents affecting the Duration of Base Fees in the Hands of Purchasers.* |

The title sufficiently explains the scope of the paper.

| | | |
|---|---|---|
| XIX. | 372 | SPRAGUE. *Solution of a Problem in Apportionment.* |

A special case of probably infrequent occurrence, where a father and son who have successive life interests in an estate, make an arrangement to borrow.

| | | |
|---|---|---|
| XX. | 385 | MACFADYEN. *On the carrying out of Reversionary Transactions by Life Assurance Companies.* |

A demonstration is given of a formula for an advance in consideration of a reversionary charge, the amount of which is small if the reversion falls in within $n$ years, but is increased if the reversion falls in thereafter. Absolute and contingent reversionary charges, also reversionary annuities, are treated on these lines. In the discussion the general opinion seemed to be that the method would not likely be largely adopted in practice.

| | | |
|---|---|---|
| XXV. | 55 | GODEFROI. **On Mortgages of Reversionary Interests.** |

A purely legal paper. The risks that a mortgagee runs are clearly pointed out; and the principles the Court goes upon, and the views it would likely take in certain circumstances, are discussed. The position of a person taking a transfer of a mortgage is also considered.

| | | |
|---|---|---|
| XXVII. | 107 | SPRAGUE. **Further Remarks on the Valuation of Reversionary Life Interests.** |

A certain case which was submitted to six Actuaries is cited. An advance was wanted on security of a reversionary life interest, and the amount of the reversionary annuity required and of the corresponding redemption money as quoted by the Actuaries is given, also the results brought out by four recognised formulas. The latter part of the paper is taken up with an explanation of a plan for purchasing reversions, allowing a power of repurchase for a limited time, and formulas are given for absolute and contingent reversions and reversionary annuities.

Mr. Sunderland in an interesting letter (p. 225) examines the difference between some of the formulas in the above paper.

| Vol. | Page. | |
|---|---|---|
| XXX. | 9 | NIGHTINGALE. *Formulas and Tables of Values for Life Interests and Reversions.* |

The chief characteristic of these formulas and tables are that complete annuities are allowed for. The adoption of Government annuity tables is advocated. The tables are given at different rates of interest, and with male and female lives separate, and include absolute and contingent reversions, life interests, and reversionary life interests.

XXX. 536 PATERSON. *Contingent Reversions, Reversionary Life Interests, and Life Interests in Possession.*

New formulas for contingent reversions and reversionary life interests are given, the premium being taken as payable only in the event of the life-tenant predeceasing the reversioner, the effect being to bring out larger values than by the usual formulas. The case of a life-interest with deferred premiums is also discussed. (See also vol. xxxi, Warner.)

XXXI. 280 **WARNER. Reversionary Interests contingent on survival of the Reversioner.**

A modification of Dr. Sprague's formulas is advocated, the new feature being that the premiums are commuted at office rate. Tables are given showing the results as brought out by the new formulas, also by those of Jellicoe and Dr. Sprague. A demonstration is given to show that Mr. Paterson's formulas (vol. xxx) practically result in valuing partly at net rates. Finally the following legal question is discussed:—'Whether or not a power of attorney granted on creation of a base fee by a tenant-in-tail in remainder, for the purpose of enabling the grantee, on the life-tenant's death, to enlarge the base fee into a fee simple, is irrevocable and valid.'

XXXV. 245 PATERSON. *Increasing Reversionary Charges.*

Formulas are given for charges increasing annually for a limited number of years. Tables are appended giving examples of such charges, with the corresponding level charges in the cases of absolute and contingent reversions, and reversionary life interests.

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## DISCUSSION.

THE PRESIDENT (Mr. G. M. Low) said that the paper divided itself into four sections. The first dealt with the practical treatment of reversions when offered for sale to an office; and, although that section might appear to be in great part elementary, yet it contained much information that would be useful to students of the subject, and even to those who could claim to have some experience in dealing with reversionary transactions. The second section dealt with the question of formulas; but, as Mr. Campbell, in reading his paper, had passed lightly over that section, they were perhaps absolved from entering in the discussion upon what had proved a fruitful field of controversy. Then the author gave them this most interesting, though limited, mortality experience, which to his mind was one of the most interesting features of the paper, because hitherto they had had no guide whatever as to the mortality prevailing among life-tenants. He confessed that it had seemed to him not unlikely that the Carlisle Table would give as near an approximation to the actual mortality as any other, because these lives did not, like the annuitants of the offices or of the Government, come and pit their valuation of their own lives against the seller of the annuity. But, as it had turned out, if this experience were an indication of the general experience of such lives, they seemed to be better even than the Government annuitants. He thought it was specially useful that these results—limited though they were—should have been made known; because it was calculated to drop a seed into the minds of those engaged in this business, and might some day lead to a more extended investigation; while even now it might serve to check the zeal of those who were bidding too high for reversions in the market—beyond the real value, as some of them thought. Lastly, there was appended to the paper a most interesting feature in the shape of a bibliography, giving references to and brief descriptions of the principal papers that had been written on this subject, which should make the paper especially valuable to the younger members of the profession.

Mr. D. DEUCHAR considered the paper alike creditable to the author and to the office in which he had received his training. With reference to the author's remarks as to the higher rate of interest on reversions as compared with the other investments of an office, he thought they must feel that it was doubtful if they had got any advantage whatever above ordinary investments in the rate of interest, when the light mortality was brought into account. He thought the subject of mortality was most important; and hoped that this paper would be the precursor of similar investigations by all the offices, and that thus the matter might be settled definitely. The author had certainly made out a *primâ facie* case for attributing exceptional longevity to life-tenants. A valuable suggestion had been made in the paper in regard to the terms of the contract where there was a power of repurchase. He thought Mr. Campbell was right in saying that the existing forms of contract had not been brought directly under the notice of the law courts; but he was able to say that, in a case connected with his own company, a transaction which originally took the form of a sale with right

to repurchase had been challenged. The case came before Mr. Justice Romer in the Chancery Court, and the deed was perused by the judge. He (Mr. Deuchar) was there as a witness. The original transaction was a sale, with right to repurchase within seven years. About a year after the sale, the seller came forward, saying he wanted more money and would like to sell out and out. The company agreed to give a further sum on his releasing his right to repurchase, and this alteration was carried through. But in the course of two or three years the life-tenant died, and the seller brought the matter before the Court of Chancery, with a view to having the deed annulled. He stated that he was an ignorant man, that he had understood that the first transaction was a mortgage, and that he had then altered it to a sale with right to redeem within seven years. But he was put under cross-examination for two hours, and the judge declared that he did not believe his statements, that the terms of the contract seemed perfectly clear, and that the company had in no way misled the seller or treated him unfairly. Although two witnesses for the company were called upon, they were only about five minutes each in the witness-box, without any cross-examination on the part of the opposing counsel; and the case resulted in a nonsuit, with costs to the company. He did not know that this case was important or conclusive, but had thought it right to mention it. He would only add that, when a future writer on this subject compiled a similar bibliography, he would certainly have to mention in heavy type the important paper that had just been brought before them.

Mr. G. Douglas welcomed the paper as a most practical one. In these days of advanced actuarial science, he thought they were just a little apt to give too much place to theoretical questions, forgetful of the fact that in their business theory and practice must go together, neither being allowed to predominate too much over the other. He agreed with the President that the greatest interest attached to the investigation into the mortality of life-tenants, and certainly the exceptionally low mortality shown was striking. It came to this, that offices buying reversions at prices based on the Carlisle Table at 5 per cent interest, would not make anything like that rate out of them, a large portion of the interest being absorbed in meeting the lighter mortality experienced. It occurred to him, however, that the extreme longevity of the life-tenants might be somewhat exaggerated, owing to the nature of the materials on which the figures were based. He gathered that Mr. Campbell had included only two classes of transactions—those where there was no power of repurchase, and those under which there was such a power, but which power was not exercised. If he had excluded all cases in which there was a power of repurchase and that power *was* exercised, the tendency must, he thought, be to lighten the mortality. It was no doubt unavoidable to omit transactions repurchased *in the lifetime* of the life-tenants, as of course all trace of the life would be lost after repurchase. But many of these repurchases would probably be occasioned by reason of the health of the life-tenant having greatly deteriorated; and in consequence of this the borrower would get together somehow the money required to redeem his reversion, or would go to another office and re-sell his reversion with an extended power of repurchase, and pay off the first office, which would lose all trace of the life thereafter. The repurchases on the *death* of the life-tenant had likewise to be omitted, and this might also tend to unduly lighten the mortality overhead. If they took an extreme case, and assumed that the author had dealt only with his second class of transactions—namely, those with a power of repurchase but which had not been exercised—the consequence would have been that he would have had no experience at all of the first five years from purchase, supposing five years to be the option period. He would have dealt alone with the lives

which had survived the first five years; but he (Mr. Douglas) thought it did not follow that because a light mortality was experienced after the first five years, the experience during these years would have been equally favourable. The option-of-repurchase system would doubtless in many cases be selected because the life-tenants were known to be indifferent lives, among which there was likely to be an undue proportion of very early deaths; but by bringing the experience under observation only after the first five years, the worst lives would escape observation altogether. He would like to know whether the author had considered that point—as to whether the material used would not have a tendency to make the mortality appear lighter than it really was. He did not know whether it was to have been expected that the life-tenant mortality should compare with that of the Carlisle Table. That table included both male and female lives, but those dealt with by Mr. Campbell were female lives only; and it was well known that, at the ages generally involved in reversionary transactions, female mortality was lighter than male. He had been under the impression that it was the practice of offices valuing by Dr. Sprague's formula and using the Carlisle Table, to write two or three years off the age when the life was a female. It would have been interesting if a comparison had been given on these lines. The expected deaths would have been considerably fewer; but how near they would have approached to the actual facts it was impossible to say. They could gather, however, from Mr. Campbell's diagram that, if the Carlisle curve were shifted three ages to the right, it would bring the two curves not very far from one another. Four ages to the right would be better; but if, as he thought, the life-tenant mortality had been slightly underestimated, a *three* years' deduction from the age under the Carlisle Table would give a mortality very nearly approximating to that of life-tenants. He thought it a pity that the male lives had not been dealt with in any way; for, although, owing to the paucity of facts, the construction of a mortality table was not practicable, it would have been of interest had a short table been given (similar to the one applicable to females) showing which table of mortality came nearest to giving the number of deaths experienced. They would then have had some indication of which was the better table to employ, while at present they had none. Mr. Jellicoe's formula allowed for the sex of the life-tenant, as naturally, in setting up an annuity, one would use the annuity-value applicable to the sex of the life involved. Although not directly connected with the matter of reversions, he thought Mr. Campbell's figures would be interesting in another direction—namely, in disentail compensation cases, in which the interests to be valued were practically reversions expectant on the death of the life-tenant and the successive heirs; and he thought the lives of heirs of entail would at any rate be no worse than those of life-tenants in ordinary reversions. He understood that it was the practice of some actuaries, in valuing these interests, to use the Carlisle Table without making any allowance for the sex of the lives involved; but with the author's figures before them, it would be seen that account should be taken of the sex of the preceding heirs in calculating the compensations to be paid to the heirs following. A mortgage of a reversion had one advantage over a purchase of the whole reversion, as there was always a margin to come and go upon; but, if they knew the borrower to be impecunious, they should not lend at all. In view of that, he suggested that the proposal form should have an additional question as to the other means and estate of the borrower. Another useful question would be as to whether the transaction had been or was being offered to any other company. Many reversions were simply hawked about from one office to another, the unfavourable points being carefully kept in the background, in the hope that some office might ultimately be found willing to undertake what others had declined. In other cases, two or

more offices would often be applied to at the same time, simply with the view of pitting one office against another in the matter of price. When a power of repurchase was given, it was best in the negotiations not to talk of an advance, but of the purchase of the reversion or of a charge thereon; and certainly in the deed it would be advisable to state certain redemption prices for each year or half-year, without mentioning how these were made up, and saying nothing about interest or premiums. With regard to the "name and arms" risk, he might mention that, when a lending office covers a risk of that kind by means of a policy, it usually takes from the borrower his own personal covenant that he will not fail to retain or assume the name and arms in question. A man would naturally not throw away anything unless it was to be to his pecuniary advantage in some other way. For example, a borrower might marry a wealthy lady with a name and arms of her own, which he would have to assume; and if his personal covenant had been given, he could be called upon to make good the loss on the "name and arms" policy. In estimating the value of a fund in reversion, the question arose as to what should be done when the fund included investments which were not open to the office to purchase in the usual way. Were they to be left out of account altogether? or was the office justified in lending on security of them? He supposed the proper course would be to buy only a reversionary charge of such an amount as would be covered by the investments that the office was empowered to lend upon. Not only had the costs of realisation to be allowed for; but, in the case of a purchase, the costs of purchase, including charge for any distringases, had also to be deducted from the price before quotation, as in ordinary course the vendor would naturally expect the sum quoted to be the net sum which he would receive. In the case of an advance on the other hand, the loan would be quoted subject to payment of the expenses thereout.

Mr. T. Wallace said that among the various important points dealt with in the paper, the author had drawn attention to the necessity of scrutinising carefully the powers of investment possessed by trustees. Sometimes cases were met with where the powers were very wide—so wide that the trustees might take up securities which, in ordinary circumstances, assurance companies would not look at. In such cases, the character, the position, and the circumstances of all the parties interested—life-tenant, reversioner, and trustees—formed important elements for consideration; but, however favourable these might be, there was always considerable risk. Instances were not unknown where trustees had sold first-class investments, and, for the sake of increasing the income of the life-tenant, entered into very speculative securities, with the result that there had been a loss on realisation. It might be that in some cases the trustee would be liable personally, but in others he was not; and, he thought, it would be an undesirable and unpleasant position for an office to find itself in court, prosecuting a trustee for the amount of loss sustained. He had heard it suggested that an undertaking granted by the trustees, not to change the investments without the consent of the purchaser, would be an effectual safeguard. Such an undertaking would be legally binding upon the trustees who granted it, but it would not bind their successors, and would not be effectual. He considered that all cases where wide powers of investment had been given to trustees, were speculative and involved a large amount of risk, and should be avoided. If they did go into them, they should make very liberal allowance for possible—nay probable—loss. In a contingent reversion, where a life assurance was required, the medical examination of the reversioner should take place at an early stage; for the negotiations often extended over a long period, giving a lot of trouble and causing considerable expense, which were altogether wasted if it afterwards turned out that the reversioner was not an insurable life. Certainly the

examination should be made before any expense was incurred. With regard to the rate of mortality among life-tenants, it would be a matter of opinion as to whether the data were sufficient in magnitude to afford trustworthy results, but he thought the author had shown that the life-tenants completely eclipsed annuitants in their proverbial tenacity of life.

Mr. J. WATT, W.S., said that some of the technicalities dealt with in the paper were entirely English, such as the doctrine, "once a mortgage, always a mortgage.", which had no application in Scotland. The only rule he would lay down was this, that the office should never deal with the reversioner personally, but through an independent solicitor of his own choosing; and that it should insist that the reversioner should get the document before it had to be signed—if possible, a day or two before—so that there could be no doubt of his having at least an opportunity of knowing what was in it. As to the mortality experience, commenting on one of the points raised by Mr. Douglas, he said that Mr. Campbell had probably wholly excluded the first five years in repurchase cases, as he had told them he had taken no cases where a power of repurchase was exercised. On the question of bookkeeping, it appeared to him that the objection which had been taken that, unless an interest account were set up, no interest would be carried into the revenue account, and the average return on investments would thus be misstated, might easily be overcome in two ways, either by taking the increment in value and treating it as interest, or by excluding the amount invested in reversions from the amount on which the calculation is based. The cardinal point was that, whatever the method of bookkeeping, the ultimate result to the company was the same.

Mr. A. E. SPRAGUE said that, as a matter of fact, money laid out in the purchase of an absolute reversion produced no annual interest. It was sunk for an indefinite time, but increased slowly in value from year to year until the death of the life-tenant, when it was returned with (generally) a large addition. He was of opinion that the published accounts of an office should be in strict accordance with the actual facts of their business; and that, of the three recognised methods of dealing with reversions in their books, the first was the only one in such accordance, and that it therefore was the proper method to adopt. By method 2, credit was taken for interest which had not been received, was not due, and had not even accrued. Method 3 was open to the same objection, and to the more serious charge that it stated part of the assets in the balance-sheet at a figure larger than they would realise, assuming a fair market price to have been given for the reversions. Taking the figures given in the paper, and assuming that they represented market values, if method 3 were employed, the 30 reversions on the lives of persons aged 95 would appear in the balance-sheet as worth £3000, while, as a matter of fact, they would only fetch £2576 if sold; so that their value would be overstated to the extent of £424. He was unable to see how such a proceeding could be justified; and was of opinion that, if any annuity fund were set up in the books, the proper way of stating the account was to enter the £3000 in the balance-sheet by means of two separate items—say, "Reversions Purchased, £2576", and "Annuities set up in connection with reversions, £424". Further, it was impossible to foretell what sum would be received when the reversion fell in: at the best they could only make an estimate, and any difference between the estimate and the amount realised must be treated according to method 1. The object of methods 2 and 3 was to include the profit on reversions, or the greater part of it, in the item "Interest and Dividends" in the revenue account, in order that the public might be able to calculate from that item and the amount of the funds at the beginning and end of the year, the rate of interest earned on the funds. The usual

method of calculating the rate from these three items was to use Hardy's formula; but he ventured to think that, in the case of an office transacting a large business in reversions, the result given by the application of that formula had practically no meaning. What the public were concerned with, was the rate at which the funds were being improved; and, as this improvement consisted partly of interest and partly of profit on reversions and other investments, it could not generally be ascertained from the items taken into account in Hardy's formula. In calculating the rate of interest, therefore, account should be taken of any profit or loss on revaluation or realisation of reversions and other investments, as well as of the interest and dividends. It was true that the profit on reversions would probably fluctuate from year to year, but the same was the case with other investments; so that, if the rate of interest were calculated from year to year, it would as a rule show considerable fluctuations. If, however, the transactions of a whole quinquennium were considered, these fluctuations would in a manner be averaged, and the true rate at which the funds were improved during that quinquennium could be calculated. Passing to the table of rates of mortality among life-tenants, he had attempted a monetary comparison of it with the Carlisle Table. In order to do this, it was necessary to know the rates of mortality for ages above 86, and for those ages he had taken the values given by Dr. Thiele in his adjustment of the Peerage Females Experience, as they joined on fairly smoothly at age 86 to the rates prevailing among the life-tenants. From these data he had constructed a mortality table, and had calculated the values of $A_x$ at 5, 4, and 2½ per cent. The results were interesting, and showed a great difference from the corresponding Carlisle values. At age 60 the value of a reversion to £1000 by the Carlisle Table and 5 per cent interest, was about £527; while by the life-tenants table and the same rate of interest, it was only about £458. At age 70 the figures were £651 and £593 respectively; and at age 80 they were £761 and £713. These were sufficient to show that, if they purchased reversions on a Carlisle basis, they would realise a much smaller rate of interest than was anticipated in their calculations. After making a note of comparisons at different ages, he had arrived at the conclusion that the difference between the anticipated and realised rates would, speaking generally, be about 1 per cent. More exactly, the values of $A_x$ by the life-tenants table and 5 per cent interest were about the same as the Carlisle 6¼ per cent; the life-tenants 4 per cent were very nearly the same as Carlisle 5 per cent; and the life-tenants 2½ per cent nearly the same as Carlisle 3½ per cent. The life-tenants 2½ per cent values also corresponded roughly with those deducted from the Government Female Annuitants (select) 2¾ per cent.

Mr. J. Nicoll said that the author of the paper had not referred to the case that sometimes arose, of people offering reversions for sale simply to get a valuation of them, which had to be guarded against. In addition to the papers mentioned by Mr. Campbell, he would point out that in Dr. Sprague's presidential address to the Institute of Actuaries in 1883 (*J. I. A.*, xxiv, 229), the practical working of reversions was pretty fully gone into. Dr. Sprague had expressed the opinion that, in lending upon a life interest, the fund, in view of possible changes in it, and the reduction of the income thereon, should be assumed to be already invested in consols. He had further shown that where the interest on a mortgage of an absolute reversion fell into arrear, three courses were open to the lender —interest might be allowed to accumulate; or he might foreclose, which was not desirable, as it was a tedious and expensive process; or he might sell the reversion. Dr. Sprague had considered the most satisfactory way of dealing with contingent reversions to be by way of partial sale. Some other points had not been dealt with by Mr. Campbell; for instance, where

a life-tenant had a power of appointment, an appointment of a share of the fund should at once be made to the reversioner, in order to give a good security. Again, in regard to "tacking": if a lender advances money upon first mortgage and later on upon a third mortgage, he may under certain circumstances tack the third on to the first, to the detriment of the second mortgagee. Mr. Campbell had said it would not be safe to comply with the request of a borrower who did not wish the trustees to know of his loan; but later on (page 87) had mentioned that, in regard to mortgages and real property, there were no means by which a purchaser can prevent trustees from realising without his knowledge. Were not these statements rather inconsistent? In an address delivered to the Insurance Institute of Ireland on 2 October 1896, Mr. Deuchar had pointed out that, in the case of reversions to real estate in England, trustees are not bound to take cognisance of notice in regard to a loan on or purchase of a reversion to the estate, and such notice has no legal effect. He (Mr. Nicoll) understood that a distringas would simply prevent the Government or a company from transferring stock or paying dividends on it for a period of eight days, during which application might be made to the court for a restraining order or an injunction to prevent the registered holder dealing with the stock. Then a distringas ordinarily gives priority to the person obtaining it; though, if a second incumbrancer at the time of taking his security has had notice of the prior incumbrance, he gains no priority by obtaining a stop order. But, if he had no such notice at the outset, a stop order obtained after he gets notice of the prior incumbrance would give him priority over a charge earlier in date for which no notice had been given or stop order obtained. With regard to the very interesting investigation Mr. Campbell had made into the mortality of life-tenants, the numbers were perhaps rather too small to form a reliable guide in practice; but it was to be hoped that the investigation might at least suggest to the offices the desirability of combining their experience in this respect, as they had done in regard to assurances and annuities. He would venture to suggest another reason why life-tenants may have a better experience than annuitants. The life-tenant, besides having what every annuitant has—the inducement to live supplied by the desire to draw as long as possible his or her life-interest—has sometimes also, through family misunderstandings or other circumstances, the not very charitable desire to keep his or her successor out of the income as long as he or she can, more especially if the reversioner be a spendthrift. If it be granted that the desire to continue in the enjoyment of the income has an influence in increasing the longevity of the ordinary annuitant, it may not be unreasonable to suppose that this additional motive has the effect of still further prolonging life in the case of life-tenants.

Mr. W. A. Robertson said there were one or two points in connection with the paper, upon which he would like to have an expression of opinion from Mr. Campbell. The first was as to the nature of his data. General conclusions based upon the experience of individual offices were sometimes unsatisfactory. He believed that the feature of the office from which the data was derived was the "five years' system", the effect of which was to give to the vendors of reversionary interests any undue profit from mortality which might be expected to arise from the early falling-in of their interests. This might have the effect of leaving a much smaller and more select class, meeting with such rates of mortality as would hardly be confirmed by the experience of another office which did not adopt the system referred to. Again, is not selection, after all, a possible explanation of the difference between the rates of mortality of the life-tenants and of the Office Annuitants Experience? When an aggregate was substituted for a select comparison, he thought that, among other things, it was

assumed that the proportion of the numbers entering at the various ages was approximately the same in both experiences. But by referring to Mr. Campbell's data, it would be seen that no fewer than 39 out of 226 entrants entered at ages over 75 (about 17 per cent of the whole); and they were distributed fairly well. He doubted whether the ratio of entrants at these ages would be so great in the Office Annuitants Experience. He would like to refer to one point in connection with the Carlisle Table, as he had not seen it mentioned elsewhere. It would be remembered that the data that formed that experience were the numbers living in January 1780 and December 1787, a period of eight years, and the deaths during the nine years 1779-1787. In order to get a comparison, Milne added the two enumerations together and multiplied by four, and deducted one-ninth from the deaths. But the number living at the beginning of the period was 7677, and at the end of the period it was (or was assumed to be) 8677, indicating an increasing population. Hence the deaths in 1779, which Milne desired to deduct, must have been the fewest in the group; and in deducting one-ninth of the whole, he deducted too many, and accordingly underestimated the rates of mortality all through.

Mr. Campbell, in reply, said that he thought a little consideration would show that it would have been wrong to include the experience of the five years during which a power of repurchase could be exercised; because, in calculating the value of the reversion, the question of mortality does not enter into the first five years at all. The price given for the reversion subject to a power of repurchase, was got by taking the value for the age five years older, and discounting that at interest only. It was to be expected that when the reversion became the absolute property of the office, the life would be a good life, and consequently a low rate of mortality would be brought out; but that was the rate that had to be reckoned with in reversionary transactions. He did not quite appreciate the difficulty raised by Mr. Nicoll. There was nothing inconsistent.

Mr. Nicoll said that his point was, what was the use of giving notice to the trustees at all?

Mr. Campbell replied that, although they might change the investments as they pleased, they would be prevented from paying to beneficiaries. Some of the other points mentioned by Mr. Nicoll as not being touched upon in the paper, had occurred to him; but to deal with all such questions would have necessitated a full treatise on the subject. He regretted if he had given the impression that there were no practical papers on reversions in the *Journal* of the Institute. That was not so; but, taken all round, he thought the tendency in recent years had been to produce papers dealing with theory and formulas rather than with practical points.

The President, in moving a hearty vote of thanks to Mr. Campbell for his paper, hoped that, as he had admitted there were various questions not exhausted on this occasion, the Faculty might look forward to a second paper by him dealing with the subject.

Mr. Campbell writes to us as follows:—

I shall be obliged if you will allow me to refer to a point which arose in the discussion, but with which it was impossible to deal without referring to the figures.

Mr. Robertson suggested that the comparison between the rates of mortality among the female Life-tenants and the female Office Annuitants may be erroneous, by reason of a possible difference in the proportion of new entrants, and consequently of select lives. His suggested comparison of the number of entrants after 75 with the total number of entrants would, however, not be suitable, if for no other reason than that the Annuitants begin to enter at a much earlier age than the Life-tenants. As stated on

page 93, I had satisfied myself that the comparison of the two-fo[illegible] quite sound; and the following figures showing the ratio of the [illegible] the exposed to risk for quinquennial groups will, I think, ju[illegible] view :—

| Ages. | Percentage of New Entrants to Exposed to Risk. | |
|---|---|---|
| | Office Annuitants. | Life-tenants. |
| 60—64 | 16·3 | 12·9 |
| 65—69 | 11·5 | 11·1 |
| 70—74 | 7·7 | 7·6 |
| 75—79 | 5·1 | 7·6 |
| 80—84 | 3·3 | 3·3 |

No. 5 Vol. I.

# TRANSACTIONS

OF THE

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CONSTITUTED 1856.

INCORPORATED 1868.

On Premiums deduced from the Mortality Experience (1863-1893) of British Life Offices —By James Chatham, F.F.A., F.I.A.

With Abstract of Discussion thereon.

Correspondence.

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# TRANSACTIONS

OF THE

# FACULTY OF ACTUARIES.

No. 5.—Vol. I.

**On Premiums deduced from the Mortality Experience (1863-1893) of British Life Offices — By James Chatham, F.F.A., F.I.A.**

**With Abstract of Discussion thereon.**

**Correspondence.**

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TRANSACTIONS

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FACULTY OF ACTUARIES.

*On Premiums deduced from the Mortality Experience* (1863-1893) *of British Life Offices. By* JAMES CHATHAM, F.F.A., F.I.A., *Secretary and Actuary to the Scottish Life Assurance Company.*

[Read before the Faculty, 10 March 1902.]

THE distinguishing feature of the New Experience which differentiates it from all others, is the division of the observations into various classes of assurance; and when our President did me the honour of asking me to contribute a paper, I thought I could not do better than invite your attention to the premiums which, according to that experience, ought to be charged for these classes, as it is of the greatest importance to insurance companies generally that the risks which they undertake should be at adequate rates. Some of us have suspected, others have known, that the rate of mortality varied according to the class of assurance, and that one table of mortality applied to all was not satisfactory; but it is now possible for the first time to arrive at definite conclusions on the subject, and to determine in many cases what the difference between them really is.

A considerable time must necessarily elapse before the graduated values are available, if indeed some of the classes are ever graduated at all; and I have therefore based my calculations upon the unadjusted facts, which will, I think, always give them a value of their own, as graduation not infrequently modifies, if it does not altogether remove, some feature of the observations. By the basis adopted all the results of individual years are gathered up in one value, each with its proper weight assigned to it; and, if

need be, these values can be adjusted. This method has the weight of at least one great authority to recommend it. Professor De Morgan in his Essay on Probabilities says: "Until observa-
" tions of human mortality become more extensive and correct,
" I prefer the tables of mean duration [expectation of life] to all
" others. The events of single years are subject to considerable
" error, and generally present such varieties of fluctuation, that
" it has become usual to take some arbitrary and purely hypo-
" thetical mode of introducing regularity. This practice cannot
" be too strongly condemned, since the tables thereby lose some
" of their value as representations of physical facts, without any
" advantage ultimately gained. For if by using the raw result
" of experiments, tables of annuities were rendered unequal and
" irregular, it would be as easy, and much more safe, to apply the
" arbitrary method of correction to the money results themselves,
" than to introduce it at a previous stage of the process."

In several of the classes the experience is so large that there is little or no necessity for graduation. This is particularly the case in the whole life classes for males, and there I have used the unadjusted facts in the select tables. These tables show for each age at entry the number assuring, and what became of them in the first and each subsequent year of assurance—whether they died, withdrew, or were existing at the termination of the observations. No exception can be taken to this method of procedure, as duplicate lives were eliminated for each age at entry, and the lives traced from the period when they came under observation until they passed out of it.

In some classes, however, the observations are not sufficiently numerous to permit of this being done, and in these cases I have joined on the select tables at the end of ten years to the aggregate table excluding the first ten years' experience. An aggregate table is one in which all the lives of the same age are grouped together, irrespective of the length of time for which they have been assured; but in the table used, the experience of the first ten years after entry is omitted, and from it is obtained what is regarded as the "ultimate" rate of mortality. This involves the assumption that selection is exhausted after ten years from entry, but otherwise the method is correct, as duplicate lives were generally eliminated in forming the aggregate table.

In other classes the numbers during the first ten years in the select tables are so small that it has been considered desirable to group the two adjacent ages below and above each quinquennial age at entry in order to obtain more regular results. This involves the further assumption that the duplicate lives at these adjacent ages are subject to the same rate of mortality as the other lives, which is probably not the case; but as the period is short, and

the number of duplicate lives in each group cannot be large, the error will probably be small. As the average age of the group differs in most cases from the central age at entry, owing to the number of entrants varying at each age, a correction by first differences has been made to allow for it. This applies of course only to the select portion.

These three methods may be designated "True Select", "Select and Ultimate", and "Grouped Select and Ultimate" respectively. For one of the classes—whole life assurances, males—the premiums have been calculated by each of these three methods, in order that the differences between them may be noted. In the endowment assurance class the premiums have been calculated by the second and third methods, and in the remaining classes by the third method only.

The classes of assured lives dealt with are as follows:—

| | |
|---|---|
| (1) Whole life, males— With profits. Without profits. (2) Whole life, females— With profits. Without profits. | With premiums payable uniformly throughout life. |
| (3) Ascending scale, with and without profits, males. | |
| (4) Limited payment, ,, ,, | |
| (5) Endowment assurances, ,, ,, | |
| (6) Contingent-survivorship, without profits, ,, | |
| (7) Temporary assurances, ,, ,, | |

In the first three classes the quinquennial ages at entry have been taken as 22, 27, 32 . . ., as in one or two cases the rate of mortality is given in the experience for groups of which these are the central ages; while in the other classes the ages have been taken as 20, 25, 30 . . ., as in two of them these are the central ages given in the experience, and besides, these ages are more convenient in the case of endowment assurance premiums. It has not been possible in the last two classes to make the correction for age by first differences, referred to above, as the number of entrants at each age is not given. It should be mentioned that in ascending scale, limited payment, contingent survivorship, and temporary assurances, duplicate lives were not eliminated in forming the aggregate tables.

The rate of interest I have adopted in all the calculations is $3\frac{1}{2}$ per cent, as I think that is a fair rate to employ for the calculation of premiums. The rates of premium are tabulated per cent throughout the paper. When stated in decimals, they are net premiums, and when in £ s. d., they are office premiums, except in Tables 2 and 5.

## Whole Life Assurances—Males.

The observations in the *With Profits* section relate only to policies with premiums payable uniformly throughout life, and are very numerous, the years of life numbering 8,647,246, and the deaths 177,279. As stated above, the net annual premiums have been calculated by each of the three methods, and the results are given in the following table:—

**With Profits.**

Table 1.—100 $\pi_x$, *Whole Life With Profits—Males.*

| Age. | True Select. | Select and Ultimate. | Grouped Select and Ultimate. |
|---|---|---|---|
| 22 | 1·349 | 1·354 | 1·350 |
| 27 | 1·517 | 1·542 | 1·530 |
| 32 | 1·761 | 1·786 | 1·792 |
| 37 | 2·072 | 2·085 | 2·099 |
| 42 | 2·509 | 2·542 | 2·512 |
| 47 | 3·056 | 3·051 | 3·056 |
| 52 | 3·712 | 3·765 | 3·774 |
| 57 | 4·564 | 4·536 | 4·627 |
| 62 | 5·689 | 5·876 | 5·893 |
| 67 | 7·554 | 7·727 | 7·555 |

These results agree, as a rule, very closely with each other, and this shows that, when the observations are numerous, it is to a great extent immaterial which method is used. It will be seen, however, that in every case, except two, the True Select premiums are less than the Select and Ultimate, showing very clearly that selection is not exhausted after ten years from entry; but the difference is small—on the average about 1½ per cent. It will also be seen that in every case, except one where they are equal, the True Select premiums are less than the Grouped Select and Ultimate. This is in part attributable to the same cause, namely, that selection is not exhausted after ten years from entry, but other two factors require to be taken into account. On the one hand, the inclusion of duplicate lives in the groups during the first ten years after entry probably reduces the rate of mortality, as these lives are likely to be better, on the average, than the other lives, from the fact of their having passed further medical examinations. On the other hand, it is well known that the rate of mortality more nearly increases in geometrical than arithmetical progression; and consequently the grouping of the lives at different ages during the ten years after entry, has a tendency to increase the rate of mortality, notwithstanding the correction applied on account of age. These two forces therefore operate in opposite directions. The figures seem to indicate that the former is the more powerful at young ages, where duplicate lives are numerous and the rate

of mortality increases very slowly; while the latter has more effect at old ages, where duplicate lives are few, and the rate of mortality increases more rapidly. On the whole, the Grouped Select and Ultimate premiums agree more closely with the True Select premiums than do the Select and Ultimate.

We may now see how the rates here brought out compare with those charged by offices for this class of business. In the following table I give the highest, average, and lowest office premiums, along with the True Select net premium and the difference between it and the average office premium.

TABLE 2.—*Net and Office Premiums, Whole Life With Profits—Males.*

| Age. | True Select Net Premium. | Office Premiums—With Profits. | | | Difference, (4)—(2). |
|---|---|---|---|---|---|
| | | Highest. | Average. | Lowest. | |
| (1) | (2) | (3) | (4) | (5) | (6) |
| 22 | £1 7 0 | £2 6 2 | £2 0 6 | £1 16 3 | £0 13 6 |
| 27 | 1 10 4 | 2 11 1 | 2 5 5 | 2 1 0 | 0 15 1 |
| 32 | 1 15 3 | 2 17 4 | 2 11 6 | 2 6 10 | 0 16 3 |
| 37 | 2 1 5 | 3 5 3 | 2 19 3 | 2 13 8 | 0 17 10 |
| 42 | 2 10 2 | 3 15 8 | 3 9 0 | 3 1 4 | 0 18 10 |
| 47 | 3 1 1 | 4 8 5 | 4 1 6 | 3 11 5 | 1 0 5 |
| 52 | 3 14 3 | 5 5 10 | 4 18 9 | 4 5 5 | 1 4 6 |
| 57 | 4 11 3 | 6 14 1 | 6 2 10 | 5 5 9 | 1 11 7 |

The differences run, as a rule, very regularly; and I estimate that, on the average, they will up to age 42 provide, roughly speaking, for a simple reversionary bonus of about 35s. per annum, or a compound reversionary bonus of about 30s. per annum. After that age the differences would provide a larger bonus. This is on the theoretical assumption that the expenses of conducting the business are met out of miscellaneous sources of profit, such as interest over that assumed in the calculations, lapsed policies, etc. I find the average rate of bonus actually declared by offices adopting these methods of distribution is 26s. 6d. simple, and 28s. compound, so that the assumption is not very wide of the mark as regards the latter. One would hardly have expected the rate in the case of the compound reversionary bonus to be higher than in the simple, but probably the rate of premium is higher also.

I thought it would also be interesting to see how the low rates charged by some offices for discounted bonus policies compare with the True Select premiums when loaded for expenses. The formula used for loading them is Dr. Sprague's well-known one,

$$1{\cdot}075\left(\varpi + \frac{{\cdot}01}{a} + {\cdot}00125\right),$$

which allows for an initial commission of £1 per cent on the sum assured, a constant of 2s. 6d. per annum for initial office expenses, and a percentage of 7½ for annual office expenses, including renewal commission. The results are given in the following table:—

TABLE 3.—*Discounted Bonus Premiums, Whole Life With Profits—Males.*

| Age. | True Select Loaded Premiums. | Office Premiums. | | |
|---|---|---|---|---|
| | | Highest. | Average. | Lowest. |
| 22 | £1 12 9 | £1 18 6 | £1 13 5 | £1 9 8 |
| 27 | 1 16 5 | 2 2 8 | 1 16 11 | 1 12 8 |
| 32 | 2 1 8 | 2 8 0 | 2 1 11 | 1 17 2 |
| 37 | 2 8 5 | 2 14 7 | 2 8 5 | 2 3 0 |
| 42 | 2 17 11 | 3 3 6 | 2 17 1 | 2 11 4 |
| 47 | 3 9 10 | 3 16 0 | 3 8 6 | 3 2 3 |
| 52 | 4 4 1 | 4 12 4 | 4 4 0 | 3 17 3 |
| 57 | 5 2 7 | 5 15 4 | 5 5 5 | 4 18 5 |

It will be seen that the average office premiums agree very closely, on the whole, with the True Select loaded premiums, and it may be concluded that there is a reasonable chance of the expectations formed at the outset regarding this class of policy being realised in the case of those offices charging the average or a higher premium; but it is likely that considerable difficulty will be experienced by those offices charging a lower premium.

**Without Profits.**

The observations in the *Without Profits* section relate to the same class of policy, but are not nearly so numerous, as in the one with profits, the years of life numbering 659,111 only, and the deaths 18,492. The net premiums have also been calculated by each of the three methods, and the results are given in the following table:—

TABLE 4.—100 $\pi_x$, *Whole Life Without Profits—Males.*

| Age. | True Select. | Select and Ultimate. | Grouped Select and Ultimate. |
|---|---|---|---|
| 22 | 1·462 | 1·498 | 1·431 |
| 27 | 1·650 | 1·656 | 1·621 |
| 32 | 1·933 | 1·934 | 1·888 |
| 37 | 2·146 | 2·219 | 2·238 |
| 42 | 2·594 | 2·685 | 2·706 |
| 47 | 3·312 | 3·260 | 3·289 |
| 52 | 3·948 | 3·886 | 3·951 |
| 57 | 5·223 | 5·365 | 5·033 |
| 62 | 5·971 | 5·974 | 6·226 |
| 67 | 6·794 | 7·226 | 7·781 |

Here, notwithstanding the smallness of the observations, the results corroborate to a great extent what we found in the with profits section. In every case, except two, the True Select premiums are less than the Select and Ultimate, the difference on the average being nearly 2 per cent. When we turn to the Grouped Select and Ultimate premiums, however, we find that the first three, and other two, are less than the True Select, and in this respect they differ from the with profits section. This cannot be due to the duplicate lives, as these are much more numerous in the with profits section than in the without, the percentage in the former being 22 as against 10 in the latter; but after all the difference is small.

A table of office premiums similar to that in the with profits section is given below, the True Select premium loaded as above, however, being substituted in the last column :—

TABLE 5.—*Net and Office Premiums, Whole Life Without Profits—Males.*

| Age. | True Select Net Premium. | Office Premiums—Without Profits. | | | True Select Loaded Premium. |
|---|---|---|---|---|---|
| | | Highest. | Average. | Lowest. | |
| 22 | £1 9 3 | £1 15 11 | £1 14 0 | £1 10 8 | £1 15 2 |
| 27 | 1 13 0 | 2 1 7 | 1 18 1 | 1 14 7 | 1 19 3 |
| 32 | 1 18 8 | 2 6 11 | 2 3 6 | 1 19 8 | 2 5 5 |
| 37 | 2 2 11 | 2 14 2 | 2 10 6 | 2 6 3 | 2 10 0 |
| 42 | 2 11 11 | 3 3 2 | 2 19 7 | 2 15 1 | 2 19 9 |
| 47 | 3 6 3 | 3 16 8 | 3 11 5 | 3 7 0 | 3 15 4 |
| 52 | 3 19 0 | 4 14 8 | 4 7 6 | 4 3 6 | 4 9 2 |
| 57 | 5 4 6 | 6 0 7 | 5 9 9 | 5 4 6 | 5 16 10 |

There has been a tendency of late to reduce the premiums for this class of business, but the results here brought out will probably check it. In every case, except one, the average office premium is less, sometimes substantially less, than the True Select loaded premium, and it is evident that some offices will have to revise their rates and increase them, unless they are prepared to conduct that part of their business at a loss.

**Comparison of With and Without Profits.**

The Grouped Select and Ultimate premiums only are available for all classes, and it will be convenient to use these in all comparisons. They are better adapted too for this purpose, as they run, on the whole, more regularly than the others. They are given in the following table for the with and without profit sections we have dealt with, and the difference between them is also given. The premiums according

to Dr. Sprague's Select 3½ per cent Tables are placed alongside for comparison.

TABLE 6.—*Net Premiums, Whole Life With and Without Profits—Males.*

| Age. | With Profits. | Without Profits. | (3)—(2) | Sprague's Select. |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| 22 | 1·350 | 1·431 | ·081 | 1·514 |
| 27 | 1·530 | 1·621 | ·091 | 1·675 |
| 32 | 1·792 | 1·888 | ·096 | 1·921 |
| 37 | 2·099 | 2·238 | ·139 | 2·237 |
| 42 | 2·512 | 2·706 | ·194 | 2·662 |
| 47 | 3·056 | 3·289 | ·233 | 3·209 |
| 52 | 3·774 | 3·951 | ·177 | 3·934 |
| 57 | 4·627 | 5·033 | ·406 | 4·928 |
| 62 | 5·893 | 6·226 | ·333 | 6·255 |
| 67 | 7·555 | 7·781 | ·226 | 8·075 |

There are substantial differences between the with and without profit rates, varying from 1s. 7d. at age 22 to 8s. 1d. at age 57, and I estimate that, up to age 42, these would of themselves, roughly speaking, provide a simple reversionary bonus of about 4s. per annum, or a compound reversionary bonus of about 3s. 6d. per annum. This, I think, shows that an office cannot afford to allow another office to reassure with profits a policy effected with it without profits, unless perhaps in a case where the guaranteeing office by independent enquiries is satisfied that the life is quite up to the standard of its with profits class. Turning to Dr. Sprague's rates, we find that in half the cases they are greater than even the without profit rates, and that in the cases where they are less, they are only slightly less. One would naturally have expected that, if the mortality remained the same, all his rates would have lain between the with and without profit rates, and as a rule, nearer the former, because the $H^M$ experience on which they were based comprised not only the with profit experience, but also the without. It is quite evident, therefore, that there has been a decided improvement in the rate of mortality, and I estimate the monetary value of it at over 5 per cent on the premium.

## WHOLE LIFE ASSURANCES—FEMALES.

The observations in the *With Profits* section also relate only to policies with premiums payable uniformly throughout life. The number is somewhat less than in the last one, the years of life being 553,886, and the deaths 15,789. The premiums have been calculated by the third method only, and

With Profits.

are given in the following table, those for males being placed alongside for comparison :—

TABLE 7.—100 $\pi_x$, *Whole Life With Profits—Males and Females.*

| Age. | Females. | Males. | Difference. |
|---|---|---|---|
| 22 | 1·547 | 1·350 | + ·197 |
| 27 | 1·662 | 1·530 | + ·132 |
| 32 | 1·853 | 1·792 | + ·061 |
| 37 | 2·071 | 2·099 | − ·028 |
| 42 | 2·469 | 2·512 | − ·043 |
| 47 | 2·932 | 3·056 | − ·124 |
| 52 | 3·566 | 3·774 | − ·208 |
| 57 | 4·495 | 4·627 | − ·132 |
| 62 | 5·749 | 5·893 | − ·144 |
| 67 | 7·725 | 7·555 | + ·170 |

Up to age 32 the premiums for females are greater than those for males, and after that they are less, with the exception of the last. These premiums, however, show only the concentrated effect of the mortality during the whole of life, and I thought it desirable to see how the differences would be affected if the mortality in each case up to age 50 only were taken into account. In the following table the net annual premiums for temporary assurances on the lives of males and females ceasing at age 50 are given, along with the differences :—

TABLE 8.—100 $\pi^1_{x:\overline{50-x}|}$, *Whole Life With Profits—Males and Females.*

| Age. | Females. | Males. | Difference. |
|---|---|---|---|
| 22 | 1·096 | ·782 | ·314 |
| 27 | 1·061 | ·805 | ·256 |
| 32 | 1·075 | ·877 | ·198 |
| 37 | 1·015 | ·900 | ·115 |
| 42 | 1·103 | ·910 | ·193 |

Here it will be seen that the differences for the period mentioned are very much greater than for the whole of life, and the practice of a number of offices in charging females an extra of 5s. per cent per annum up to age 50 is amply justified by the results of the new experience. It is sometimes urged that it is quite unnecessary to charge it in some cases; but it is obvious that, if this course be adopted, it would be necessary to charge a higher rate in other cases to compensate for it. No doubt a larger reduction than 5s. could be given after age 50, but the difference is not so great as to constitute any real hardship, and the rule is a convenient one in practice.

The class of policy in the *Without Profits* section is similar to that in the last, but the years of life and deaths are smaller, being only 122,481 and 4116 respectively. Net premiums corresponding to those in the last section are given in the following table :—

Without Profits.

TABLE 9.—100 $\pi_x$, *Whole Life Without Profits—Males and Females.*

| Age. | Females. | Males. | Difference. |
|---|---|---|---|
| 22 | 1·574 | 1·431 | + ·143 |
| 27 | 1·729 | 1·621 | + ·108 |
| 32 | 1·941 | 1·888 | + ·053 |
| 37 | 2·186 | 2·238 | − ·052 |
| 42 | 2·500 | 2·706 | − ·206 |
| 47 | 2·944 | 3·289 | − ·345 |
| 52 | 3·699 | 3·951 | − ·252 |
| 57 | 4·494 | 5·033 | − ·539 |
| 62 | 5·626 | 6·226 | − ·600 |
| 67 | 7·401 | 7·781 | − ·380 |

The same feature occurs here as in the with profits section. Up to age 32 the premiums for females are greater than those for males, and after that age they are less. The difference, however, between the two sexes is less at first and afterwards much greater than in the corresponding section with profits, showing that the female rate of mortality in this section does not follow the same law as the male. I shall refer to this again presently.

Net annual premiums for temporary assurances ceasing at age 50, corresponding to those tabulated in the other section, are given in the following table :—

TABLE 10.—100 $\pi^1_{x:\overline{50-x}|}$, *Whole Life Without Profits—Males and Females.*

| Age. | Females. | Males. | Difference. |
|---|---|---|---|
| 22 | 1·111 | ·872 | ·239 |
| 27 | 1·138 | ·912 | ·226 |
| 32 | 1·180 | ·988 | ·192 |
| 37 | 1·166 | 1·095 | ·071 |
| 42 | 1·058 | 1·262 | − ·204 |

The differences here, of course, are not so great as in the with profits section. The rates for males, however, are so high that they cannot be regarded as a true index with which to ascertain the real difference between the sexes.

We may now compare the net premiums for females with profits with those for without profits, and this is done in the following table :—

Comparison of With and Without Profits.

TABLE 11.—100 $\pi_x$, *Whole Life With and Without Profits—Females.*

| Age. | With Profits. | Without Profits. | Difference. |
|---|---|---|---|
| 22 | 1·547 | 1·574 | ·027 |
| 27 | 1·662 | 1·729 | ·067 |
| 32 | 1·853 | 1·941 | ·088 |
| 37 | 2·071 | 2·186 | ·115 |
| 42 | 2·469 | 2·500 | ·031 |
| 47 | 2·932 | 2·944 | ·012 |
| 52 | 3·566 | 3·699 | ·133 |
| 57 | 4·495 | 4·494 | – ·001 |
| 62 | 5·749 | 5·626 | – ·123 |
| 67 | 7·725 | 7·401 | – ·324 |

It will be seen that the difference between the two rates is, as a rule, small; indeed in three cases—at ages 57, 62, and 67—the rates without profits are slightly lower than the rates with profits. This shows that the rate of mortality among females who effect without profit policies is only slightly higher than among those who effect with profit policies, and is in marked contrast to what we found in the case of the males. Probably a certain proportion of both males and females in the case of without profit policies are of extravagant habits, but these in the former lie in directions which are generally more detrimental to health.

## ASCENDING SCALE ASSURANCES, WITH AND WITHOUT PROFITS—MALES.

In this class the published tables make no distinction between with and without profit assurances, which is rather unfortunate in view of the difference in the rate of mortality shown to exist in these two sections. There were 23,280 entrants in the class, of whom 18,673 effected policies with profits, and 4607 without profits, so that the latter is about 20 per cent of the whole. The years of life are 207,709, and the deaths 3064. The net annual premiums payable uniformly throughout life are as follows, the whole life with and without profits premiums being placed on either side for comparison:—

TABLE 12.—100 $\pi_x$, *Ascending Scale, and Whole Life With and Without Profits—Males.*

| Age. | With Profits. | Ascending Scale. | Without Profits. |
|---|---|---|---|
| 22 | 1·350 | 1·449 | 1·431 |
| 27 | 1·530 | 1·609 | 1·621 |
| 32 | 1·792 | 1·800 | 1·888 |
| 37 | 2·099 | 2·144 | 2·238 |
| 42 | 2·512 | 2·617 | 2·706 |
| 47 | 3·056 | 3·133 | 3·289 |
| 52 | 3·774 | 3·963 | 3·951 |
| 57 | 4·627 | 4·863 | 5·033 |

With two small exceptions, the ascending scale rates lie between the with and without profits—on the average nearer the latter than the former—and this shows that the mortality in the ascending scale class is not so good as that in the with profits.

It should be noted that these rates apply principally to the with profits class, and that they would no doubt be higher for the without profits.

## Limited Payment Assurances, With and Without Profits—Males.

In this class the exposed to risk and the deaths in the with and without profit policies are also tabulated together. The number of entrants is 36,839, of which 35,020 were with profits and 1819 without, so that the latter form only 5 per cent of the whole. The years of life are 410,251, and the deaths 5050. The net premiums deduced are given in the following table, along with those by Dr. Sprague's Select Life Tables for comparison :—

Table 13.—$100_m\pi_x$, *Limited Payments—Males, and Sprague's Select.*

| Age. | 1 Payment. | | 10 Payments. | |
|---|---|---|---|---|
| | New Experience. | Sprague's Select. | New Experience. | Sprague's Select. |
| 20 | 25·874 | 30·318 | 3·058 | 3·657 |
| 25 | 28·251 | 32·122 | 3·341 | 3·858 |
| 30 | 30·815 | 34·917 | 3·641 | 4·195 |
| 35 | 34·210 | 38·305 | 4·058 | 4·616 |
| 40 | 38·512 | 42·281 | 4·586 | 5·121 |
| 45 | 42·854 | 46·793 | 5·124 | 5·723 |
| 50 | 48·099 | 51·685 | 5·856 | 6·413 |
| 55 | 51·940 | 57·054 | 6·285 | 7·255 |

| Age. | 20 Payments. | | 30 Payments. | |
|---|---|---|---|---|
| | New Experience. | Sprague's Select. | New Experience. | Sprague's Select. |
| 20 | 1·839 | 2·226 | 1·461 | 1·783 |
| 25 | 2·014 | 2·349 | 1·604 | 1·889 |
| 30 | 2·197 | 2·565 | 1·759 | 2·079 |
| 35 | 2·457 | 2·843 | 1·987 | 2·331 |
| 40 | 2·810 | 3·194 | 2·309 | 2·665 |
| 45 | 3·186 | 3·637 | 2·678 | 3·109 |
| 50 | 3·715 | 4·191 | 3·226 | 3·699 |
| 55 | 4·101 | 4·941 | 3·703 | 4·530 |

It will be seen that the New Experience rates are in every case less, substantially less, than Sprague's Select. Roughly speaking, the 1 and 10 payment rates are about 10 per cent less, and the 20 and 30 payment, 15 per cent less. There has been a tendency in recent years to increase the rates for this class of business, but these figures seem to show that this is unnecessary. It should be borne in mind, however, that duplicate lives were not eliminated in forming the aggregate table, and this no doubt has the effect of making the mortality appear rather better than it would otherwise be.

In order to measure the mortality in this class with the others, I have calculated the uniform annual premium payable throughout life, and give it in the following table along with that for the whole life with profits class, which has up to the present been the class in which we have found the mortality lightest. In the last-mentioned class it has been necessary to interpolate for the required ages, and this has been done correct to second differences :—

TABLE 14.—100 $\pi_x$, *Limited Payment, and Whole Life With Profits—Males.*

| Age. | Limited Payment. | Whole Life, With Profits. | Difference. |
|---|---|---|---|
| 20 | 1·180 | 1·301 | ·121 |
| 25 | 1·332 | 1·448 | ·116 |
| 30 | 1·506 | 1·682 | ·176 |
| 35 | 1·758 | 1·963 | ·205 |
| 40 | 2·118 | 2·331 | ·213 |
| 45 | 2·535 | 2·818 | ·283 |
| 50 | 3·134 | 3·471 | ·337 |
| 55 | 3·655 | 4·236 | ·581 |

I consider these rates show the superiority of the limited payment class of policyholders in a very marked degree. In every case the rate for them is less than for the whole life class, the difference on the average being about 10 per cent.

It should be noted here also that these rates apply almost entirely to the with profits class, and there is but little doubt that the rates in the without profits would be higher. If the premiums are calculated by the whole life without profits data, the 1 payment rates given above are increased by about 11 per cent, the 10 payment by about 13 per cent, the 20 payment by about 16 per cent, and the 30 payment by about 18 per cent—in fact the rates are similar to Sprague's Select.

## ENDOWMENT ASSURANCES, WITH AND WITHOUT PROFITS—MALES.

We now come to perhaps the most important, certainly the most interesting, class in the New Experience, namely, endowment

assurances: important, because of the large amount of business now transacted under it; interesting, because of the various theories which have been advanced from time to time regarding its effect upon life assurance institutions generally. In this class also the exposed to risk and deaths are tabulated for with and without profit policies together. The total number of entrants is 140,414, of whom 116,604 effected policies with profits, and 23,810 without profits, so that the latter is about 17 per cent of the whole. The years of life are 947,753, and the deaths 6021. The smallness of these numbers is due to the fact that this class has only recently advanced rapidly in popular favour. As previously mentioned, the net premiums were deduced by the second and third methods, but they both agreed so closely that the latter only has been given in the following table, those by Dr. Sprague's tables being placed alongside for comparison. The other rates given (Limited Payment) I will explain presently.

TABLE 15.—100 $\pi_{x:\overline{n}|}$, *Endowment Assurance, Sprague's Select, and Limited Payment—Males.*

| Age. | 10 Years. | | | 20 Years. | | |
|---|---|---|---|---|---|---|
| | New Experience. | Sprague's Select. | Limited Payment. | New Experience. | Sprague's Select. | Limited Payment. |
| 20 | 8·460 | 8·679 | 8·439 | 3·701 | 3·960 | 3·725 |
| 25 | 8·426 | 8·630 | 8·445 | 3·689 | 3·931 | 3·746 |
| 30 | 8·442 | 8·632 | 8·433 | 3·742 | 3·964 | 3·747 |
| 35 | 8·474 | 8·669 | 8·480 | 3·804 | 4·039 | 3·799 |
| 40 | 8·530 | 8·731 | 8·526 | 3·894 | 4·172 | 3·915 |
| 45 | 8·588 | 8·849 | 8·574 | 4·070 | 4·390 | 4·053 |
| 50 | 8·787 | 9·028 | 8·794 | 4·411 | 4·728 | 4·342 |
| 55 | 9·118 | 9·335 | 8·721 | ... | ... | ... |

| Age. | 30 Years. | | | 40 Years. | | |
|---|---|---|---|---|---|---|
| | New Experience. | Sprague's Select. | Limited Payment. | New Experience. | Sprague's Select. | Limited Payment. |
| 20 | 2·220 | 2·500 | 2·265 | 1·570 | 1·870 | 1·620 |
| 25 | 2·238 | 2·499 | 2·296 | 1·623 | 1·906 | 1·679 |
| 30 | 2·323 | 2·571 | 2·327 | 1·757 | 2·031 | 1·755 |
| 35 | 2·432 | 2·703 | 2·426 | ... | ... | ... |
| 40 | 2·606 | 2·922 | 2·614 | ... | ... | ... |

Here also the New Experience rates are considerably less than Sprague's Select. The 10 year rates are, on the average, about 2½ per cent less; the 20 year, 6½ per cent; the 30 year, 10 per cent; and the 40 year, 15 per cent.

The same features, although in a modified degree, are observable in the following table of single premiums, the percentages being about $\frac{3}{4}$, 3, 6, and 10 respectively :—

TABLE 16.—100 $A_{x:\overline{m}|}$, *Endowment Assurance, Sprague's Select, and Limited Payment—Males.*

| Age. | 10 Years. | | | 20 Years. | | |
|---|---|---|---|---|---|---|
| | New Experience. | Sprague's Select. | Limited Payment. | New Experience. | Sprague's Select. | Limited Payment. |
| 20 | 71·442 | 71·963 | 71·391 | 52·255 | 53·939 | 52·417 |
| 25 | 71·361 | 71·845 | 71·405 | 52·181 | 53·756 | 52·556 |
| 30 | 71·398 | 71·851 | 71·377 | 52·532 | 53·966 | 52·562 |
| 35 | 71·476 | 71·939 | 71·490 | 52·941 | 54·432 | 52·911 |
| 40 | 71·611 | 72·081 | 71·601 | 53·522 | 55·234 | 53·651 |
| 45 | 71·749 | 72·352 | 71·715 | 54·622 | 56·492 | 54·517 |
| 50 | 72·210 | 72·748 | 72·227 | 56·597 | 58·297 | 56·222 |
| 55 | 72·947 | 73·406 | 72·058 | ... | ... | ... |

| Age. | 30 Years. | | | 40 Years. | | |
|---|---|---|---|---|---|---|
| | New Experience. | Sprague's Select. | Limited Payment. | New Experience. | Sprague's Select. | Limited Payment. |
| 20 | 39·628 | 42·498 | 40·108 | 31·708 | 35·614 | 32·387 |
| 25 | 39·827 | 42·495 | 40·442 | 32·432 | 36·057 | 33·179 |
| 30 | 40·717 | 43·198 | 40·767 | 34·193 | 37·517 | 34·166 |
| 35 | 41·836 | 44·423 | 41·765 | ... | ... | ... |
| 40 | 43·533 | 46·347 | 43·604 | ... | ... | ... |

I thought it would be interesting to see how the mortality in this class compared with the other classes, and I therefore calculated endowment assurance premiums by the limited payment data, and the results are inserted in the columns headed "Limited Payment" in the above tables. It will be seen that the premiums thus obtained agree on the whole very well with the true endowment assurance premiums; but if we examine them more closely, we find that in the 10-year rates, whether single or annual, 5 are less and 3 greater; 20-year, 3 less and 4 greater; 30-year, 1 less and 4 greater; and 40-year, 1 slightly less and 2 greater. Although the true endowment assurance rates for 10 years are slightly higher, on the average, than the rates deduced from the limited payment data, yet the position is gradually reversed as the duration is extended, until at 40 years we find that practically all the true endowment assurance rates are less. But there are two points to which I wish to draw attention: (1) the inclusion of duplicate lives in the limited payment data, which no doubt reduces the rate of mortality in the ultimate table, at all events

at the earlier portion; and (2) the much larger percentage of without profit policies included in the endowment assurance data, which no doubt increases the rate of mortality. The percentage in it is 17 against 5 in the limited payment data. I am of opinion, therefore, that the superiority of endowment assurance policyholders over limited payment policyholders is placed beyond all question.

We may now compare the new experience net annual premiums with those charged by offices for this class of policy with participation in profits; and, in doing so, I thought that the best way to institute a fair comparison was to load them with a bonus, as that of course forms an important element in the calculation, on account of its value being much greater in the case of an endowment assurance than a whole life policy. We found that the differences between the True Select and Average Office Premiums for whole life with profit policies provided, roughly speaking, for a simple reversionary bonus of about 35s. per annum, or a compound one of about 30s. per annum, and I have adopted the former rate in the calculations. The results are given in the following table for all the durations we have dealt with; but, unfortunately, in two of them one office premium only is readily obtainable. The two referred to, however, are, comparatively speaking, unimportant:—

TABLE 17.—*Office Premiums for Endowment Assurances.*

| Age. | New Experience Loaded Premiums. | Office Premiums—With Profits. | | |
|---|---|---|---|---|
| | | Highest. | Average. | Lowest. |
| | | 10 Years. | | |
| 40 | £9 19 7 | £11 12 9 | £10 10 3 | £10 0 6 |
| | | 20 Years. | | |
| 30 | £4 19 2 | £5 6 8 | £5 0 3 | £4 14 10 |
| 35 | 5 0 6 | 5 8 7 | 5 2 5 | 4 17 5 |
| 40 | 5 2 5 | 5 12 3 | 5 5 9 | 4 19 11 |
| 45 | 5 6 1 | 5 18 11 | 5 11 0 | 5 2 7 |
| | | 30 Years. | | |
| 20 | £3 4 11 | £3 12 3 | £3 3 9 | £2 19 4 |
| 25 | 3 5 4 | 3 12 8 | 3 5 2 | 3 0 11 |
| 30 | 3 7 3 | 3 13 11 | 3 7 6 | 3 2 9 |
| 35 | 3 9 8 | 3 17 2 | 3 11 0 | 3 6 7 |
| | | 40 Years. | | |
| 25 | £2 10 5 | £2 18 0 | £2 11 1 | £2 7 6 |

The loaded premiums agree, on the whole, very well with the average office premiums, and this shows, I think, that the rates charged by offices for the two classes are fairly consistent, when the same reversionary bonus is allotted to each.

But there is one point to which I should like to direct attention. Endowment assurances are of, comparatively speaking, recent origin, and no doubt at first, and for some time afterwards, this class of assurance was selected by persons who believed in their prospects of longevity; in fact, they would exercise, in a marked degree, the very selection we have found to be so prominent a feature of the New Experience. Now, however, this class of policy, from being the option of the few, has become the fashion of the many, and that of itself is bound to increase the rate of mortality in it. If the experience could be divided into two sections, the one applicable to the earlier, and the other to the later, period of the observations, it would probably be found that the rate of mortality had increased during the period under review. Endowment assurances now occupy very much the same position as whole life policies did formerly, and the experience of the latter will, no doubt, be more applicable in future to the former than their own experience in the past. I have calculated the endowment assurance premiums by the ordinary whole life with profits data, and find that this would increase the above rates by about 4 per cent in the case of the 20 years' endowment, and about 7½ per cent in the 30 and 40 years' endowment. It should be kept in view, however, that the endowment assurance data contains a small proportion of without-profit policies, and this, as we have seen, tends to affect the mortality. I have also calculated the endowment assurance premiums by the ordinary whole life without profits data, and I find that they agree very closely with Sprague's Select.

When this class of policy was becoming more popular, fears were expressed as to the effect it would have upon life offices generally; but up to the present that effect cannot be said to be injurious. It must be borne in mind, however, that the advantages derived by life offices from good old lives during the past will not be experienced in this class, on account of the limited duration of the policies. Its effect in this direction has not been fully experienced yet.

## Contingent-Survivorship Assurances—Males.

This class may not be so readily recognised under its double-barrelled appellation in the New Experience, as it is more commonly known by one or other of the names forming it. It

is a very interesting class, on account of the attention which has been directed towards it, in the belief that the lives forming it are decidedly below the average. Attempts have been made at various times to determine the risk, notably by the late Mr. Sunderland in his paper, "On Risk Premiums for Survivorship Assurances", *J.I.A.*, xxvii, 81. He used Dr. Sprague's Select Life Table for the insured lives, and Mr. A. J. Finlaison's Government Female Annuitants' Analysed Table of 1884 for the counter lives, slightly modifying the former so as to make it coincide exactly with the latter as regards the period during which selection is allowed for. He says: "There is an impression in the minds of "many actuaries which may be roughly expressed by saying that, "in comparing the vitality of lives insured under different classes "of policies, *e.g.*, survivorship, ordinary whole term, endowment "assurances, etc., the higher the rate of premium the better the "life. This impression, probably, has some foundation in fact, "and, if so, the vitality of lives insured under survivorship policies "may fall slightly below the standard of Mr. Sprague's Tables. "The inferiority could not be serious if proper care be exercised "by the assuring offices, but, assuming it exists, the errors arising "in the calculation of survivorship premiums from the use of "Mr. Sprague's Tables for the lives insured are in the opposite "direction to those introduced by the use of the Government "Female Annuitants' Table for the counter lives, where the latter "overestimates the chances of life. The Government Female "Annuitants' Analysed Table appears perfectly safe for all cases. "It takes into account the effect of selection which I think is in "operation, and should be taken account of in the case of "the counter lives. No other table extant combines these "qualifications." It will be observed that Sunderland thought that the Government Female Annuitants' Table overestimates the chances of life of the counter lives. He indicates that among the counter lives there would be a considerable number of male lives, whose vitality would be inferior to that of female annuitants; but Mr. Neil Campbell, in an interesting paper which he read before you at the last meeting, dealing with the mortality among life-tenants, who form the bulk of these counter lives, showed that a large proportion—about three-fourths—of these lives were females, and that they were better lives than the Female Government Annuitants, and even the New Experience Female Annuitants. When, however, the male lives were mixed with the females, the effect was to increase the rate of mortality throughout. Having regard to all the circumstances, I think the best table to adopt on the present occasion for the counter life is the New Experience Female Annuitants, and I have used it in precisely the same way as I have done the tables of assured lives.

The data in the New Experience relating to the insured lives are given for quinquennial groups of ages at entry, and separately for the first ten years of assurance only. The number of entrants for that period is 3338, the years of life being 15,586, and the deaths 176. After ten years, I used the whole of life without profits data, as being the most appropriate. Probably the lives in that class are rather better on the whole than those in the contingent-survivorship class. We can only compare them for the first ten years, and in the following table are given the annual premiums for ten-year temporary assurances obtained from the contingent-survivorship and whole life without profits data respectively :—

TABLE 18.—100 $\pi^{1}_{x:\overline{10}|}$, *Contingent-Survivorship, and Whole Life Without Profits—Males.*

| Age. | Contingent Survivorship. | Whole Life Without Profits. |
|---|---|---|
| 20 | ·593 | ·765 |
| 25 | ·751 | ·678 |
| 30 | 1·049 | ·764 |
| 35 | ·903 | ·939 |
| 40 | 1·383 | 1·184 |
| 45 | 1·560 | 1·554 |
| 50 | 1·664 | 1·854 |
| 55 | 1·229 | 2·689 |
| 60 | 1·712 | 3·599 |

The number of observations in the contingent-survivorship class below age 25 and above age 45 is so small, that no reliance can be placed upon the figures deduced from them. If we confine our attention to the ages from 25 to 45 both inclusive, we find that at four of them the temporary premium derived from the contingent-survivorship data is greater than the whole life without profits one, and that at the fifth it is only slightly less. In all probability the mortality in the contingent-survivorship class will continue to be heavy even after 10 years, but there is no other table available for our purpose than the whole life without profits one.

In the following table are given the annual premiums for $x$ against $y$ calculated in this way, namely, using for the insured lives the contingent-survivorship data for the first ten years after entry, and the whole life without profits data after that period, and for the counter lives, the New Experience Female Annuitant data. The premiums obtained by Sunderland are placed below for comparison. It should be mentioned, however, that these were calculated at 4 per cent.

TABLE 19.—100 $\pi^1_{x:y}$, *Contingent-Survivorship and Female Annuitants, and Sunderland's.*

| x | y | | | | | |
|---|---|---|---|---|---|---|
| | 30 | 40 | 50 | 60 | 70 | 80 |
| | NEW EXPERIENCE, 3½ per cent. | | | | | |
| 20 | ·962 | ·855 | ·759 | ·676 | ·611 | ·567 |
| 30 | 1·546 | 1·397 | 1·248 | 1·110 | ·995 | ·851 |
| 40 | ... | 2·099 | 1·874 | 1·609 | 1·393 | 1·223 |
| 50 | ... | ... | 2·864 | 2·435 | 2·029 | 1·786 |
| 60 | ... | ... | ... | 3·420 | 2·600 | 1·924 |
| | SUNDERLAND, 4 per cent. | | | | | |
| 20 | 1·108 | 1·024 | ·954 | ·902 | ·859 | ·806 |
| 30 | 1·402 | 1·266 | 1·115 | ·977 | ·857 | ·747 |
| 40 | ... | 1·903 | 1·669 | 1·405 | 1·161 | ·955 |
| 50 | ... | ... | 2·803 | 2·395 | 1·938 | 1·534 |
| 60 | ... | ... | ... | 4·485 | 3·776 | 3·036 |

If we except $x=20$ and 60, where the observations are very small, the New Experience rates are in every case greater than Mr. Sunderland's—on the average about 10 per cent higher.

The same features are observable in the following table of single premiums for $x$ against $y$:—

TABLE 20.—100 $A^1_{x:y}$, *Contingent-Survivorship and Female Annuitants, and Sunderland's.*

| x | y | | | | | |
|---|---|---|---|---|---|---|
| | 30 | 40 | 50 | 60 | 70 | 80 |
| | NEW EXPERIENCE, 3½ per cent. | | | | | |
| 20 | 16·90 | 13·68 | 11·10 | 8·02 | 5·39 | 3·29 |
| 30 | 25·19 | 21·03 | 17·39 | 12·73 | 8·59 | 4·88 |
| 40 | ... | 29·31 | 24·69 | 17·79 | 11·74 | 6·91 |
| 50 | ... | ... | 34·61 | 25·42 | 16·50 | 9·86 |
| 60 | ... | ... | ... | 33·73 | 20·71 | 10·58 |
| | SUNDERLAND, 4 per cent. | | | | | |
| 20 | 17·72 | 15·32 | 12·69 | 9·90 | 6·96 | 4·49 |
| 30 | 21·76 | 18·57 | 14·69 | 10·70 | 6·96 | 4·17 |
| 40 | ... | 26·16 | 21·00 | 14·97 | 9·29 | 5·29 |
| 50 | ... | ... | 31·95 | 23·84 | 14·91 | 8·33 |
| 60 | ... | ... | ... | 38·68 | 26·46 | 15·65 |

We may now compare the annual premiums charged by offices for this class of business with the rates we have brought out; and I think a fair addition to make for expenses is a constant of 5s. and a loading of 10 per cent. The results are as follows :—

TABLE 21.—*Office Premiums for Contingent-Survivorship Assurances.*

| x | y = 30 | y = 40 | y = 50 | y = 60 |
|---|---|---|---|---|
| | **New Experience Loaded Premium.** | | | |
| 20* | £1 6 8 | £1 4 4 | £1 2 2 | £1 0 4 |
| 30 | 1 19 6 | 1 16 3 | 1 12 11 | 1 9 11 |
| 40 | ..... | 2 11 8 | 2 6 9 | 2 0 11 |
| 50 | ..... | ..... | 3 8 6 | 2 19 1 |
| 60* | ..... | ..... | ..... | 4 0 9 |
| | **Highest Office Premium.** | | | |
| 20 | £1 10 3 | £1 8 4 | £1 6 4 | £1 4 4 |
| 30 | 2 0 11 | 1 18 3 | 1 15 4 | 1 12 6 |
| 40 | ..... | 2 13 8 | 2 9 8 | 2 5 4 |
| 50 | ..... | ..... | 3 13 3 | 3 6 11 |
| 60 | ..... | ..... | ..... | 5 12 0 |
| | **Average Office Premium.** | | | |
| 20 | £1 6 2 | £1 4 2 | £1 2 2 | £1 0 8 |
| 30 | 1 15 5 | 1 11 11 | 1 8 7 | 1 5 11 |
| 40 | ..... | 2 6 4 | 2 0 8 | 1 15 7 |
| 50 | ..... | ..... | 3 5 4 | 2 16 5 |
| 60 | ..... | ..... | ..... | 5 3 3 |
| | **Lowest Office Premium.** | | | |
| 20 | £1 2 10 | £1 0 11 | £0 18 10 | £0 17 2 |
| 30 | 1 11 11 | 1 8 6 | 1 5 8 | 1 2 4 |
| 40 | ..... | 2 3 0 | 1 17 10 | 1 12 7 |
| 50 | ..... | ..... | 3 0 0 | 2 11 8 |
| 60 | ..... | ..... | .... | 5 0 2 |

* The number of observations at these ages is very small, and the rates are too low.

On glancing at the above table, one cannot help being struck with the great difference between the rates charged by offices for this class of risk, as it occasionally amounts to no less than 40 per

cent. It will be seen that the New Experience loaded premiums are, if we except ages 20 and 60, substantially higher than the average office premium, and this shows that the office premiums are, as a rule, too small. It is evident, therefore, that most offices will require to revise their rates and increase them.

### Temporary Assurances—Males.

The data in this class also are given for quinquennial groups of ages at entry, and separately for the first ten years of assurance only. The number of entrants relating to that period is 11,592, the years of life being 36,489, and the deaths 391. The net premiums are as follows, those by Dr. Sprague's table, and those deduced by me from the Gotha Experience (*J.I.A.*, xxix, 157), being placed alongside for comparison :—

Table 22.—100 $\pi^{1}_{x:\overline{n}|}$, *Temporary, Sprague's Select, and Chatham's Select—Males.*

| Age. | 1 Year. | | | 3 Years. | | |
|---|---|---|---|---|---|---|
| | New Experience. | Sprague's Select. | Chatham's Select. | New Experience. | Sprague's Select. | Chatham's Select. |
| 20 | ·819 | ·483 | ... | ·764 | ·761 | ... |
| 25 | ·267 | ·435 | ·290 | ·612 | ·667 | ·482 |
| 30 | ·335 | ·435 | ·377 | ·365 | ·645 | ·527 |
| 35 | ·579 | ·473 | ·560 | ·689 | ·688 | ·626 |
| 40 | ·340 | ·531 | ·647 | ·415 | ·767 | ·738 |
| 45 | ·837 | ·618 | ·696 | 1·060 | ·898 | ·861 |
| 50 | 1·002 | ·763 | ·928 | 1·632 | 1·131 | 1·295 |
| 55 | 1·341 | 1·005 | 1·237 | 1·488 | 1·538 | 1·799 |

| Age. | 5 Years. | | | 10 Years. | | |
|---|---|---|---|---|---|---|
| | New Experience. | Sprague's Select. | Chatham's Select. | New Experience. | Sprague's Select. | Chatham's Select. |
| 20 | 1·137 | ·865 | ... | 1·889 | ·908 | ... |
| 25 | ·868 | ·762 | ·561 | ·674 | ·821 | ·654 |
| 30 | ·554 | ·741 | ·605 | ·753 | ·869 | ·719 |
| 35 | ·738 | ·812 | ·683 | ·511 | ·951 | ·797 |
| 40 | ·615 | ·905 | ·826 | ·754 | 1·122 | 1·007 |
| 45 | ·917 | 1·097 | ·988 | ·877 | 1·415 | 1·285 |
| 50 | 1·857 | 1·399 | 1·491 | 1·908 | 1·837 | 1·909 |
| 55 | 1·749 | 1·919 | 2·085 | 1·606 | 2·558 | 2·667 |

On looking at the table, one cannot help being struck with the high rates for age 20 all through, and the sudden drop to more

normal rates after that age. For one year, the lowest rate is at age 25; for three and five years, age 30; and for ten years, age 35. I think we may fairly assume that the rate decreases to age 30, and after that increases.

Another feature is that Dr. Sprague's rates are, as a rule, too high at first, and too low afterwards. This is particularly noticeable in the one-year rates. The rates deduced by me from the Gotha Experience agree fairly well, on the whole, with the New Experience rates.

I have calculated the single premium rates also, because they are occasionally required, and I give them in the following table along with Dr. Sprague's rates and my own:—

TABLE 23.—100 $A^{1}_{x:\overline{n}|}$, *Temporary, Sprague's Select, and Chatham's Select—Males.*

| Age. | 3 Years. | | | 5 Years. | | |
|---|---|---|---|---|---|---|
| | New Experience. | Sprague's Select. | Chatham's Select. | New Experience. | Sprague's Select. | Chatham's Select. |
| 20 | 2·201 | 2·192 | ... | 5·229 | 3·983 | ... |
| 25 | 1·772 | 1·925 | 1·392 | 4·003 | 3·515 | 2·596 |
| 30 | 1·060 | 1·862 | 1·521 | 2·574 | 3·420 | 2·798 |
| 35 | 1·991 | 1·985 | 1·805 | 3·405 | 3·744 | 3·153 |
| 40 | 1·205 | 2·211 | 2·125 | 2·854 | 4·165 | 3·803 |
| 45 | 3·044 | 2·586 | 2·478 | 4·207 | 5·037 | 4·539 |
| 50 | 4·678 | 3·250 | 3·714 | 8·429 | 6·391 | 6·790 |
| 55 | 4·256 | 4·405 | 5·139 | 7·953 | 8·698 | 9·405 |

| Age. | 10 Years. | | |
|---|---|---|---|
| | New Experience. | Sprague's Select. | Chatham's Select. |
| 20 | 15·472 | 7·530 | ... |
| 25 | 5·608 | 6·835 | 5·489 |
| 30 | 6·335 | 7·231 | 6·022 |
| 35 | 4·277 | 7·891 | 6·653 |
| 40 | 6·331 | 9·261 | 8·345 |
| 45 | 7·233 | 11·568 | 10·566 |
| 50 | 15·142 | 14·808 | 15·333 |
| 55 | 12·811 | 20·120 | 20·845 |

The features observed in the single premium rates are very much the same as those in the annual.

The observations are small, and the rates deduced from them too irregular to be used as they are. I have, therefore, graduated

the annual premiums, and loaded them with a constant of 5s. and a percentage of 10. The results are given in the following table :—

TABLE 24.—1·1 (100 $\pi^{1}_{x:\overline{n}|}$ + ·25), *Temporary—Males.*

| Age. | One year. | Three years. | Five years. | Ten years. |
|---|---|---|---|---|
| 20 | £0 18 3 | £1 2 5 | £1 5 4 | £1 7 6 |
| 25* | 0 16 1 | 0 19 7 | 1 2 0 | 1 4 10 |
| 30* | 0 14 9 | 0 18 8 | 1 1 4 | 1 4 5 |
| 35* | 0 15 8 | 0 19 5 | 1 2 3 | 1 5 4 |
| 40 | 0 16 9 | 1 1 9 | 1 4 8 | 1 8 4 |
| 45 | 1 0 4 | 1 6 2 | 1 9 8 | 1 15 10 |
| 50 | 1 5 6 | 1 12 4 | 1 18 6 | 2 7 3 |
| 55 | 1 12 9 | 2 2 5 | 2 11 8 | 3 2 11 |

* The Rates proposed for these ages are given below.

The decrease between 25 and 30, and the increase between 30 and 35 are, as a rule, so small that, instead of the rates for ages 25, 30, and 35 given above, I would suggest a uniform premium of 16s. 6d. for one year, 20s. for three years, 22s. 6d. for five years, and 25s. for ten years. When this is done, the rates will be found to be very similar to what is generally charged by offices. Rates below age 25 would require to be dealt with specially. I have taken the temporary assurance premiums of ten representative offices, and find the average rates to be as follows :—

TABLE 25.—*Average Office Premiums for Temporary Assurances.*

| Age. | One year. | Three years. | Five years. | Ten years. |
|---|---|---|---|---|
| 20 | £0 17 2 | £0 19 4 | £0 19 10 | £1 1 4 |
| 25 | 0 18 1 | 1 0 3 | 1 0 7 | 1 2 1 |
| 30 | 0 18 11 | 1 1 0 | 1 1 8 | 1 3 5 |
| 35 | 1 0 1 | 1 2 3 | 1 3 6 | 1 5 10 |
| 40 | 1 2 0 | 1 4 5 | 1 6 5 | 1 9 10 |
| 45 | 1 4 9 | 1 7 11 | 1 10 9 | 1 16 2 |
| 50 | 1 9 0 | 1 13 4 | 1 18 1 | 2 5 9 |
| 55 | 1 15 5 | 2 2 3 | 2 9 8 | 3 2 0 |

It will be seen that these rates, on the whole, do not differ greatly from those I propose should be charged.

I have now dealt with all the classes I proposed to take up, and we may now consider what conclusions can be drawn from the results submitted. I think it has been proved beyond all question, that the rate of mortality varies according to the class of assurance. Dealing with males only, we have seen that they range themselves

very much in the following order: endowment assurance, limited payment, whole life, ascending scale, temporary assurance, contingent-survivorship. The general principle seems to be, the higher the rate of premium, the lower the rate of mortality.

Another conclusion to be drawn is that the selection exercised against an office by a proposer in the choice of his class of assurance is much greater than has generally been supposed, and this will probably have the effect of causing offices to exercise greater discrimination than hitherto in those classes where the mortality has been shown to be heavy. I am doubtful, however, whether it is possible by this means to reduce the rate of mortality materially, at all events in several classes; and this leads me to the third conclusion, namely, that in some classes, notably the whole life without profits and the contingent-survivorship, it will be necessary to increase the rates of premium hitherto charged. In one or two classes it is possible that a reduction might be made, but I think that any one will hesitate to reduce premiums in face of a rate of interest which has a tendency to fall, and a rate of expenditure which has a tendency to increase.

I had hoped to deal with the question of policy values, but I have found it impossible to do so properly in the limited time at my disposal. The present paper has the advantage of dealing with one branch of the subject only, and I may at a future time have the opportunity of inviting your consideration to the other branch of it.

## DISCUSSION.

THE PRESIDENT (Mr. G. M. LOW) said that it had become a commonplace to say that they were much indebted to the author of a paper for his trouble, but he thought that on this occasion, when they considered the great pains Mr. Chatham had bestowed on his paper, and the labour of calculating these rates of premium—in many of the tables, for select lives of each separate age at entry—the compliment of saying that they were indebted to him for the trouble he had taken was not by any means an empty one. As gratitude was said to arise from a sense of favours to come, he thought they would look forward with the greatest interest to Mr. Chatham's promised paper on the even more important question of policy-values. He took it that the present paper would take rank as the first important contribution to their knowledge of the wider questions arising out of the New Experience; for, although they had had Mr. Moir's interesting and suggestive paper at the beginning of the session, Mr. Chatham had gone over a more extended field, and had brought his results to practical issues of the greatest importance.

Mr. A. HEWAT thought the paper had come very opportunely, because he supposed there was no subject interesting the minds of actuaries more at the present time than this one of the New Experience. Mr. Chatham had done well in introducing the premium, which was the item that brought them into contact with the public, and which, along with its component parts, formed the most important element in their calculations. He was one of those who had not lost faith in the old masters, and he sympathised a good deal with Professor De Morgan's remarks on graduation. He had always felt that graduation was apt to alter the trend of the statistics that supplied the experience. He considered one part of De Morgan's remark important, namely, his advice to apply the graduation to the "money results". With regard to the rate of interest used by Mr. Chatham in the calculation of the premiums, namely 3½ per cent, he hoped they might long be able to get that as a minimum rate. He would himself have preferred a 3 per cent basis. As to the premiums charged by the offices, he supposed Mr. Chatham had taken the participating rates irrespective of the bonus systems. He found, taking ages 22 to 57, the difference between the True Select net premiums in the paper and the average office premiums given, meant a loading ranging from 50 to 34 per cent. Twenty years ago he had prepared a paper on Bonuses for the old Actuarial Society, in which his average premium of 41 offices was very near the average premium as now given. But he had not found them give the 35s. bonus Mr. Chatham had arrived at. On the basis of the $H^M$ 3½ per cent net premium for an increasing assurance, he had found that at age 20 at entry the average office premium was sufficient to give a simple bonus of 30s.; at 30, 29s.; at 40, 28s.; and at 50, 26s. Sixteen offices, giving simple bonuses, he then found had declared bonuses averaging 27s. 9d., which came very near to Mr. Chatham's 26s. 6d. Taking offices giving compound bonuses, for ages at entry 20 to 50 and durations 5 to 25 years, the bonuses he then found had ranged from

25s. to 34s. 4d., which agreed fairly well with Mr. Chatham's average of 28s. Discounted-bonus schemes were one of his pet aversions. He had always thought the plan to be a pandering to the demand for cheapness, and was of opinion that it did not bring the best class of customers. He did not think it quite fair to start on the assumption that the mortality among such low-premium rated policyholders was to be the same as that of the full-premium class, and then discount that. It would be seen from Table 4 on page 114, that the True Select net premium for without-profit policies was higher than the similar premium for with-profit policies. Discounted-bonus premiums were, as a rule, less than without-profit rates, and if they were calculated on the without-profit mortality, as they should be, they ought to be higher than at present. Indeed, Mr. Chatham's True Select net premiums for without-profit policies showed that the lowest office rates allow practically no loading at all. He endorsed all that Mr. Chatham had said as to the probability of the expectations regarding that class of policy being realised; and thought it was at variance with sound principle to tamper with the sum assured or the premium on the face of the policy, after it was once issued. With regard to the formula given by Mr. Chatham for loading the premiums, he observed that 1 per cent on the sum assured was allowed for, and a constant of 2s. 6d. per annum for *initial* expenses. But as these initial expenses were not annual, he thought it would be better to make a larger addition for the first year's commission and the other initial expenses. He was afraid that the tendency mentioned by Mr. Chatham, to reduce the premium rates for without-profit policies, was the result of over-competition. As such policies were frequently for large sums, in connection with financial transactions, there was apt to be a temptation to shut an eye and three-quarters to careful selection. One such policy might possibly be equal to the whole business from one branch office in a year, and they had to consider, with such cases before them, whether the heavy mortality was compensated for by the smaller proportionate initial expense. With reference to the case mentioned by Mr. Chatham, it was not usually thought permissible for a reassuring office to deliberately make independent enquiries. It would appear as if there were two standards of selection—one by the office offering a without-profit proposal, and the other by the office accepting a with-profit proposal; but he thought they should assume that the standard of each office is equally good irrespective of class of assurance. On the question of the female extra, he thought Mr. Chatham might have put his sentence in the negative form —that the practice of *not* charging the 5s. per cent is *not* justified by the New Experience. In the case of a publican risk, the agent was apt to say, "This is an entirely exceptional case, and of course an extra premium is not needed from a man of this kind who, I know, is temperate", overlooking the fact that it is these "good" cases that keep the extra rate so low as it is, and that, if they were to be exempt, the extra might be at least doubled for the remainder. In the same way with female risks, they could not eliminate the good from the not so good, and ought therefore to charge the uniform extra rate in all cases. Some actuaries of the old school used to say that, to be equitable, the extra should be refunded after age 50! He did not agree with that, and Mr. Chatham had proved clearly that the extra rate of 5s. was barely enough to cover risk and expenses. Coming to the ascending-scale class, in which the mortality was higher than in the whole-life, he remarked that the man who was not very sure of himself, and who had private doubts as to his successfully passing the medical scrutiny, would choose the ascending-scale policy, and when called upon to pay the higher premium, after say 5 or 7 years, would probably say he had not been "lucky". That was another indication of selection against the office,

because few really sound proposers would burden themselves in later days with so large an addition to their premium. As for endowment assurances, he found that in the valuation returns about the year 1870, less than 5 per cent of the policies then existing belonged to that class; in 1893, the date up to which the New Experience was brought, the proportion was 29 per cent; and he supposed that it would now be something like 35 or even 40 per cent. He fancied that those of them who might take part in the next mortality investigation, say up to 1923, would find considerable changes in the rate of mortality in endowment assurances. He was, however, not quite sure that, as hinted, the rate would approach that of the whole-life class, as the 1 per cent commission might tend to keep it down. The agent would probably not allow a man to insure for say £600 under the endowment assurance class, when he might get him to take a whole-life policy for £1000 at about the same premium, and thus earn £10 commission instead of £6. He thought they must also look upon a proposer preferring an endowment assurance, as making a selection (in favour of the office, however, as well as of himself) somewhat similar to that of the annuitant (who selects against the office), who would not buy an annuity unless he (or she) felt pretty safe to live a good many years to enjoy it. The same was the case with the endowment-assurance proposer whose father lived past the three score years and ten, and who was himself looking forward to a long life. But the man whose father and mother and a few brothers and sisters had died of consumption, would try some low-rated plan—certainly not an endowment assurance. It would be interesting to set down the mortality of endowment assurances alongside that of annuitants. Mr. Chatham seemed to think that where endowment assurances are preferred, they would not in future get profits out of very old lives. He (Mr. Hewat) would prefer, however, to get his "profits" earlier, more frequently, and in the long run, more largely under the more profitable endowment assurance plan. These very old lives, of say over 85, are few, and under compound and increasing bonus schemes, they get too large a bonus. It was one advantage of endowment assurances, that they could see the end of the affair. Some people, who were not actuaries, did not understand what the "profit" on endowment assurances really is, for at the maturity of the policy they would take a compound interest table and try to see how much they had "made" or "lost". Of course every premium paid, after meeting the initial expenses, yields a profit, no matter how soon a claim might arise; and he thought the "profit" on endowment assurances would, on the whole, be better than that on whole-of-life policies. However debatable that point may still be, it had been proved to demonstration that the mortality was better, and he thought it would not be difficult to demonstrate that so also are the interest and loading profits. With regard to contingent-survivorship assurances, he was afraid that there also the rates were regulated very much by competition. The transactions were usually large and the rate of premium small; and frequently they were on the lives of persons physically and financially weak. They had this doubtful advantage, however—they swelled the amount of new business, irrespective of the profit to be derived, and that was a temptation to be guarded against. He was glad to see Mr. Chatham recommending offices to revise and increase their rates. As to the promised paper on policy-values, he would suggest that it should come from some younger member, fresher from the kindly attentions of the examiners, and then perhaps they would have the benefit of Mr. Chatham's criticism of it, and of other older members. In closing, Mr. Hewat thanked Mr. Chatham for his interesting paper, and congratulated the Faculty on having secured such a valuable contribution to their first series of Transactions.

Mr. J. J. M'Lauchlan said that the question of the rates of premium

was vital in insurance business, and much of the progress made had consisted of the evolution in various ways of premium rates. At first no attempt was made to discriminate between persons of different ages. Subsequently the element of age was introduced; then the medical examination; and now they occasionally, in a rough way, made some allowance for the class of insurance. Mr. Chatham had now brought before them, in a very admirable and thorough manner, the results of the New Experience as regards the mortality among persons insured under different classes of policies, and in future there would be no difficulty in giving effect to the special mortality found in each class of insurance. The causes that seemed to him to influence the mortality of the different classes, were (1) the desire of the person effecting the insurance to get the most he could for his money, choosing the particular class that he would make most of, and (2) the desire of persons effecting insurances for security purposes, to get the necessary protection at as low a cost as possible. There was also the effect of the class of society to which a proposer belonged. He thought it had been shown by Mr. Stenhouse (*Transactions of the Actuarial Society of Edinburgh*, vol. ii, p. 260) that there was a lighter mortality among persons insured under the larger policies. That would, no doubt, be found to be the case for provident policies and not for financial policies. It was an interesting circumstance that, in the case of females, the mortality in the without-profits class was practically the same as in the with-profits, being in great contrast to the male mortality in that respect. Mr. Chatham had suggested that in the without-profits class, a certain proportion of both males and females were of extravagant habits, but that those of the former lay in directions which are generally more detrimental to health. He (Mr. M'Lauchlan) thought that there were some other considerations to be taken into account. Judging from the figures given by Mr. A. E. Sprague in his paper read before the Actuarial Society in 1896 (*Transactions of the Actuarial Society of Edinburgh*, vol. iii, p. 389), it appeared that among policies on female lives the proportion of financial policies was twice as large as it was among policies on male lives, and this was what might be expected. Females in the insuring classes did not usually earn an income, and it was therefore seldom necessary to insure their lives for provident purposes; but it was often necessary for financial purposes. It might be expected that the larger proportion of these financial policies would have an unfavourable effect on the female mortality, but he thought that, while a financial policy on the life of a male was frequently an indication of extravagance, it was often otherwise in the case of females. In Scotland they were in the way of making a widow's life-interest alimentary; but in England the practice was not the same. There were many cases where a son induced his mother to insure her life in order to raise money for his use. That, of course, was an indication of extravagance on the part of the son and not of the mother, and of course such cases would not have the same prejudicial effect on the mortality. He had often seen cases where a mother had been placed in straitened circumstances in consequence of having anticipated her life-interest; but the mortality, even in cases of this kind, would be very different from that of young men borrowing on their expectancies. The comparison given by Mr. Chatham of the mortality of endowment assurances and limited payments was very interesting. A different conclusion had been reached from that arrived at by Mr. Moir, whose calculations were based on the first year's experience. Mr. Chatham attached considerable importance to the fact that a much larger percentage of without-profit policies is included in the endowment-assurance data, as tending to increase the rate of mortality. He (Mr. M'Lauchlan) thought it ought not to be assumed that the existence of

without-profit policies in a body of endowment-assurance policies would have the same effect in increasing the rate of mortality, as if the endowment assurances were whole-life policies. The fact that a number of the without-profit whole-life policies were effected for financial purposes, had a most important effect in bringing about the difference in the rates of mortality that had been observed. He had compiled, from the general summaries given in the blue-books, the following table giving information as to the average amounts of policies under different classes :—

TABLE SHOWING THE NUMBER AND AMOUNT OF ASSURANCES IN FORCE UNDER CERTAIN CLASSES, AS SHOWN BY THE RETURNS OF THE COMPANIES, WITH THE AVERAGE AMOUNT OF THE POLICIES.

| Class. | | With Profits. | | | Without Profits. | | |
|---|---|---|---|---|---|---|---|
| | | Number. | Amount. | Average Policy. | Number. | Amount. | Average Policy. |
| Whole life, | 1888 | 665,914 | £338,628,167 | £509 | 79,921 | £55,015,539 | £688 |
| | 1894 | 722,609 | 352,150,940 | 487 | 122,957 | 61,839,843 | 503 |
| | 1900 | 783,426 | 376,616,030 | 481 | 139,068 | 67,929,938 | 488 |
| Limited payments, | 1888 | 24,854 | 14,096,205 | 567 | 4,256 | 1,979,781 | 465 |
| | 1894 | 29,020 | 17,383,206 | 599 | 5,349 | 2,576,142 | 482 |
| | 1900 | 43,614 | 26,797,404 | 615 | 8,707 | 3,802,180 | 437 |
| Endowment assurances, | 1888 | 116,831 | 24,838,964 | 213 | 24,500 | 6,904,291 | 282 |
| | 1894 | 340,000 | 59,580,714 | 175 | 30,808 | 9,401,566 | 310 |
| | 1900 | 774,264 | 129,515,961 | 168 | 46,726 | 14,627,270 | 313 |
| Contingent-survivorship, | 1888 | 57 | 66,985 | 1,175 | 2,716 | 4,646,902 | 1,711 |
| | 1894 | 35 | 32,269 | 922 | 3,156 | 5,327,759 | 1,689 |
| | 1900 | 30 | 55,759 | 1,859 | 3,895 | 5,678,840 | 1,458 |

The small average amount of the endowment assurances indicated that there were very few financial policies among them. He was inclined to think that the large percentage of endowment assurances without profits was to be accounted for by the fact that these were introduced into insurance practice much earlier than the corresponding policies with profits. In fact, one large office issued none of the latter to this day. It had often happened that the man who took out his endowment assurance without profits was taking the only policy he could get in that form, and there was not the same selection against the company, or the same reason to suppose that the mortality would differ much from that of endowment assurances with profits. The question of the probable future mortality of the class was of interest, as bearing on the question of whether it was possible or desirable to reduce premiums. It was important to remember that their experience of this class of assurance extended over a very limited period. He had put down in the table the amount of endowment assurances existing at three different dates, and it would be seen that the endowment assurances had increased from about £32,000,000 in 1888 to about £69,000,000 in 1894 and about £144,000,000 in 1900. It was just possible that the general conditions affecting the mortality of assured lives might have had some peculiarity about them in the comparatively short period during which the endowment assurances came into the experience. To be able to predict future mortality with confidence, they must have an average extending over a certain time as well as over a certain number of cases. He concluded by expressing the

opinion that the paper before them was a long and important step in the direction of making the New Experience available for practical purposes.

Mr. D. PAULIN said he was much impressed with the practical value to the profession of Mr. Chatham's admirable paper. The ultimate results were most interesting and instructive, but he would have liked if the enquiry had gone further and had discussed the causes which led to these results. He would like to know if the reason for the differing mortality experience in the various classes of assurances was owing to an attitude of mind on the part of proposers at the time of proposing, or to conditions of life effeiring to the sections of the insuring community who selected the cheap or dear premiums. As regards low-rated contingent-survivorship policies on the lives of borrowers, it was not difficult to diagnose how these lives came to be under-average. But as between ordinary whole-life policies, and limited-payments and endowment assurances, it was more difficult to arrive at a reason. Could it be that those who were able to afford the higher premiums, were in a better and more comfortable condition of life and were able to command greater medical skill, and so lived longer? The holder of an endowment-assurance policy had certainly an object in living, and it had yet to be proved that the mental attitude to death has not something to do with the duration of life. He instanced a case in his own experience, where a gentleman attributed his recovery from an illness which was thought by the doctors and his family to be hopeless, to his having taken out an endowment-assurance policy ten years before. He hoped Mr. Chatham would put the Faculty under still greater obligation by applying his mind to the further elucidation of the many problems with which this great investigation brought them face to face.

Mr. J. M. WARDEN said, with reference to Mr. Chatham's suggestions as to the probable future mortality of endowment assurances (page 125), that he had had to look into the mortality of endowment assurances on two occasions. The first period was for five years ending in 1893—the final year of the New Experience. There were 18,750 years of life approximately, the average age at exposure being 34½, and the average duration of the assurance 3¼ years. The deaths were 72, giving the rate of mortality overhead as ·0038, which corresponded fairly well with the New Experience as to endowment assurances. In the second period, 1893-8, there were 33,855 years of life approximately, the average age was 36, and the average duration 4 years; and the number of deaths being 197, the rate of mortality was ·0058. The differences in age and duration having been so trifling, one would hardly have expected so great an increase in the mortality. Bearing in mind especially that the tendency in recent years had been to effect endowment assurances for shorter periods than formerly, he could not help thinking that the difference might be caused by an influx of endowment-assurance policyholders from a somewhat lower level of society than before. Though the figures he had been able to give were very rough, he thought they tended to confirm Mr. Chatham's idea that the rate of mortality among endowment assurances was already increasing. It was curious to observe from Tables 1 and 4, that the net premiums at age 67 for whole-life without-profit policies, by the True Select method and the Select and Ultimate method, were less than the corresponding premiums for with-profit policies. The comparison in Table 6 being according to the Grouped Select and Ultimate method, the point was not referred to in the paper. He supposed that, this being the extreme age for which the rates had been calculated, it was simply an accident, due to the observations being few. With regard to the formula for loading premiums, as Mr. Hewat had pointed out, an annual constant did not appear to be a suitable way of providing for the initial office expenses, and its capital value at age 20 would be much larger than at age 60. He was

under the impression that, in the formula given by Dr. Sprague at a time when the first year's commission formed the bulk of the initial expenses, the constant of 2s. 6d. per £100 was intended to provide an annual profit to the office. The initial expenses being now much greater than allowed for by the 1 per cent in the formula, the method of loading would require to be reconsidered.

Mr. A. E. SPRAGUE said that he thought De Morgan's remark hardly justified their neglecting graduation nowadays, because he expressly said it only applied "until observations of human mortality become more extensive and correct". Observations were not only more extensive than in De Morgan's day, but more correct; and graduation was really necessary for their purposes. No workman would set to work with a rough tool, and the mortality table should be made smooth before they started to calculate premiums. One strong objection to graduating monetary values occurred in the case of a complicated benefit. If calculated from an ungraduated mortality table, the various errors in it might be cumulative and produce a very large final error; and he did not suppose Mr. Chatham would construct a whole table of complicated benefits in order to adjust a single value. He differed also from Mr. Chatham as to an office reassuring with profits a policy effected without profits. It was news to him that there was a different standard for different classes. If a life was not good enough to be accepted at the ordinary rate in the with-profit class, he did not think any office would take it at ordinary rates under a without-profit policy. If there was a doubt as to eligibility, the tendency was rather to transfer him from the without-profits class, and make him take an endowment assurance. There was one point that indicated that this investigation into the mortality of the different classes would not be so useful as one would think. It was a great anomaly that a man should be able to take a limited-payment policy at a cheaper premium than the whole-life premium; but a comparison of tables 1 and 13 showed that for ages above (about) 40, the net premium (with-profits class) payable for the whole of life was actually larger than the net premium payable for only 30 years; and from tables 6 and 13 it would be seen that, for all ages above (about) 30, the whole-life premiums for without-profits were larger than those payable for 30 years only.

Mr. J. NICOLL thought that, in the comparisons given on page 113 as regards bonuses, it would have served the author's purpose better if the premiums of the offices giving simple bonuses had been kept apart from those of offices giving compound bonuses. Mr. Chatham had shown that the premiums charged by some offices for without-profit policies were much too low, but this was only what they already knew even from the $H^M$ Table. One way to check the tendency to charge inadequate premiums would be to rescind the rule under which it is understood that offices are not to be referred to by name in papers and discussions at the meetings. He could scarcely see the force of the remarks in connection with the reassurance point referred to on page 116 of the paper. Granting, as is shown by the New Experience, that assurers who effect policies under the without-profits scale are as a class subject to higher mortality than those who assure with profits, it would still seem to be to the advantage of the reassuring office to get from the principal office in such a case the with-profits, in place of the without-profits, rate of premium. There was an impression that lives under reassured policies, both with and without profits, were scarcely so good as those under direct policies. The question was perhaps worth investigating, but in one case he had heard of, where an investigation had been made into such assurances, the result did not show that the lives were worse than those under direct policies. He thought Mr. Chatham was not quite comfortable in his own mind that endowmen

assurances are always so desirable for the offices as is sometimes supposed. The remarks on the subject, in a paper by the late Mr. Newbatt, were pretty convincing—the chief objections to endowment-assurance business being (1) the greater difficulty entailed upon the office in investing the larger premiums payable for such policies, to the detriment of their ordinary assurances; (2) the loss of profit on mortality in after years on these superior lives; and (3) the greater expense incurred in securing new entrants to take the place of these endowment-assurance policyholders who retire at the end of a comparatively short term. Probably this class of business has not been in existence for a sufficiently long time to show the full force of these objections to it.

Mr. S. F. M. Cumming supposed that unadjusted data had been used in the construction of premiums to a larger extent on this occasion than ever before; and although the plan worked well enough in regard to whole-life policies with profits, it did not apply quite so well to the without-profit class. If the premiums for that class were set down in a curve, it would be found that they did not run regularly, the differences of the figures in the second column of Table 5 being as follows:—3s. 9d., 5s. 8d., 4s. 3d., 9s., 14s. 4d., 12s. 9d., and 25s. 6d. He presumed that it was the intention to smooth out the loaded premiums afterwards; but thought it was much better to adhere to the usual plan and graduate the mortality table at the outset. It would then be suitable for all kinds of calculations that might be required. He thought the most important question suggested by the paper was whether the premiums for different classes of assurances should in future be based upon different bodies of facts. At present nearly all premiums were derived from the Institute Experience in some form or other —either the $H^M$ or the improved Select form—except perhaps the contingent-survivorship premiums, which were the most elastic premiums they had to deal with. If in that class they combined a little extra mortality in the assured life with a little lighter mortality in the counter life, the result was a great difference in the premium; and therefore they need not be surprised that the difference in the premiums charged was sometimes as much as 40 per cent.

Mr. W. R. Halliday thought that, if the picture drawn by Mr. Hewat of a somewhat doubtful life carefully studying prospectuses for the smallest available rate of premium were a true one, they might expect such a process of selection to result in the effecting of larger assurances, rather than the tender of smaller premiums to the offices. The higher rate of mortality shown in the paper would in these classes tend to fall on the larger policies, and the strain on the funds would be greater than is indicated by the mortality. On the question of female lives, there was considerable divergency of practice among the offices. The extra mortality was very variable. It was extremely heavy at the beginning, and gradually decreased. A uniform charge of 5s., while more adequate than the extras sometimes charged, was hardly fair as between individual entrants. He remembered once noticing in a paper a suggestion that in cases of first confinement there was a considerable selection exercised against the office. Whether that was so, he was not aware; but he thought the statistics of the present paper suggested it, and, if so, more discrimination might be desirable than was implied in the uniform 5s. charge.

Mr. W. A. Robertson said that the first table given in the paper showed not only that the effect of selection is not exhausted in ten years, but also that it is more enduring at the older than at the younger ages, the difference between the True Select and the Select and Ultimate net premiums practically increasing throughout. There would therefore seem to be two ways by which to calculate premiums—either to base them upon the experience of the particular age at entry (grouping if necessary from the point

of view of data), or to join the select rates on to an aggregate experience after a varying term of years. He thought the comparison of the whole-life premiums, based on their own and on the limited-payment experience, was particularly instructive; but possibly conclusions based on Mr. Chatham's method of calculation did not state the whole case. An aggregate table derived from whole-life statistics might be expected to exhibit lighter mortality at the older ages than one based on the limited-payment statistics, on account of the different incidence of the ages at entry in the two classes. Comparatively little limited-payment business was done at ages over 50; but this was not the case with whole-life assurances, which would therefore get the benefit of their experience at the older ages. The same remark would apply to a comparison of $H^{M(10)}$ experiences. Hence he would infer that, if whole-life premiums based on limited-payment Select and Ultimate data were lower than the corresponding whole-life premiums based on their own experience, a True Select comparison might be expected to exhibit an even greater difference. Two experiences might give identically the same select rates of mortality, and yet when combined to form an aggregate experience, the resulting rates of mortality might be wholly different. Endowment-assurance policyholders might be superior to limited-payment policyholders; but he did not think it necessarily followed from the figures given by Mr. Chatham. Taking the annual premiums for a 10 years' assurance, if they left out age 55, where the large difference would seem to be accidental, the rates formed from the endowment-assurance and limited-payment data were practically the same. For the longer terms, the so-called superiority of endowment assurances began to assert itself, but it was just at that very time that they began to mature and disturb the rates of mortality. Now these lives whose policies were maturing had given the class the benefit of the light years of mortality, and were now relieving the class by withdrawing when their mortality would be most felt. On the other hand, the limited-payment class continued to contribute the heavy mortality after their assurances were paid up. He thought the paper clearly showed that the only sound basis for office premiums was the True Select method. This New Experience should in future form the practical basis of their premium calculations; but in applying the results of the individual observations to their respective classes, they must guard against the introduction of anomalies. They were told that the non-profit whole-life premiums require to be increased, while the limited-payment and endowment-assurance premiums might properly be diminished. The combined effect of these changes might give rise to anomalies. Again, if joint-life premiums were based on the single-life statistics, they might take it that no increase would be justifiable for joint-life risks. On the other hand, the contingent premiums fell to be materially increased; and the sum of the two contingent premiums might be out of all proportion to the joint-life premium.

Mr. Chatham, in reply, said he quite agreed with Mr. Hewat's criticism of the discounted-bonus premiums. There was no doubt the mortality of that class would more nearly approach that of without-profits, and perhaps, therefore, the premiums should have been calculated on the without-profit data; but the results were bad enough on the with-profit basis. The fact pointed out by Mr. Hewat that whole-life without-profit premiums were very low in a number of offices, might be due to the offices having founded all their premiums—including the without-profits—upon their own general experience. The point he had mentioned as to reassurance, and which had been discussed far more than he expected, arose in this way, that two offices to which a certain reassurance of a without-profits policy was offered on the with-profits plan, both took the same view without any consultation, namely, that the mortality among without-profit policies was higher than in the case

of with-profit policies, and that the former class of policy was not entitled to participate in the bonuses of the latter class. The principal office, however, maintained that in this particular case the life was quite up to the standard of the with-profits class, and allowed independent enquiries to be made. The results were quite satisfactory, and accordingly the desired policies were granted. He thought the figures he had given showed that there is a difference in the standard of with- and without-profit lives. With regard to Mr. Hewat's remark that the old lives took too much bonus in proportion to the premiums contributed, he should say that was the fault of the bonus system and not of the premiums. He was glad to see the concurrence of opinion as to the charge of 5s. per cent on females. Although, as pointed out by Mr. Halliday, the extra risk was higher at some ages than at others, he hesitated to disturb the present well-known rule. He agreed with Mr. M'Lauchlan that the mortality of endowment assurances without profits would not likely be so high as in the whole-life class without profits. He had given the figures for the last-mentioned class as showing the maximum. He thought it was a combination of attitude of mind and position in society that accounted for the differences existing in the various classes. There was first the unconscious selection against the office, in that the assured, generally speaking, chose the class that appeared most advantageous to himself; and then those who could afford a higher premium were in better circumstances and had better conditions of life than those who wished to get in at the lowest possible rates. Mr. Warden's figures as to the mortality among endowment assurances during two periods were interesting, as confirming a suggestion in the paper. He had not observed the apparent anomaly with regard to the rates at age 67; but doubtless it was due to the observations being rather too small at the high ages. With reference to Mr. Sprague's remarks on graduation, he would point out that De Morgan's statement was that, until observations were more extensive, he preferred the expectation of life. Mr. Sprague had referred to some of the premiums in the different classes being inconsistent with each other; but it must be remembered that in some cases the loading would no doubt modify these. With reference to Mr. Cumming's remark as to irregularities in the series of premiums, it was certainly intended that they should be graduated. They had, however, served his purpose without being graduated, and it should be borne in mind that the rates used for comparative purposes were not the True Select, but the Grouped Select and Ultimate. He had no doubt Mr. Robertson would concede that the period of selection would vary, and that selection would vary also with class. He did not imply that the 10 years' period should be fixed irrevocably; but he could not quite follow Mr. Robertson's remarks as to aggregate tables, because it had been shown that the difference was extremely small—1½ per cent in the case of with-profits, and 2 per cent in the case of without-profits—so that the difference must be very small when they took ten years as the period during which selection lasted.

The President said that one lesson they had learned from this meeting, was that if such papers as this were to be read in future sessions—papers giving rise to so much interesting discussion—their rules of procedure would have to be revised, and a time limit for speakers introduced. Something had been said about the "younger mind", and the desirableness of its applying itself to such questions as Mr. Chatham had dealt with. He found the younger mind sometimes too apt to think that every possible subject had been exhausted; but the fact was that on no subject connected with their profession had the last word been said, or would it, perhaps, ever be said. If any one, young or old, would take hold of any subject, he would find its possibilities grow upon him, and there would be no difficulty, after a little reflection and consideration, in saying something useful and

interesting about it. He moved a hearty vote of thanks to Mr. Chatham for the paper he had read, and for the great amount of useful information he had put before them and elicited from the other speakers. If any members had been prevented from joining in the discussion by its being brought to a close at that hour, they might send in their observations to the editor, and any such communications found to be suitable would appear in the published Transactions. This was the last meeting of the present session. In all five meetings had taken place, and he thought the members would agree that they had been most successful, and had entirely fulfilled the anticipations that had been formed in regard to them.

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# CORRESPONDENCE.

## MR. CHATHAM'S PAPER—THE REASSURANCE QUESTION.

*To the Editor of the Transactions of the Faculty of Actuaries.*

SIR,—Inasmuch as there appears to be a difference of opinion regarding the advisability of an office "*B*" agreeing to reassure *with* profits a without-profit policy effected with another office "*A*", I shall be glad if you will permit me to add the following to what has been said on the subject.

It is a principle now pretty generally accepted, that a reassuring office should not be asked to run a greater risk, in proportion to the sum assured, than the office effecting the reassurance. I do not say that the principle is a correct one; but, if it be accepted, it appears to follow that Mr. Chatham's conclusion is sound.

In the discussion which followed the reading of the paper, Mr. Sprague drew attention to the fact that offices will as readily accept first-class lives on the one scale as on the other—not having separate standards for the two. This is admitted, and renders it, at first sight, not apparent why "*B*" (in our hypothetical case) should refuse to grant a participating policy when the proposer has proved his eligibility to the satisfaction of "*A*'s" medical officer. It must, however, be borne in mind that "selection"—in so far as it affects the question under discussion—is exercised not by the office but by the proposer. In this instance the latter has selected the without-profit scale, and therefore it may reasonably be assumed that he will be subject to the heavier mortality apparently existing among the holders of non-participating policies. Now, Mr. Chatham's figures indicate that "*B*" would run the risk of having its bonus loading to a considerable extent expended in counteracting the effect of that higher mortality. "*A*" does not run this risk, and hence if equality, as between the two offices, is to be maintained, "*B*" cannot agree to "*A*'s" proposition, unless satisfied that the table selected by the proposer has been chosen for some reason other than that supposed usually to influence the choice.

It only remains to be added that, while the above reasoning holds good, yet circumstances alter cases. The premium scales of the two offices may render the conclusion inapplicable in their case. It is just in those cases, however, that the reassuring office is *not* likely to be asked to consider the question.

The point is perhaps rather of theoretical interest than of practical importance.

I am, SIR,
Your obedient Servant,
JOHN EDGAR.

22 GEORGE STREET,
EDINBURGH, 12 *March* 1902.

No. 1. Vol.

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# TRANSACTIONS

OF THE

# FACULTY OF ACTUARIES.

No. 6.—Vol. I.

**On Policy Values deduced from the Mortality Experience (1863-1893) of British Life Offices—By James Chatham, F.F.A., F.I.A.**

**With Abstract of Discussion thereon.**

**Correspondence.**

LONDON:
CHARLES & EDWIN LAYTON,
FARRINGDON STREET.
1902.

TRANSACTIONS

OF THE

FACULTY OF ACTUARIES.

*On Policy Values deduced from the Mortality Experience* (1863-1893) *of British Life Offices. By* JAMES CHATHAM, F.F.A., F.I.A., *Secretary and Actuary to the Scottish Life Assurance Company.*

[Read before the Faculty, 10 November 1902.]

IN the Paper which I had the honour of submitting to you at the close of last session, "On Premiums deduced from the Mortality Experience of British Life Offices", I expressed the hope that at a future time I might have an opportunity of inviting your consideration to the other branch of the subject, namely, policy values. This hope I am now able to realise.

I have based my calculations upon the "Grouped Select and Ultimate" rates of mortality obtained from the unadjusted observations in the manner explained in the paper referred to. The period during which selection was assumed to exist was, it will be remembered, ten years; and the observations at five consecutive ages at entry were grouped together, and the results assumed to relate to the central age of the group. A correction to first differences was made, as the average age of the group differed in most cases from the central age at entry, owing to the number of entrants varying at each age. The net premiums are given in that paper, and the annuity values from which they were deduced have been used in the calculation of the policy values. Any additional annuity values required were calculated in the same way as before, a correction to first differences being made for the age, when necessary. Notwithstanding that the unadjusted observations have been used, the results will be found to be very regular.

The rate of interest used in the calculations is, as before, $3\frac{1}{2}$ per cent. This is a higher rate than is now usually employed in valuations; but the adoption of a lower rate would have necessitated the recalculation of all the various functions, and this did not seem to be worth while for the object I have in view. A difference of a half per cent in the rate of interest matters little for comparative purposes, so long as the same rate is used throughout the calculations.

The select policy values thus obtained form the standard for the comparisons given in this paper. They are based upon the experience of each section in the New Experience to which the class of policy dealt with relates. I have for shortness generally called them "New Select". The tables with which they are compared are the $H^M$ and the $O^M$, also in two cases the $H^M$ and $H^{M(5)}$ and the $O^M$ and $O^{M(5)}$.

These select values are of course the ideally correct ones to employ in the valuation of the liabilities of an insurance office. The amount of work involved, however, in the adoption of select tables in a valuation, would be so great, that I do not think it is ever likely to become general, more particularly as the period during which selection lasts has been shown by the New Experience to be greater than the period hitherto adopted, namely, five years. In connection with this, I thought it would be useful to see what the difference in premium would be, if the period during which selection lasts had been taken as five years instead of the ten years adopted in my paper. The following table shows the "Select and Ultimate" net premiums at $3\frac{1}{2}$ per cent for whole-life assurances—males—when selection is assumed to pass off in (1) 5 years; (2) 10 years:—

TABLE 1.—$100\pi_x$, Whole-Life With Profits—Males.

| Age. | Selection passing off in | | Difference. |
|---|---|---|---|
| | 5 years. | 10 years. | |
| 22 | 1·344 | 1·354 | –·010 |
| 27 | 1·542 | 1·542 | ·000 |
| 32 | 1·787 | 1·786 | ·001 |
| 37 | 2·117 | 2·085 | ·032 |
| 42 | 2·555 | 2·542 | ·013 |
| 47 | 3·076 | 3·051 | ·025 |
| 52 | 3·833 | 3·765 | ·068 |
| 57 | 4·683 | 4·536 | ·147 |
| 62 | 5·949 | 5·876 | ·073 |
| 67 | 7·870 | 7·727 | ·143 |

The differences in the last column indicate that, contrary to what has generally been supposed, selection endures for a shorter period in the case of young lives than in those of middle age. In the case of the former, it seems to be exhausted after five years from entry, whereas, in the case of the latter, the figures in my former paper show that it is not exhausted even after ten years.

Reverting to the question of the tables—select or aggregate—likely to be adopted in valuations in the future, I think there are clear indications that the latter will be used. There is a growing tendency to adopt simpler methods of calculation than formerly. For instance, it is rare now that policy values are calculated to three places of decimals, although this was quite a common practice not so many years ago. Again, a number of offices have adopted the method of valuation book-keeping advocated by me in a paper[1] read before the Institute of Actuaries some years ago, and the number doing so is yearly increasing. The method there recommended is not adapted for the employment of select tables, although the $H^M$ and $H^{M(5)}$, and the $O^M$ and $O^{M(5)}$ tables may be used.

The classes of assured lives I shall deal with on this occasion are as follows:—

(1) Whole-life, males—
With profits,
Without profits,
(2) Whole-life, females—
With profits,
Without profits,
} With Premiums payable uniformly throughout life.

(3) Limited payment, with and without profits, males.
(4) Endowment assurances, " " "

These are all the classes dealt with in my former paper, with the exception of ascending scale, contingent-survivorship, and temporary assurances.

The policy values were calculated for ages at entry 22, 27, 32, . . . in the case of Whole-Life Assurances, and for ages 20, 25, 30, . . . in the case of Limited-Payment and Endowment Assurances; the durations being 5, 10, 15, 20, 30, 40, 50. Tables of these values, along with the $H^M$ and the $O^M$, are given at the end of this paper, as they may prove useful for future reference; but throughout the body of the paper I shall employ ratios only, as they show more readily the differences between the policy values according to the various tables. As already stated, the principal tables used for comparison are the $H^M$ and the $O^M$; and in all cases I have given the ratio per cent of the policy values I have deduced from the New Experience to the

[1] *J.I.A.*, xxxii, 393.

values according to these tables, thus showing at a glance whether the standard values I have adopted—the select policy values deduced by me—are greater or less than the values with which they are compared. In all the classes, except those relating to females, I have given three ratios,

(1) New Select to $H^M$,
(2) New Select to $O^M$,
(3) $O^M$ to $H^M$;

but in the first class the last-mentioned ratio has been given for one section only, namely, the with profits section. The ratios of the New Select to the $H^M$ and $H^{M(5)}$ and to the $O^M$ and $O^{M(5)}$ have also been given for that section, and for endowment assurances. In the case of females, the first two ratios only have been given, as it is unnecessary to repeat the third.

Before I proceed there is one point to which I should like to draw attention. In all the tables the usual policy values have been calculated. Bonuses have not been taken into account, and they will no doubt—particularly when the amounts are large—appreciably modify the results of actual valuations. The allowance for the distribution of the premium income throughout the year, usually two-thirds, has also not been taken into account, but this will not affect the results materially.

## Whole-Life Assurances—Males.

*With Profits.*—This is the class which contains the largest number of observations, and I now submit the following tables of ratios :—

Table 2.—Whole-Life With Profits—Males—Ratio per cent of New Select to $H^M$.

($H^M$ *represented by* 100.)

| Duration. | Age at Entry. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 117·5 | 118·5 | 117·7 | 116·1 | 113·4 | 113·9 | 111·3 | 112·6 | 117·2 |
| 10 | 107·5 | 110·5 | 109·7 | 108·9 | 108·0 | 106·8 | 106·0 | 109·7 | 107·6 |
| 15 | 103·8 | 106·7 | 104·5 | 105·0 | 103·7 | 103·4 | 103·3 | 103·8 | 104·1 |
| 20 | 103·0 | 103·5 | 102·8 | 102·5 | 102·0 | 102·1 | 100·9 | 102·2 | 102·5 |
| 30 | 100·9 | 101·2 | 100·7 | 101·0 | 100·1 | 100·2 | 100·3 | 101·4 | 97·3 |
| 40 | 100·0 | 100·4 | 99·7 | 99·9 | 100·0 | 100·4 | 97·5 | 92·5 | |
| 50 | 99·4 | 99·7 | 99·8 | 100·2 | 97·9 | 94·0 | | | |

TABLE 3.—Whole-Life With Profits—Males—Ratio per cent of New Select to $O^M$.

($O^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 106·3 | 110·9 | 112·2 | 112·3 | 113·5 | 113·0 | 112·3 | 114·0 | 117·6 |
| 10 | 99·2 | 104·6 | 105·6 | 107·4 | 107·6 | 106·9 | 107·1 | 110·5 | 110·3 |
| 15 | 97·2 | 102·1 | 102·3 | 103·9 | 103·9 | 104·0 | 104·1 | 105·7 | 105·3 |
| 20 | 97·7 | 100·6 | 101·2 | 102·1 | 102·5 | 102·6 | 102·4 | 103·3 | 102·7 |
| 30 | 98·2 | 99·9 | 100·4 | 101·0 | 101·1 | 100·9 | 100·5 | 100·0 | 100·2 |
| 40 | 98·9 | 100·0 | 100·2 | 100·4 | 100·1 | 99·4 | 99·6 | 100·8 | |
| 50 | 99·4 | 99·9 | 99·7 | 99·3 | 99·5 | 100·3 | | | |

TABLE 4.—Whole-Life With Profits—Males—Ratio per cent of $O^M$ to $H^M$.

($H^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 110·5 | 106·9 | 104·9 | 103·4 | 99·9 | 100·7 | 99·1 | 98·7 | 99·6 |
| 10 | 108·3 | 105·7 | 103·9 | 101·4 | 100·3 | 99·8 | 99·0 | 99·3 | 97·6 |
| 15 | 106·7 | 104·6 | 102·2 | 101·1 | 99·9 | 99·5 | 99·3 | 98·1 | 98·8 |
| 20 | 105·4 | 102·9 | 101·6 | 100·4 | 99·6 | 99·6 | 98·5 | 99·0 | 99·9 |
| 30 | 102·7 | 101·3 | 100·4 | 99·9 | 99·0 | 99·3 | 99·8 | 101·4 | 97·1 |
| 40 | 101·1 | 100·4 | 99·5 | 99·6 | 99·8 | 101·0 | 97·9 | 91·8 | |
| 50 | 100·0 | 99·9 | 100·0 | 100·9 | 98·3 | 93·7 | | | |

If we examine table 2, we find that for duration 5 and age at entry 22, the select reserve according to the New Experience, that is to say, the proper reserve, is 17·5 per cent greater than the $H^M$, this percentage being much higher than at any subsequent duration. A similar feature occurs at every other age at entry. This, of course, is due to the fact that the standard employed is a "select" value, and that the table used in the comparison is an "aggregate" one; and it shows that when an office employs an aggregate table in its valuation, a considerable allowance is thereby made for the heavy initial ex-

penses incurred in the acquisition of new business. It will be seen that the same thing occurs in table 3; and as it is a feature of all the tables where a "select" value is compared with an "aggregate" one, I will not refer to it again.

But there is another very obvious feature in the tables, namely, that the ratios as a rule diminish as the duration extends; that is to say, that an office employing the $H^M$ or $O^M$ table in its valuations gradually approaches to the reserve required by a select valuation. It takes a considerable time, about 40 years, until the policy values of the $H^M$ and the $O^M$ tables come up to the select values of the New Experience, but still they finally do so, and that in a gradual manner. The total reserve by the former tables will, of course, always be too small in comparison.

There is still another feature, however, to which I should like to direct your attention, and that is the manner in which the ratios for the same duration increase or diminish as the age at entry becomes older. If we except age at entry 22, the ratios of the New Select to the $H^M$ in table 2 have a tendency, as a rule, to diminish and then increase; the ratios of the New Select to the $O^M$ in table 3 almost invariably increase; while the ratios of the $O^M$ to the $H^M$ as steadily decrease. Those features are no doubt due to the incidence of the mortality in the $H^M$ and the $O^M$ tables. We shall find afterwards that they occur more or less in the other tables.

The $H^M$ is generally believed to be a good table for valuation purposes—about 70 per cent of the offices adopt it—and the ratios in table 4 show that the $O^M$ gives higher values at the younger ages at entry and lower at the older. In the valuation of an average office, the total reserves by the two tables will probably not differ much from each other.

Briefly, it may be said that the select policy values by the New Experience are in almost every case higher than the $H^M$, and these in turn are, in the majority of cases, higher than the $O^M$. Where the age at entry and duration together do not exceed 50, however, the $O^M$ values are, with one exception, greater than the $H^M$, and upon this it depends whether the reserve according to the $O^M$ table will be greater or less than that according to the $H^M$, subject always to any modification which the bonuses may introduce.

As about 30 per cent of the offices use the $H^M$ and $H^{M(5)}$ tables in their valuations, I thought it would be interesting to see how the values by those tables and by the corresponding $O^M$ and $O^{M(5)}$ compare with my standard values. Similar ratios for them, therefore, are given in the following tables:—

TABLE 5.—Whole-Life With Profits—Males—Ratio per cent of New Select to $H^M$ and $H^{M(5)}$.

($H^M$ *and* $^{M(5)}$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 80·8 | 93·3 | 97·7 | 101·3 | 101·5 | 104·4 | 105·0 | 107·8 | 113·8 |
| 10 | 93·9 | 99·8 | 101·8 | 102·5 | 103·1 | 103·6 | 103·7 | 108·1 | 106·5 |
| 15 | 96·4 | 101·2 | 100·1 | 101·6 | 101·6 | 101·8 | 102·3 | 103·0 | 103·5 |
| 20 | 98·7 | 100·0 | 100·2 | 100·8 | 100·8 | 101·4 | 100·3 | 101·8 | 102·2 |
| 30 | 99·0 | 99·9 | 99·9 | 100·4 | 99·7 | 99·9 | 100·1 | 101·0 | 96·6 |
| 40 | 99·3 | 100·0 | 99·4 | 99·7 | 99·8 | 100·1 | 97·0 | | |
| 50 | 99·2 | 99·6 | 99·7 | 99·9 | 97·5 | | | | |

TABLE 6.—Whole-Life With Profits—Males—Ratio per cent of New Select to $O^M$ and $O^{M(5)}$.

($O^M$ *and* $O^{M(5)}$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 82·1 | 94·9 | 102·2 | 105·9 | 109·2 | 110·1 | 110·5 | 113·1 | 117·3 |
| 10 | 91·2 | 99·5 | 102·3 | 105·2 | 106·2 | 106·1 | 106·7 | 110·3 | 110·2 |
| 15 | 93·9 | 99·9 | 100·8 | 102·9 | 103·3 | 103·7 | 103·9 | 105·7 | 105·3 |
| 20 | 96·0 | 99·5 | 100·4 | 101·6 | 102 3 | 102·5 | 102·4 | 103·3 | 102·7 |
| 30 | 97·6 | 99·5 | 100·2 | 101·0 | 101·1 | 100·9 | 100·5 | 100·0 | 100·2 |
| 40 | 98·8 | 99·9 | 100·2 | 100·3 | 101·1 | 99·4 | 99·6 | 100·8 | |
| 50 | 99·4 | 99·9 | 99·7 | 99·3 | 99·5 | 100·3 | | | |

TABLE 7.—Whole-Life With Profits—Males—Ratio per cent of $O^M$ and $O^{M(5)}$ to $H^M$ and $H^{M(5)}$.

($H^M$ *and* $H^{M(5)}$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 98·5 | 98·3 | 95·6 | 95·6 | 92·9 | 94·8 | 95·0 | 95·3 | 97·1 |
| 10 | 103·0 | 100·2 | 99·5 | 97·4 | 97·2 | 97·6 | 97·2 | 98·0 | 96·6 |
| 15 | 102·7 | 101·3 | 99·3 | 98·8 | 98·3 | 98·2 | 98·4 | 97·5 | 98·3 |
| 20 | 102·8 | 100·5 | 99·8 | 99·2 | 98·6 | 98·9 | 98·0 | 98·6 | 99 6 |
| 30 | 101·3 | 100·4 | 99·6 | 99·5 | 98·7 | 99·0 | 99·6 | 101·0 | 96·4 |
| 40 | 100·5 | 100·1 | 99·2 | 99·3 | 99·7 | 100·7 | 97·4 | | |
| 50 | 99·8 | 99·7 | 99·9 | 100·6 | 98·0 | | | | |

As the policy values by the $H^M$ and $H^{M(5)}$ and the $O^M$ and $O^M$ tables are greater than the $H^M$ and the $O^M$ values respectively, it follows that the values by the first-mentioned tables will approximate more closely to those deduced by me from the New Experience; and this is clearly shown by the ratios in the above tables, as they are in nearly every case smaller than the corresponding ratios in the $H^M$ and the $O^M$ tables. There is no tendency in the ratios for any particular duration to diminish and then increase, such as we found to exist in the comparison with the $H^M$ table, but the other law holds good—namely, under each age at entry the ratios diminish as the duration extends. On the whole, the $H^M$ and $H^{M(5)}$ values correspond better with the select values deduced from the New Experience, and any office using these tables in its valuation may rest secure in the knowledge that the reserve made is very close indeed to that required by a valuation based upon select tables.

*Without Profits.*—Similar ratios are given for this class in the following tables :—

TABLE 8.—Whole-Life Without Profits—Males—Ratio per cent of New Select to $H^M$.

($H^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 114·4 | 118·0 | 110·8 | 108·7 | 102·5 | 108·6 | 109·7 | 107·0 | 108·5 |
| 10 | 100·3 | 109·6 | 106·7 | 102·5 | 101·6 | 101·0 | 105·2 | 103·5 | 107·8 |
| 15 | 104·0 | 104·5 | 102·6 | 102·8 | 100·5 | 100·6 | 103·3 | 102·0 | 104·4 |
| 20 | 101·8 | 101·9 | 102·7 | 101·5 | 100·4 | 100·7 | 102·0 | 101·3 | 102·7 |
| 30 | 101·1 | 101·3 | 101·1 | 100·9 | 100·4 | 100·3 | 100·9 | 100·5 | 92·5 |
| 40 | 100·4 | 100·8 | 100·7 | 100·5 | 100·2 | 100·1 | 94·3 | 91·1 | |
| 50 | 100·4 | 100·5 | 100·3 | 100·2 | 95·2 | 93·0 | | | |

TABLE 9.—Whole-Life Without Profits—Males—Ratio per cent of New Select to $O^M$.

($O^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 103·5 | 110·4 | 105·6 | 105·2 | 102·6 | 107·9 | 110·7 | 108·4 | 108·9 |
| 10 | 92·6 | 103·7 | 102·7 | 101·1 | 101·2 | 101·2 | 106·3 | 104·3 | 110·5 |
| 15 | 97·5 | 100·0 | 100·4 | 101·7 | 100·6 | 101·2 | 104·0 | 103·9 | 105·7 |
| 20 | 96·6 | 99·1 | 101·1 | 101·0 | 100·8 | 101·1 | 103·6 | 102·3 | 102·9 |
| 30 | 98·4 | 100·0 | 100·8 | 101·0 | 101·4 | 101·1 | 101·1 | 99·1 | 95·2 |
| 40 | 99·4 | 100·4 | 101·2 | 100·9 | 100·3 | 99·1 | 96·3 | 99·2 | |
| 50 | 100·4 | 100·6 | 100·3 | 99·3 | 96·8 | 99·2 | | | |

Notwithstanding that the mortality in this class is higher than in the with profits class, the policy values follow practically the same law as the with profits, the New Select being greatest, and the $H^M$ and the $O^M$ following in the same order as before. If we again leave out of consideration age at entry 22, it will be seen that the tendency of the ratios of the New Select to $H^M$ in table 8 to diminish and then increase, is even more marked than in the with profits section; and, contrary to what we found in that section, the same feature also occurs in the ratios of New Select to $O^M$ in table 9.

*Comparison of With and Without Profits.*—We may now compare the with and without profits classes; and adopting the former as the standard, the following table shows the percentage of it necessary to be reserved in the case of without profit policies:—

TABLE 10.—Whole-Life—Males—Ratio per cent of Without Profits to With Profits.

(*With Profits represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 97·4 | 99·6 | 94·1 | 93·6 | 90·4 | 95·4 | 98·5 | 95·1 | 92·6 |
| 10 | 93·3 | 99·1 | 97·3 | 94·1 | 94·0 | 94·6 | 99·3 | 94·4 | 100·2 |
| 15 | 100·2 | 97·9 | 98·2 | 97·9 | 96·9 | 97·3 | 99·9 | 98·3 | 100·3 |
| 20 | 98·9 | 98·5 | 99·9 | 99·0 | 98·4 | 98·6 | 101·2 | 99·1 | 100·2 |
| 30 | 100·2 | 100·2 | 100·4 | 99·9 | 100·3 | 100·1 | 100·6 | 99·1 | 95·0 |
| 40 | 100·5 | 100·4 | 101·0 | 100·6 | 100·2 | 99·7 | 96·7 | 98·4 | |
| 50 | 100·9 | 100·7 | 100·6 | 100·0 | 97·2 | 99·0 | | | |

Up to duration 20, the reserve required for without profit policies is generally less than for with profit, frequently substantially less. As offices almost invariably adopt the same table for both classes in their valuations, it follows that the reserve they make for without profit policies is really stronger than for with profit policies.

### Whole-Life Assurances—Females.

*With Profits.*—This class is a small one, but it presents several interesting features, and I therefore give the ratios for it in the following table :—

Table 11.—Whole-Life With Profits—Females—Ratio per cent of New Select to $H^M$.

($H^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 101·8 | 87·7 | 91·0 | 96·9 | 95·4 | 106 6 | 104·5 | 111·1 | 113·3 |
| 10 | 103·7 | 83·0 | 84·8 | 93·5 | 94·4 | 98·5 | 101·3 | 105·3 | 104·5 |
| 15 | 75·6 | 81·8 | 86·8 | 94·7 | 94·9 | 98·1 | 102·5 | 101·8 | 103·8 |
| 20 | 77·3 | 84·7 | 90·3 | 95·2 | 96·0 | 100·2 | 100·7 | 102·1 | 103·6 |
| 30 | 85·9 | 90·9 | 94·0 | 98·3 | 98·4 | 100·3 | 101·6 | 100·8 | 94·3 |
| 40 | 91·5 | 95·8 | 97·2 | 99·3 | 100·2 | 100·0 | 95·6 | 93·4 | |
| 50 | 95·8 | 98·0 | 99·4 | 99·5 | 96·0 | 94·7 | | | |

Table 12.—Whole-Life With Profits—Females—Ratio per cent of New Select to $O^M$.

($O^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 92·1 | 82·0 | 86·8 | 93·7 | 95·5 | 105·8 | 105·4 | 112·6 | 113·7 |
| 10 | 95·8 | 78·5 | 81·6 | 92·2 | 94·0 | 98·7 | 102·4 | 106·1 | 107·1 |
| 15 | 70·8 | 78·2 | 85·0 | 93·8 | 95·1 | 98·7 | 103·2 | 103·7 | 105·1 |
| 20 | 73·4 | 82·3 | 88·9 | 94·8 | 96·4 | 100·6 | 102·2 | 103·2 | 103·7 |
| 30 | 83·6 | 89·7 | 93·7 | 98·4 | 99·4 | 101·0 | 101·8 | 99·4 | 97·1 |
| 40 | 90·5 | 95·4 | 97·7 | 99·8 | 100·4 | 99·0 | 97·7 | 101·7 | |
| 50 | 95·8 | 98·1 | 99·4 | 98·7 | 97·6 | 101·0 | | | |

It will be seen that up to age at entry 42 (age 22 again excepted) the ratios in these tables do not, as in the case of the males, diminish as the duration extends, but that they first of all diminish—the younger the age at entry the greater, as a rule, the diminution—and that when the age at entry and duration are together about 47 the ratios begin to increase, and continue to do so steadily, almost without exception. This, of course, is due to the heavy mortality below age 45 which is so prominent a characteristic of female life. After age at entry 47 the ratios almost invariably diminish as the duration extends, just as in the case of the males.

It will be remembered that the ratios for the same duration in the males had a tendency to diminish and then increase as the age at entry became older, when the comparison was made with the $H^M$ table, and steadily increased throughout when the comparison was made with the $O^M$ table. In the case of the females, the ratios for each duration are generally very much lower to begin with, but rapidly increase at first, and afterwards more slowly. Looking at the matter in another way, we may say that when the age at entry and duration together do not exceed 42, the policy values according to the $H^M$ and the $O^M$ tables are greater—in some cases considerably greater—than the New Select values. Up to that age, therefore, the position of the New Select table is the reverse of what it was in the males. In the male experience, the New Select values were greatest, while in the female they are least.

*Without Profits.*—The ratios for this class are given in the following tables :—

TABLE 13.—Whole-Life Without Profits—Females—Ratio per cent of New Select to $H^M$.

($H^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 103·8 | 89·1 | 99·8 | 105·7 | 100·9 | 100·9 | 104·6 | 107·2 | 112·7 |
| 10 | 66·3 | 91·9 | 94·6 | 92·6 | 98·0 | 101·9 | 100·6 | 110·3 | 112·9 |
| 15 | 86·6 | 90·4 | 87·9 | 92·3 | 96·4 | 101·2 | 102·7 | 105·7 | 108·2 |
| 20 | 87·5 | 86·7 | 89·5 | 93·1 | 97·8 | 102·5 | 101·7 | 104·3 | 99·4 |
| 30 | 87·5 | 90·5 | 94·0 | 98·2 | 100·1 | 101·7 | 97·4 | 98·4 | 92·1 |
| 40 | 92·7 | 96·4 | 98·0 | 99·6 | 97·4 | 98·1 | 93·2 | 84·4 | |
| 50 | 97·0 | 98·6 | 96·7 | 97·5 | 94·3 | 87·6 | | | |

TABLE 14.—Whole-Life Without Profits—Females—Ratio per cent of New Select to $O^M$.

($O^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 93·9 | 83·4 | 95·1 | 102·3 | 101·0 | 100·2 | 105·5 | 108·5 | 113·1 |
| 10 | 61·2 | 87·0 | 91·0 | 91·3 | 97·7 | 102·1 | 101·7 | 111·1 | 115·6 |
| 15 | 81·4 | 86·5 | 86·0 | 91·3 | 96·5 | 101·7 | 103·4 | 107·7 | 109·5 |
| 20 | 83·0 | 84·2 | 88·1 | 92·8 | 98·2 | 103·0 | 103·3 | 105·4 | 99·5 |
| 30 | 85·2 | 89·4 | 93·7 | 98·3 | 101·1 | 102·4 | 97·6 | 97·0 | 94·8 |
| 40 | 91·7 | 95·9 | 98·4 | 100·1 | 97·6 | 97·1 | 95·2 | 91·9 | |
| 50 | 97·0 | 98·7 | 96·7 | 96·6 | 95·9 | 93·4 | | | |

As was to be expected, the ratios for this class are similar to those of the with profits class, and I need not deal with them specially.

*Comparison of With and Without Profits.*—If the with profits values are taken as the standard, the following table shows the percentage necessary in the case of the without profits:—

TABLE 15.—Whole-Life—Females—Ratio per cent of Without Profits to With Profits.

(*With Profits represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 102·0 | 101·7 | 109·6 | 109·1 | 105·8 | 94·7 | 100·1 | 96·4 | 99·5 |
| 10 | 63·9 | 110·8 | 111·5 | 99·0 | 103·9 | 103·4 | 99·3 | 104·7 | 108·0 |
| 15 | 114·9 | 110·5 | 101·3 | 97·4 | 101·5 | 103·1 | 100·2 | 103·8 | 104·2 |
| 20 | 113·1 | 102·3 | 99·1 | 97·8 | 101·9 | 102·4 | 101·0 | 102·2 | 96·0 |
| 30 | 101·8 | 99·6 | 100·0 | 99·9 | 101·7 | 101·4 | 95·8 | 97·6 | 97·6 |
| 40 | 101·3 | 100·6 | 100·8 | 100·3 | 97·2 | 98·1 | 97·5 | 90·4 | |
| 50 | 101·3 | 100·6 | 97·2 | 97·9 | 98·2 | 92·5 | | | |

The ratios in this table are not so regular as we have hitherto found them to be, and this is owing no doubt to the small numbers under observation upon which the policy values are based. For the principal ages at entry the ratios show that there is not much difference between the policy values, and this was to be expected, as the mortality in the two classes is similar.

## Whole-Life Assurances—Males and Females.

The difference in the mortality of males and females is so great at the younger ages, that it will be interesting to compare the policy values of the former with those of the latter; and in the following tables the ratios are given for with and without profit policies:—

Table 16.—Whole-Life With Profits—Ratio per cent of Females to Males.

(*Males represented by* 100.)

| Duration. | Age at Entry. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 86·7 | 74·0 | 77·4 | 83·5 | 84·1 | 93·6 | 93·9 | 98·7 | 96·7 |
| 10 | 96·5 | 75·1 | 77·3 | 85·8 | 87·4 | 92·3 | 95·6 | 96·0 | 97·1 |
| 15 | 72·8 | 76·6 | 83·1 | 90·3 | 91·5 | 94·9 | 99·2 | 98·1 | 99·7 |
| 20 | 75·1 | 81·8 | 87·9 | 92·9 | 94·1 | 98·1 | 99·8 | 99·9 | 101·1 |
| 30 | 85·2 | 89·9 | 93·4 | 97·3 | 98·3 | 100·1 | 101·3 | 99·4 | 96·9 |
| 40 | 91·5 | 95·4 | 97·5 | 99·4 | 100·3 | 99·6 | 98·1 | 100·9 | |
| 50 | 96·3 | 98·3 | 99·7 | 99·4 | 98·1 | 100·8 | | | |

Table 17.—Whole-Life Without Profits—Ratio per cent of Females to Males.

(*Males represented by* 100.)

| Duration. | Age at Entry. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| 5 | 90·8 | 75·5 | 90·1 | 97·2 | 98·4 | 92·9 | 95·3 | 100·1 | 103·9 |
| 10 | 66·1 | 83·9 | 88·6 | 90·3 | 96·5 | 100·9 | 95·6 | 106·5 | 104·7 |
| 15 | 83·5 | 86·5 | 85·7 | 89·8 | 95·9 | 100·5 | 99·4 | 103·6 | 103·7 |
| 20 | 85·9 | 85·0 | 87·1 | 91·8 | 97·5 | 101·8 | 99·7 | 103·0 | 96·8 |
| 30 | 86·6 | 89·3 | 93·0 | 97·4 | 99·7 | 101·3 | 96·5 | 97·8 | 99·6 |
| 40 | 92·3 | 95·6 | 97·3 | 99·1 | 97·3 | 98·0 | 98·9 | 92·7 | |
| 50 | 96·7 | 98·1 | 96·4 | 97·3 | 99·1 | 94·2 | | | |

It will be seen that for with-profit policies the reserve required for females is, with two or three exceptions, less than that required for males, and that, for the shorter durations at the younger ages at entry, the difference is considerable. The same features, although not so pronounced, are also seen in the without-profits table.

## Limited-Payment Assurances, With and Without Profits—Males.

This class used to be an important one in many offices, and is still so in several. The mortality in the New Experience has been shown to be so light that the policy values deduced from it cannot fail to be interesting. The ratios are given in the following tables:—

Table 18.—Limited Payments—Males—Ratio per cent of New Select to $H^M$.

($H^M$ *represented by* 100.)

| Duration. | Age at Entry. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| **10 Payments.** | | | | | | | | |
| 5 | 97·3 | 97·1 | 97·6 | 96·4 | 97·3 | 98·1 | 96·0 | 102·1 |
| **20 Payments.** | | | | | | | | |
| 5 | 103·4 | 103·8 | 106·2 | 103·3 | 103·7 | 105·6 | 98·0 | 115·0 |
| 10 | 98·2 | 98·1 | 100·2 | 100·0 | 101·2 | 103·9 | 103·4 | 112·2 |
| 15 | 95·2 | 95·6 | 96·2 | 97·9 | 98·7 | 99·6 | 100·7 | 104·4 |
| **30 Payments.** | | | | | | | | |
| 5 | 108·8 | 109·0 | 113·0 | 108·1 | 107·6 | 109·5 | 98·5 | 119·8 |
| 10 | 101·3 | 101·1 | 104·8 | 104·1 | 104·8 | 108·2 | 106·7 | 118·3 |
| 15 | 97·1 | 97·8 | 99·4 | 101·8 | 102·2 | 103·0 | 104·4 | 110·2 |
| 20 | 95·8 | 96·0 | 98·8 | 100·3 | 99·9 | 101·9 | 102·6 | 105·0 |

TABLE 19.—Limited Payments—Males—Ratio per cent of New Select to $O^M$.

($O^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| **10 PAYMENTS.** | | | | | | | | |
| 5 | 99·3 | 98·9 | 98·4 | 96·8 | 98·0 | 98·5 | 96·4 | 102·7 |
| **20 PAYMENTS.** | | | | | | | | |
| 5 | 100·4 | 102·4 | 104·2 | 101·5 | 103·5 | 105·1 | 98·1 | 115·7 |
| 10 | 97·4 | 97·4 | 99·2 | 99·6 | 101·1 | 104·0 | 103·9 | 112·4 |
| 15 | 95·7 | 95·8 | 96·4 | 98·1 | 99·1 | 100·0 | 101·1 | 104·9 |
| **30 PAYMENTS.** | | | | | | | | |
| 5 | 101·0 | 104·8 | 108·4 | 104·5 | 106·9 | 108·6 | 98·7 | 121·2 |
| 10 | 96·9 | 97·9 | 101·6 | 102·4 | 104·1 | 107·9 | 107·3 | 119·1 |
| 15 | 94·2 | 95·6 | 97·8 | 100·9 | 101·9 | 103·2 | 104·7 | 114·0 |
| 20 | 94·2 | 95·1 | 98·1 | 99·9 | 100·0 | 102·1 | 103·2 | 106·2 |

TABLE 20.—Limited Payments—Males—Ratio per cent of $O^M$ to $H^M$.

($H^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| | **10 PAYMENTS.** | | | | | | | |
| 5 | 97·9 | 98·2 | 99·1 | 99·6 | 99·3 | 99·6 | 99·5 | 99·4 |
| | **20 PAYMENTS.** | | | | | | | |
| 5 | 103·1 | 101·4 | 101·9 | 101·8 | 100·2 | 100·4 | 99·9 | 99·4 |
| 10 | 100·8 | 100·7 | 101·0 | 100·4 | 100·0 | 100·0 | 99·6 | 99·8 |
| 15 | 99·5 | 99·9 | 99·8 | 99·8 | 99·7 | 99·6 | 99·6 | 99·5 |
| | **30 PAYMENTS.** | | | | | | | |
| 5 | 107·7 | 104·0 | 104·2 | 103·4 | 100·7 | 100·9 | 99·8 | 98·9 |
| 10 | 104·5 | 103·3 | 103·1 | 101·6 | 100·6 | 100·3 | 99·5 | 99·3 |
| 15 | 103·0 | 102·4 | 101·6 | 100·9 | 100·2 | 99·9 | 99·7 | 98·9 |
| 20 | 101·7 | 101·0 | 100·7 | 100·3 | 99·8 | 99·9 | 99·4 | 98·9 |

The ratios in tables 18 and 19 follow the laws we have found to be so general, namely, they diminish as the duration extends, and increase as the age at entry becomes greater. It will be seen from the ratios that the policy values of the $H^M$ and the $O^M$ tables approximate much more closely to those of the New Select than we found in the case of the whole-life with profits policies. If the $H^M$ and the $O^M$ tables, therefore, are good for the valuation of whole-life policies, it follows that they are better for limited-payment policies. The values by the $O^M$ table are generally greater than those of the $H^M$.

It should be borne in mind that these comparisons are made while the premiums are still payable. After these have ceased, the values by the $O^M$ Table will be less than by the $H^M$ Table, as the mortality is lighter, and the New Select values will be least of all.

## Endowment Assurances, With or Without Profits—Males.

This class is undoubtedly the most important of all, as not only are the policy values large, but the amount of business done under it is now so great that it bids fair to outstrip the Whole-Life class. The ratios for it are given in the following tables:—

Table 21.—Endowment Assurances—Males—Ratio per cent of New Select to $H^M$.

($H^M$ *represented by* 100.)

| Duration. | Age at Entry. 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|---|---|---|---|---|---|---|---|
| **10 Years.** | | | | | | | |
| 5 | 101·0 | 101·0 | 101·4 | 101·6 | 101·6 | 102·0 | 102·1 |
| **20 Years.** | | | | | | | |
| 5 | 102·9 | 103·1 | 105·2 | 106·2 | 105·3 | 108·0 | 107·5 |
| 10 | 100·9 | 101·9 | 102·9 | 103·3 | 103·4 | 105·0 | 105·0 |
| 15 | 100·6 | 101·0 | 101·2 | 101·3 | 101·6 | 102·0 | 102·6 |
| **30 Years.** | | | | | | | |
| 5 | 104·9 | 105·8 | 109·5 | 111·1 | 109·2 | | |
| 10 | 101·2 | 104·2 | 106·2 | 106·7 | 107·1 | | |
| 15 | 101·5 | 103·5 | 103·3 | 103·5 | 105·1 | | |
| 20 | 101·5 | 102·0 | 101·8 | 102·6 | 103·0 | | |
| **40 Years.** | | | | | | | |
| 5 | 106·5 | 107·8 | 113·0 | | | | |
| 10 | 100·9 | 105·8 | 108·6 | | | | |
| 15 | 101·8 | 105·4 | 104·7 | | | | |
| 20 | 102·6 | 103·4 | 102·8 | | | | |
| 30 | 101·1 | 101·8 | 101·8 | | | | |

TABLE 22.—Endowment Assurances—Males—Ratio per cent of New Select to $O^M$.

($O^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | |
|---|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| | | | | 10 YEARS. | | | |
| 5 | 100·3 | 100·5 | 100·9 | 101·2 | 101·4 | 101·8 | 101·9 |
| | | | | 20 YEARS. | | | |
| 5 | 100·2 | 101·5 | 103·3 | 104·4 | 104·6 | 107·1 | 107·0 |
| 10 | 99·4 | 100·7 | 101·7 | 102·5 | 102·9 | 104·5 | 104·8 |
| 15 | 99·9 | 100·5 | 100·7 | 100·9 | 101·3 | 101·8 | 102·4 |
| | | | | 30 YEARS. | | | |
| 5 | 98·7 | 102·3 | 105·7 | 107·6 | 108·0 | | |
| 10 | 97·8 | 101·3 | 103·3 | 104·8 | 106·1 | | |
| 15 | 99·0 | 101·4 | 101·6 | 102·3 | 104·4 | | |
| 20 | 99·9 | 100·8 | 100·8 | 101·9 | 102·6 | | |
| | | | | 40 YEARS. | | | |
| 5 | 97·8 | 102·4 | 107·5 | | | | |
| 10 | 95·1 | 101·1 | 104·2 | | | | |
| 15 | 97·2 | 101·5 | 101·9 | | | | |
| 20 | 99·0 | 100·8 | 100·9 | | | | |
| 30 | 99·6 | 100·7 | 101·1 | | | | |

TABLE 23.—Endowment Assurances—Males—Ratio per cent of $O^M$ to $H^M$.

($H^M$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | |
|---|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| | | | | 10 YEARS. | | | |
| 5 | 100·8 | 100·5 | 100·5 | 100·5 | 100·2 | 100·3 | 100·2 |
| | | | | 20 YEARS. | | | |
| 5 | 102·7 | 101·7 | 101·8 | 101·7 | 100·7 | 100·9 | 100·5 |
| 10 | 101·4 | 101·1 | 101·2 | 100·8 | 100·6 | 100·5 | 100·2 |
| 15 | 100·7 | 100·6 | 100·5 | 100·4 | 100·3 | 100·2 | 100·2 |
| | | | | 30 YEARS. | | | |
| 5 | 106·3 | 103·4 | 103·6 | 103·3 | 101·1 | | |
| 10 | 103·5 | 102·8 | 102·8 | 101·8 | 101·0 | | |
| 15 | 102·5 | 102·1 | 101·7 | 101·3 | 100·7 | | |
| 20 | 101·6 | 101·2 | 101·0 | 100·8 | 100·4 | | |
| | | | | 40 YEARS. | | | |
| 5 | 108·9 | 105·4 | 105·2 | | | | |
| 10 | 106·1 | 104·7 | 104·3 | | | | |
| 15 | 104·7 | 103·8 | 102·8 | | | | |
| 20 | 103·6 | 102·6 | 102·0 | | | | |
| 30 | 101·4 | 101·1 | 100·8 | | | | |

The same law holds good in this case as in the case of most of the other tables—the ratios in tables 21 and 22 diminish as the duration extends, and increase as the age at entry becomes greater. Here also the policy values according to the $H^M$ Table approximate more closely to those of the New Select than in the case of whole-life policies—in fact they are more like the limited-payment policies in that respect. But it will be seen that the values according to the $O^M$ Table approximate still more closely to those of the New Select. If therefore the $O^M$ Table is a good table for the valuation of whole-life policies, it again follows that it is better

still for the valuation of endowment assurances. The ratios in table 23 show that in every case the value according to the $O^M$ Table is greater than that of the $H^M$ Table. I may mention that I formed an aggregate table from the unadjusted observations in the endowment assurance experience, and calculated policy values from it, but the results were not closer to the New Select than those of the $O^M$ Table, which, as you are aware, was deduced from the whole-life with profits experience on the lives of males.

I thought it would be desirable to see how the policy values by the $H^M$ and $H^{M(5)}$ and the $O^M$ and $O^{M(5)}$ Tables compared with the New Select, and the ratios for them are given in the following tables :—

TABLE 24.—Endowment Assurances—Males—Ratio per cent of New Select to $H^M$ and $H^{M(5)}$.

($H^M$ *and* $H^{M(5)}$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | |
|---|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| | **10 YEARS.** | | | | | | |
| 5 | 100·2 | 100·7 | 101·1 | 101·4 | 101·4 | 101·7 | 101·8 |
| | **20 YEARS.** | | | | | | |
| 5 | 96·6 | 99·9 | 102·2 | 103·7 | 102·6 | 104·7 | 104·6 |
| 10 | 100·1 | 101·2 | 102·4 | 102·7 | 102·7 | 104·4 | 104·3 |
| 15 | 100·4 | 100·9 | 101·1 | 101·2 | 101·5 | 101·9 | 102·5 |
| | **30 YEARS.** | | | | | | |
| 5 | 89·4 | 96·6 | 100·5 | 103·2 | 101·5 | | |
| 10 | 98·0 | 101·1 | 103·5 | 103·9 | 104·1 | | |
| 15 | 100·3 | 102·5 | 102·3 | 102·3 | 104·0 | | |
| 20 | 101·2 | 101·7 | 101·4 | 102·3 | 102·6 | | |
| | **40 YEARS.** | | | | | | |
| 5 | 80·1 | 90·7 | 96·9 | | | | |
| 10 | 93·7 | 99·0 | 102·8 | | | | |
| 15 | 98·3 | 102·4 | 101·7 | | | | |
| 20 | 101·0 | 101·8 | 101·2 | | | | |
| 30 | 100·7 | 101·6 | 101·6 | | | | |

TABLE 25.—Endowment Assurances—Males—Ratio per cent of New Select to $O^M$ and $O^{M(5)}$.

($O^M$ *and* $O^{M(5)}$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | |
|---|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| | | | | 10 YEARS. | | | |
| 5 | 99·8 | 100·2 | 100·7 | 101·0 | 101·3 | 101·7 | 101·8 |
| | | | | 20 YEARS. | | | |
| 5 | 95·9 | 98·4 | 101·3 | 102·9 | 103·4 | 106·0 | 106·1 |
| 10 | 98·7 | 100·2 | 101·4 | 102·2 | 102·6 | 104·3 | 104·6 |
| 15 | 99·8 | 100·4 | 100·6 | 100·9 | 101·3 | 101·8 | 102·4 |
| | | | | 30 YEARS. | | | |
| 5 | 89·0 | 94·7 | 100·3 | 103·6 | 104·9 | | |
| 10 | 95·1 | 99·4 | 101·9 | 103·7 | 105·1 | | |
| 15 | 98·3 | 100·8 | 101·2 | 101·9 | 104·1 | | |
| 20 | 99·7 | 100·7 | 100·6 | 101·7 | 102·5 | | |
| | | | | 40 YEARS. | | | |
| 5 | 80·6 | 89·7 | 98·4 | | | | |
| 10 | 89·8 | 97·3 | 101·4 | | | | |
| 15 | 95·2 | 100·1 | 100·7 | | | | |
| 20 | 98·2 | 100·1 | 100·3 | | | | |
| 30 | 99·5 | 100·6 | 101·0 | | | | |

TABLE 26.—Endowment Assurances—Males—Ratio per cent of $O^M$ and $O^{M(5)}$ to $H^M$ and $H^{M(5)}$.

($H^M$ *and* $H^{M(5)}$ *represented by* 100.)

| Duration. | AGE AT ENTRY. | | | | | | |
|---|---|---|---|---|---|---|---|
| | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| | | | | 10 YEARS. | | | |
| 5 | 100·4 | 100·5 | 100·4 | 100·4 | 100·2 | 100·0 | 100·0 |
| | | | | 20 YEARS. | | | |
| 5 | 100·7 | 101·5 | 100·9 | 100·8 | 99·3 | 98·8 | 98·6 |
| 10 | 101·5 | 100·9 | 101·0 | 100·6 | 100·0 | 100·1 | 99·7 |
| 15 | 100·6 | 100·5 | 100·5 | 100·3 | 100·2 | 100·2 | 100·1 |
| | | | | 30 YEARS. | | | |
| 5 | 100·5 | 102·0 | 100·2 | 99·6 | 96·8 | | |
| 10 | 103·1 | 101·7 | 101·6 | 100·2 | 99·1 | | |
| 15 | 102·1 | 101·7 | 101·1 | 100·4 | 100·0 | | |
| 20 | 101·5 | 101·0 | 100·7 | 100·5 | 100·1 | | |
| | | | | 40 YEARS. | | | |
| 5 | 99·3 | 101·2 | 98·5 | | | | |
| 10 | 104·3 | 101·8 | 101·3 | | | | |
| 15 | 103·2 | 102·3 | 101·0 | | | | |
| 20 | 102·9 | 101·6 | 100·9 | | | | |
| 30 | 101·2 | 100·9 | 100·6 | | | | |

The ratios in tables 24 and 25 are of course less than in the tables (21 and 22) where the New Select are compared with the $H^M$ and the $O^M$, that is to say, the values by the $H^M$ and $H^{M(5)}$ and the $O^M$ and $O^{M(5)}$ approximate more closely to the select values. As a rule the values by both these tables fall short of what is required, but on the whole the $O^M$ and $O^{M(5)}$ values come nearest.

This appears from the ratios in table 26 of the $H^M$ and $H^{M(5)}$ to the $O^M$ and $O^{M(5)}$, which show that the policy values by the latter are, as a rule, the greater.

We may now briefly summarise the conclusions arrived at in the paper.

Dealing first of all with whole-life with profits assurances on the lives of males, we have seen that the $O^M$ table gives reserves which differ, on the whole, very little from those of the $H^M$. The former table gives larger values than the latter at the younger ages at entry, and smaller at the older. Speaking some time ago, I gave the results of a valuation I had made of this class of policy in a young office at 3 per cent interest. Leaving out of account bonus, the reserve by the $O^M$ table was 1·22 per cent greater than that required by the $H^M$ table, that is to say, if the $H^M$ reserve was 100, the $O^M$ reserve was 101·22. If, however, bonus was taken into account, the $O^M$ reserve was reduced from 101·22 to 100·46, which is a very material reduction. I added that old offices with large bonuses would have very little difficulty indeed in passing to the $O^M$ table. Since then an old office with large bonuses has made a valuation of this class, and it is stated that the reserve required by the new tables is actually less than that required by the old Institute tables.

We have also seen that both the $H^M$ and $O^M$ policy values fall considerably short for a long time of the values required on a select basis. Even after the lapse of ten years from entry, the policy values fall short by about 8 per cent, and it is not until about forty years have elapsed that they come up to the required standard. This is rather disappointing, but perhaps it is inseparable from the employment of an aggregate table. One would have preferred, however, if the required standard had been reached more quickly. Even the $H^M$ and $H^{M(5)}$ values are, as a rule, under the standard, but they come up to it more rapidly, namely, in about half the time. The $O^M$ and $O^{M(5)}$ table is not nearly so good in this respect as the $H^M$ and $H^{M(5)}$.

On the other hand, the reserve made by the $H^M$ and the $O^M$ tables for without profit assurances, males, and for assurances on the lives of females, is larger in proportion, in a number of cases even larger than is necessary, and this will, to some extent at all events, make up for the deficiency above mentioned.

There is one point which can hardly have escaped notice, and that is, that I have frequently been obliged to exempt age at entry 22 in my remarks. This applies to all the whole-life assurances, whether male or female, with or without profits. It is evident that there is some special feature in the mortality of the group of which that represents the central age.

Coming now to limited-payment assurances, we have found that the policy values of the $H^M$ and the $O^M$ tables approximate much more closely to the required standard, than in the case of whole-life with profits assurances. The $O^M$ table is on the whole better than the $H^M$.

Lastly, we have seen that for endowment assurances the policy values of the $H^M$ and the $O^M$ tables approximate even more closely to the required standard, the latter being the nearer of the two. The $H^M$ and $H^{M(5)}$ and the $O^M$ and $O^{M(5)}$ tables give better results still, but even they fall short of the required standard. The $O^M$ and $O^{M(5)}$ is in this class superior to the $H^M$ and $H^{M(5)}$.

In conclusion, I may remind you that bonuses will affect the comparisons we have been making. As the mortality in the $O^M$ and the $O^{M(5)}$ Tables is, as a rule, lighter than in the $H^M$ and the $H^{M(5)}$ respectively, it follows that for whole-life assurances the reserve required for bonuses by the first-mentioned tables will be less, so that the differences between the policy values will not be so great when bonuses are taken into account. When, however, we compare the $H^M$ and the $O^M$ values with the New Select, the value of the bonuses in the last-mentioned comes between the $H^M$ and the $O^M$, and bonuses would therefore bring the $H^M$ policy values nearer the New Select, and take the $O^M$ further away.

In the case of limited-payment and endowment assurances, the reserve for bonuses by the New Select tables is less than by the $H^M$ and the $O^M$, and therefore the policy values by the last-mentioned tables are brought nearer to those of the New Select, and the $H^M$ nearer to the $O^M$, when bonuses are taken into account.

It should be borne in mind that the reversionary bonuses from a given cash surplus will be larger by the $O^M$ than by the $H^M$ Table. The reversionary bonus by the New Select in similar circumstances would be less than the $O^M$ in the case of whole-life participating assurances, and greater in the case of limited-payment and endowment assurances.

TABLE 27.—WHOLE LIFE WITH PROFITS—MALES.

*Values of* 100 ${}_nV_x$ *at* $3\frac{1}{2}$ %, *according to* (1) NEW SELECT; (2) $H^M$; *and* (3) $O^M$ TABLES.

| Table. | Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| New | 5 | 5·005 | 6·049 | 7·097 | 8·342 | 9·984 | 11·779 | 13·788 | 16·322 | 19·356 |
| $H^M$ | | 4·261 | 5·103 | 6·032 | 7·184 | 8·801 | 10·346 | 12·388 | 14·499 | 16·520 |
| $O^M$ | | 4·709 | 5·457 | 6·328 | 7·428 | 8·793 | 10·420 | 12·276 | 14·314 | 16·457 |
| New | 10 | 9·833 | 11·968 | 14·025 | 16·722 | 19·690 | 22·903 | 26·594 | 31·401 | 35·299 |
| $H^M$ | | 9·147 | 10·827 | 12·783 | 15·353 | 18·286 | 21·452 | 25·091 | 28·624 | 32·792 |
| $O^M$ | | 9·909 | 11·440 | 13·286 | 15·567 | 18·297 | 21·417 | 24·833 | 28·415 | 32 015 |
| New | 15 | 15·180 | 18·396 | 21·384 | 25·305 | 29·419 | 33·961 | 38·709 | 44·134 | 48·535 |
| $H^M$ | | 14·627 | 17·284 | 20·459 | 24·111 | 28·366 | 32·841 | 37·467 | 42·537 | 46·629 |
| $O^M$ | | 15·610 | 18·018 | 20·910 | 24·365 | 28·326 | 32·665 | 37·203 | 41·747 | 46·081 |
| New | 20 | 21·373 | 25·381 | 29·487 | 34·354 | 39·543 | 44·860 | 50·085 | 55·562 | 59·748 |
| $H^M$ | | 20·761 | 24·518 | 28·688 | 33·512 | 38·752 | 43·986 | 49·656 | 54·368 | 58·285 |
| $O^M$ | | 21·879 | 25·226 | 29·152 | 33·650 | 38·586 | 43·746 | 48·898 | 53 799 | 58·205 |
| New | 30 | 35·513 | 41·179 | 46·918 | 53·050 | 58·890 | 64·281 | 68·947 | 72·708 | 76·034 |
| $H^M$ | | 35·211 | 40·711 | 46·581 | 52·544 | 58·837 | 64·157 | 68·752 | 71·683 | 78·137 |
| $O^M$ | | 36·172 | 41·240 | 46·745 | 52·504 | 58·248 | 63·694 | 68·584 | 72·703 | 75·870 |
| New | 40 | 51·454 | 57·981 | 63·905 | 69·586 | 74·424 | 78·062 | 81·511 | 84·744 | |
| $H^M$ | | 51·468 | 57·682 | 64·099 | 69·660 | 74·451 | 77·757 | 83·623 | 91·574 | |
| $O^M$ | | 52·022 | 57·937 | 63·795 | 69·346 | 74·332 | 78·549 | 81·862 | 84·089 | |
| New | 50 | 66·990 | 72·749 | 77·544 | 81·321 | 84·772 | 87·737 | | | |
| $H^M$ | | 67·382 | 72·945 | 77·717 | 81·172 | 86·610 | 93·382 | | | |
| $O^M$ | | 67·388 | 72·852 | 77·742 | 81·889 | 85·181 | 87·497 | | | |

TABLE 28.—WHOLE-LIFE WITHOUT PROFITS—MALES; AND WITH AND WITHOUT PROFITS—FEMALES.

*Values of* 100 $_nV_x$ *at* $3\frac{1}{2}$ %, *according to* NEW SELECT TABLES.

(1) *Without Profits*, MALES (MN); (2) *With Profits*, FEMALES (FP); *and* (3) *Without Profits*, FEMALES (FN).

| Table. | Duration. | AGE AT ENTRY. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 22 | 27 | 32 | 37 | 42 | 47 | 52 | 57 | 62 |
| MN | 5 | 4·874 | 6·028 | 6·681 | 7·811 | 9·021 | 11·289 | 13·587 | 15·517 | 17·918 |
| FP | | 4·338 | 4·474 | 5·492 | 6·962 | 8·395 | 11·025 | 12·942 | 16·114 | 18·708 |
| FN | | 4·424 | 4·549 | 6·021 | 7·596 | 8·881 | 10·437 | 12·952 | 15·536 | 18·610 |
| MN | 10 | 9·178 | 11·863 | 13·643 | 15·786 | 18·518 | 21·666 | 26·405 | 29·636 | 35·363 |
| FP | | 9·489 | 8·984 | 10·842 | 14·355 | 17·206 | 21·139 | 25·427 | 30·149 | 34·277 |
| FN | | 6·064 | 9·951 | 12·090 | 14·215 | 17·868 | 21·859 | 25·250 | 31·569 | 37·023 |
| MN | 15 | 15·213 | 18·016 | 20·994 | 24·774 | 28·509 | 33·047 | 38·686 | 43·385 | 48·689 |
| FP | | 11·055 | 14·099 | 17·768 | 22·844 | 26·925 | 32·226 | 38·384 | 43·310 | 48·411 |
| FN | | 12·699 | 15·585 | 17·991 | 22·250 | 27·338 | 33·226 | 38·470 | 44 948 | 50·464 |
| MN | 20 | 21·132 | 24·995 | 29·467 | 33·998 | 38·896 | 44·220 | 50·667 | 55·057 | 59·862 |
| FP | | 16·054 | 20·772 | 25·918 | 31·901 | 37·198 | 44·002 | 49·993 | 55·501 | 60·372 |
| FN | | 18·161 | 21·252 | 25·672 | 31·214 | 37·907 | 45·035 | 50·499 | 56·698 | 57·931 |
| MN | 30 | 35·585 | 41·250 | 47·107 | 53·001 | 59·040 | 64·372 | 69·865 | 72·071 | 72·245 |
| FP | | 30·249 | 37·003 | 43·807 | 51·644 | 57·887 | 64·326 | 69·848 | 72·269 | 73·694 |
| FN | | 30·806 | 36·856 | 43·807 | 51·615 | 58·881 | 65·218 | 66·933 | 70·501 | 71·948 |
| MN | 40 | 51·694 | 58·165 | 64·544 | 69·981 | 74·565 | 77·860 | 78·816 | 83·398 | |
| FP | | 47·092 | 55·267 | 62·318 | 69·194 | 74·608 | 77·768 | 79·985 | 85·532 | |
| FN | | 47·688 | 55·584 | 62·788 | 69·882 | 72·532 | 76·306 | 77·951 | 77·289 | |
| MN | 50 | 67·619 | 73·279 | 77·982 | 81·345 | 82·412 | 86·839 | | | |
| FP | | 64·521 | 71·502 | 77·280 | 80·802 | 83·144 | 88·401 | | | |
| FN | | 65·358 | 71·894 | 75·142 | 79·148 | 81·684 | 81·758 | | | |

TABLE 29.—LIMITED PAYMENTS—MALES.

*Values of* $100\,{}_{n:m}V_x$ *at* $3\frac{1}{2}$ %, *according to* (1) NEW SELECT; (2) $H^M$; *and* (3) $O^M$ TABLES.

| Table. | Duration. | AGE AT ENTRY. | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| | | **10 PAYMENTS.** | | | | | | | |
| New | 5 | 15·040 | 16·762 | 18·597 | 20·851 | 22·852 | 25·894 | 27·215 | 31·869 |
| $H^M$ | | 15·464 | 17·256 | 19·063 | 21·102 | 23·483 | 25·877 | 28·358 | 30·721 |
| $O^M$ | | 15·141 | 16·952 | 18·899 | 21·019 | 23·322 | 25·769 | 28·228 | 30·546 |
| | | **20 PAYMENTS.** | | | | | | | |
| New | 5 | 8·159 | 9·294 | 10·492 | 11·816 | 12·793 | 14·363 | 14·730 | 18·915 |
| $H^M$ | | 7·888 | 8·950 | 9·876 | 10·955 | 12·332 | 13·605 | 15·026 | 16·448 |
| $O^M$ | | 8·131 | 9·078 | 10·065 | 11·150 | 12·358 | 13·665 | 15·011 | 16·348 |
| New | 10 | 17·118 | 19·063 | 21·583 | 23·953 | 26·884 | 30·341 | 32·924 | 38·199 |
| $H^M$ | | 17·432 | 19·428 | 21·493 | 23·962 | 26·575 | 29·193 | 31·838 | 34·043 |
| $O^M$ | | 17·570 | 19·566 | 21·709 | 24·054 | 26·584 | 29·181 | 31·700 | 33·977 |
| New | 15 | 27·328 | 30·464 | 34·080 | 38·428 | 42·730 | 47·076 | 51·261 | 56·404 |
| $H^M$ | | 28·693 | 31·862 | 35·412 | 39·240 | 43·277 | 47·274 | 50·918 | 54·040 |
| $O^M$ | | 28·563 | 31·814 | 35·346 | 39·155 | 43·128 | 47·074 | 50·725 | 53·788 |
| | | **30 PAYMENTS.** | | | | | | | |
| New | 5 | 6·027 | 6·997 | 8·046 | 8·658 | 9·966 | 11·473 | 11·874 | 16·644 |
| $H^M$ | | 5·542 | 6·421 | 7·122 | 8·010 | 9·259 | 10·477 | 12·051 | 13·893 |
| $O^M$ | | 5·968 | 6·677 | 7·422 | 8·282 | 9·322 | 10·567 | 12·032 | 13·786 |
| New | 10 | 12·371 | 13·916 | 16·032 | 18·012 | 20·488 | 23·781 | 26·459 | 32·862 |
| $H^M$ | | 12·211 | 13·759 | 15·302 | 17·311 | 19·557 | 21·971 | 24·804 | 27·775 |
| $O^M$ | | 12·765 | 14·211 | 15·782 | 17·587 | 19·676 | 22·035 | 24·669 | 27·593 |
| New | 15 | 19·305 | 21·758 | 24·721 | 28·301 | 31·680 | 35·390 | 39·439 | 45·875 |
| $H^M$ | | 19·888 | 22·245 | 24·874 | 27·789 | 31·014 | 34·352 | 37·791 | 41·627 |
| $O^M$ | | 20·488 | 22·767 | 25·268 | 28·049 | 31·084 | 34·307 | 37·664 | 41·175 |
| New | 20 | 27·619 | 30·972 | 35·380 | 39·767 | 43·627 | 48·337 | 52·584 | 57·676 |
| $H^M$ | | 28·844 | 32·254 | 35·812 | 39·668 | 43·686 | 47·428 | 51·257 | 54·953 |
| $O^M$ | | 29·332 | 32·572 | 36·073 | 39·790 | 43·609 | 47·367 | 50·940 | 54·324 |

TABLE 30.—ENDOWMENT ASSURANCES—MALES.

*Values of* $100\,{}_{n}V_{x:\overline{m}|}$ *at* $3\frac{1}{2}$ %, *according to* (1) NEW SELECT; (2) $H^M$; *and* (3) $O^M$ TABLES.

| Table. | Duration. | AGE AT ENTRY. | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| | | 10 YEARS. | | | | | | |
| New | 5 | 45·340 | 45·365 | 45·402 | 45·347 | 45·218 | 45·128 | 44·781 |
| $H^M$ | | 44·886 | 44·915 | 44·768 | 44·628 | 44·504 | 44·235 | 43·858 |
| $O^M$ | | 45·225 | 45·133 | 44·994 | 44·828 | 44·613 | 44·351 | 43·951 |
| | | 20 YEARS. | | | | | | |
| New | 5 | 18·568 | 18·726 | 18·956 | 19·049 | 19·025 | 19·503 | 19·581 |
| $H^M$ | | 18·046 | 18·158 | 18·020 | 17·937 | 18·064 | 18·054 | 18·220 |
| $O^M$ | | 18·524 | 18·458 | 18·346 | 18·250 | 18·197 | 18·214 | 18·368 |
| New | 10 | 40·431 | 40·787 | 40·969 | 41·044 | 41·015 | 41·471 | 41·354 |
| $H^M$ | | 40·084 | 40·037 | 39·810 | 39·734 | 39·651 | 39·479 | 39·400 |
| $O^M$ | | 40·658 | 40·495 | 40·280 | 40·062 | 39·870 | 39·679 | 39·471 |
| New | 15 | 67·277 | 67·479 | 67·374 | 67·240 | 67·190 | 67·025 | 66·722 |
| $H^M$ | | 66·907 | 66·795 | 66·598 | 66·392 | 66·119 | 65·689 | 65·039 |
| $O^M$ | | 67·359 | 67·170 | 66·928 | 66·645 | 66·298 | 65·884 | 65·158 |
| | | 30 YEARS. | | | | | | |
| New | 5 | 10·282 | 10·564 | 10·968 | 11·325 | 11·755 | | |
| $H^M$ | | 9·752 | 9·984 | 10·011 | 10·193 | 10·764 | | |
| $O^M$ | | 10·367 | 10·327 | 10·375 | 10·527 | 10·883 | | |
| New | 10 | 21·861 | 22·646 | 23·133 | 23·786 | 24·642 | | |
| $H^M$ | | 21·598 | 21·742 | 21·791 | 22·245 | 23·000 | | |
| $O^M$ | | 22·357 | 22·354 | 22·400 | 22·646 | 23·227 | | |
| New | 15 | 36·244 | 37·033 | 37·112 | 37·566 | 38·918 | | |
| $H^M$ | | 35·726 | 35·779 | 35·919 | 36·282 | 37·027 | | |
| $O^M$ | | 36·601 | 36·524 | 36·521 | 36·736 | 37·282 | | |
| New | 20 | 53·587 | 53·893 | 53·758 | 54·337 | 54·920 | | |
| $H^M$ | | 52·810 | 52·837 | 52·802 | 52·942 | 53·336 | | |
| $O^M$ | | 53·632 | 53·461 | 53·339 | 53·339 | 53·530 | | |

TABLE 30.—ENDOWMENT ASSURANCES—MALES—*continued.*

*Values of* $100\,{}_{n}V_{x:\overline{n}|}$ *at* $3\frac{1}{2}$ %, *according to* (1) NEW SELECT; (2) $H^M$; *and* (3) $O^M$ TABLES.

| Table. | Duration. | AGE AT ENTRY. | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| | | 40 YEARS. | | | | | | |
| New | 5 | 6·580 | 7·108 | 7·785 | | | | |
| $H^M$ | | 6·178 | 6·592 | 6·887 | | | | |
| $O^M$ | | 6·726 | 6·945 | 7·242 | | | | |
| New | 10 | 13·742 | 14·957 | 16·025 | | | | |
| $H^M$ | | 13·624 | 14·140 | 14·758 | | | | |
| $O^M$ | | 14·452 | 14·802 | 15·386 | | | | |
| New | 15 | 22·666 | 24·128 | 25·052 | | | | |
| $H^M$ | | 22·271 | 22·892 | 23·938 | | | | |
| $O^M$ | | 23·327 | 23·771 | 24·595 | | | | |
| New | 20 | 33·272 | 34·351 | 35·338 | | | | |
| $H^M$ | | 32·446 | 33·239 | 34·364 | | | | |
| $O^M$ | | 33·615 | 34·096 | 35·039 | | | | |
| New | 30 | 59·857 | 60·693 | 61·319 | | | | |
| $H^M$ | | 59·232 | 59·596 | 60·223 | | | | |
| $O^M$ | | 60·083 | 60·246 | 60·680 | | | | |

## DISCUSSION.

THE PRESIDENT (Mr. G. M. LOW), in inviting discussion, said that whatever other opinions might be expressed on the paper, there were two things on which they were entirely agreed. One was that the paper at that period of actuarial history was a most interesting one; and the other that they were greatly indebted to Mr. Chatham, who had worked out in so much detail these calculations for their benefit, and had put them together and commented on them in such an able and lucid and interesting way.

Mr. W. G. WALTON thought it might be assumed that a hundred per cent of the actuarial profession would read this paper. It was safe to say so, in the first place because Mr. Chatham had written it, and secondly because the subject was of great present interest. Now, of that hundred per cent of readers, they gathered from the paper there were no less than seventy per cent who had been basing their hopes on the $H^M$ table. He confessed that Mr. Chatham's first table of ratios was calculated to give that seventy per cent a very uncomfortable feeling at the first glance. Before they arrived at this first table, however, they were told that "Select values are, of course, the ideally correct ones to employ in the valuation of the liabilities of an insurance office". What was added to this might be true, but was not very comforting, that the adoption of such values would involve too much labour. No, the preliminary remarks were not calculated to mitigate the shock when the startling percentages of Table 2 were suddenly come upon. Some relief might be experienced when the 18 per cent by which the correct values exceed the $H^M$ values, was found to dwindle down to very small dimensions with the duration of the assurance. It would be found on investigation, however, that while Mr. Chatham's ratio mountains are really molehills, his ratio molehills, if not mountains, are not so insignificant as they look. Still there would, he thought, be no harm in prefacing this paper with a warning—"Caution: these hills are *not* dangerous"—so that the seventy per cent at least might know that there was nothing to upset them seriously. There was one point to which he would call attention before referring further to the tables of ratios. Examples were given of $3\frac{1}{2}$ per cent net premiums according to Mr. Chatham's "Select and Ultimate" table for whole-life assurances, males, when selection is assumed to pass off in five years, and also in ten years. Mr. Chatham had said that the differences given in the last column of Table 1 indicate that "contrary to what has generally been supposed, selection endures for a shorter period in the case of young lives than in those of middle age". He (Mr. Walton) did not consider the relative amounts of these premiums justified this deduction. The true explanation of the premiums at early ages under the "5 years'" table being less, or at least differing so little from the premiums under the "10 years'" table, was that, while in the "ultimate" mortality of the "5 years'" premiums the more or less select lives that had been assured for over 5 and under 10 years were included up to the end of life, they were not so included in the "ultimate" mortality of the "10 years'" table. Hence annuity values in early life were higher

under the 5 years' table than they would have been had the "ultimate" mortality not included the second 5 years of assurance; and the result was, of course, that the premiums were lower. For a similar reason the $O^M$ premium at 22 (the youngest age in Mr. Chatham's Table 1) was still lower than the premium by the "5 years'" table; while the $O^M$ premiums at the advanced ages were higher than the "5 years'" table premiums. He thought it was hardly possible to come to any satisfactory conclusion as to when the effect of selection is exhausted at various ages, merely by a comparison of tables of whole-life premiums. The way in which to settle the question effectively is to compare the rates of mortality under complete tables for each age at entry, without making any assumption whatever as to the period when selection dies out. In the $O^{[M]}$ select graduated table, where selection is assumed to end in 10 years, he found this assumption to be in accordance with the facts. But to return to the tables of ratios. The actuarial value of a policy is ascertained for the purpose of fixing the amount which can be allowed on surrender of an individual case, or for fixing the reserve which must be held against that policy—not as an individual case, but as a fraction of the whole liability of an office. Mr. Chatham dealt with policy values in this latter aspect, and this should be kept strictly in view in considering the mass of figures which had been brought before them. Further, Mr. Chatham had referred only to policies under which premiums are still payable. Now such a policy value is a very complex quantity. Indeed, it is so intricate, that the mere statement that the value of a policy amounted to so much, conveyed little idea as to whether that value was "cheap, middling, or dear". If they said a person is seventy inches high, they had a fact from which it was possible to form an opinion of the relative height of that person. They were here dealing with something within the bounds of their limited imagination. But of policy values they could form no opinion from a table of their amounts. This, however, did not appear to have any terrors for Mr. Chatham, as he had not hesitated to exhibit his figures by multiplying one extremely complex quantity by the reciprocal of another; and he (Mr. Walton) must confess that he was at a loss to form an opinion from an inspection of these tables. Notwithstanding, they were all greatly indebted to Mr. Chatham for undertaking the labour of calculating these values; and he would not quarrel with that excellent and very valuable piece of work. He did, however, question a distinctive feature of his paper—his method of presenting the results in these mysterious ratios. The method had been adopted by Mr. Ackland in his paper[1] read before the Institute of Actuaries in March last; but Mr. Ackland was good enough to give a more general view in addition, by giving tables indicating where the values exceeded or fell short of each other, by simply marking the spaces with a *plus* or *minus*. As an illustration of the difficulty he referred to, let them take Table 4, where the $O^M$ and the $H^M$ values are compared. In the third column, under age 27 at entry, they found the ratios of $O^M$ reserve to $H^M$ reserve to be as follows; and for comparison he had put beside these ratios the actual difference in the policy values per 100 assured :—

| Duration. | Ratio. | Difference. |
|---|---|---|
| 5 | 106·9 | ·354 |
| 10 | 105·7 | ·613 |
| 15 | 104·6 | ·784 |
| 20 | 102·9 | ·708 |
| 30 | 101·3 | ·529 |
| 40 | 100·4 | ·255 |
| 50 | 99·9 | ·093 |

[1] *J.I.A.*, xxxvii, 113.

The ratios gave quite a different impression from the differences. Let them bear in mind that the differences are really ratios to the sum assured, and hence have a common relationship to one another, which the ratios have not. Their position was now very different from what it was when the Institute of Actuaries published its experience thirty years ago. Then the class of business transacted was very much more limited: deduct the whole-life assurances and you practically swept the board. Now the state of affairs was very different indeed. There had been a large proportionate increase in the assurances effected other than whole-life policies; and the whole-life-premium policies bid fair to take a second place at no distant date. Again, at that time any rough-and-ready method was sufficient to discover the comparative effect of using mortality tables which differed so widely as the Institute, the Carlisle, the English Life, and even the Northampton table, which was not altogether abandoned for some considerable time afterwards. They heard nothing now, however, of the more or less hypothetical tables. They were now running on much more delicate lines in respect of the mortality basis, and in their comparisons were dealing with a much more complicated set of policies than formerly. Further, there was the highly disturbing element of bonuses. They could indeed envy the happy days when a reference to Manly's or to King's "model office" was sufficient to give a fairly good solution of the problem, for which now he feared there was no short cut left. He had no hesitation in saying this, notwithstanding the somewhat contrary remarks recently made by Mr. King at the Institute of Actuaries.[1] In view of these circumstances he did not consider that functions so complex as policy values formed the best method of comparing the tables. Possibly they might have to be content with dealing, as it were, with each side of the account separately, comparing the values of the reversions on the one hand and the values of future premiums on the other. If they must go along such lines, the policy value was now of little help. They had so long accustomed themselves to the fashion of using policy values for the purpose of judging the probable effect of changing from one basis to another, that it might take time to realise that, for the reasons he had stated, it is too complex and too insignificant an item to afford them sufficient information upon which to form an opinion. After all, however, the important feature of Mr. Chatham's paper was the numerous comparisons which he had made, under the various classes of assurance, of select values with other values. These, along with the special premiums which he had already furnished, could not fail to be of great value in considering many special points in connection with the different classes to which they referred. The general view which he was led to take was that, if they compared results derived from the former Institute Experience with those derived from the New Experience, the differences would not be found to be very material. In some offices the effect of a change to the new tables would be to show lower reserves; while in other offices he knew that a change had the effect of making higher reserves necessary.

Mr. G. C. Stenhouse thought the Faculty was to be congratulated on having among its members a man like Mr. Chatham, whom no amount of work seemed to frighten. Not only had he the energy and industry to take up an elaborate paper such as the one now submitted, but his results had been put in such form and with such delightful clearness that the paper had been highly interesting to read, and for future reference would be extremely useful to the profession. Coming to the paper, he would say that there was probably no period in an actuary's career at which his work became more engrossing than when he was required for the first time to

[1] *J.I.A.*, xxxvii, 186.

undertake the actual valuation of an office—there was certainly no time when he felt his responsibilities more. And this arose to a great extent from the fact that there was no recognised standard—that is, no single mortality table and no fixed rate of interest—by which all offices could be measured. The office might be an old one, or it might be but recently established. It might transact a large with-profit business on the whole-life scale; it might devote its attention principally to endowment assurances; it might press the claims of the discounted-bonus system; or it might transact a large non-profit or special-class business. He supposed it would now be at once admitted that the policy reserves in the different circumstances should not all be derived from the same table. Even confining attention to one office, one would, no doubt, find differences of opinion as to what table should be used, what rate of interest should be assumed, and what reserves were necessary or prudent. As Mr. Chatham had stated, in any office the ideally correct values were those derived from select tables; but he quite agreed with him that valuations by such tables were not likely to become general. And that was what made the paper so valuable. It was of great scientific interest in the first place, and in addition it was of considerable practical value. While they could not conveniently make a valuation by select tables, they had, with a paper like this before them, a standard by which, proceeding on broader lines, they could tell whether their reserves were near to or far away from correct reserves. Of course, the great interest in the paper was in the various tables, which were full of information. Taking the big classes, Mr. Chatham had pointed out that, in the case of whole-life policies, when the age *plus* duration is less than 50, the $O^M$ table gives a larger reserve—leaving bonus out of account—than the $H^M$. The former would therefore probably be a safe table for a young office. But when they came to an old office, the majority of lives were likely to be over 50, and the policies to have large bonus additions; and that being so, the $O^M$ table would not be so suitable for valuation purposes. The satisfactory feature about Mr. Chatham's examination of this part of the business was that the $H^M$ and $H^{M(5)}$ reserves are not unlike those he had obtained by using his select tables. Turning to endowment assurances, they found Mr. Chatham had done even more work than the paper showed; for he had mentioned that for the sake of his comparison he had formed an aggregate table from the endowment assurance observations. It must of course be kept in view in regard to these endowment assurances, that the observations among this class were probably on more recently selected lives, and that being so, the light mortality and other favourable results might be influenced by that cause. There again, they were told, the $H^M$ and $H^{M(5)}$ and $O^M$ and $O^{M(5)}$ values approximate to the select values. Coming to what was practical, he might be allowed to refer to a large valuation with which he had recently had a good deal to do. When it was in progress, the new tables for the whole-life class became available, and a hurried valuation of groups of policies at certain ages was made, interest being taken as 3 per cent. The general result was that the $H^M$ table gave a smaller reserve than the $O^M$, while the combined $O^M$ and $O^{M(5)}$ gave a smaller reserve than the combined $H^M$ and $H^{M(5)}$. Taking $H^M = 100$, the reserves required for original sums assured and also for bonus additions were as follows:—

| Table Employed. | Original Sum Assured. | Vested Bonus. | Sums Assured and Vested Bonus. |
|---|---|---|---|
| (1) | (2) | (3) | (4) |
| $O^M$ | 101·0 | 99·1 | 100·4 |
| $O^M$ $O^{M(5)}$ | 101·7 | 99·5 | 101·0 |
| $H^M$ $H^{M(5)}$ | 102·4 | 100·6 | 101·9 |

The figures in columns (2) and (4) for $O^M$ agreed pretty closely with some results obtained by Mr. Chatham; a reference to page 167 of the paper would show these to be 101·22 and 100·46. He went on to say that, before a final decision was come to as to what table or tables of mortality were to be employed, a valuation was made by the various tables of the total of the whole-life with-profit class of which the various groups mentioned above formed part, the original sums assured and bonuses, however, not being kept separate, but valued as a whole; and that he would quote figures showing the comparative results. Along with these he would also show figures illustrating a fact which seemed to him rather curious. Whether it was a mere coincidence or not he was not prepared to say. The total sums assured, including bonuses, under the class in question which were used in arriving at the undernoted figures, amounted to a little over £21,000,000; and it was found that the pivot on which this business swung was age at valuation $55\frac{10}{12}$. Up to and including that age there was a little over ten and a half millions of sums assured, and at all older ages there was a little under ten and a half millions; so that age $55\frac{10}{12}$ might be taken as the central age. A valuation of the policies existing at that age was made, with the following result:—

| Table Employed. | Actual Valuation, £21,000,000. | Value of Policies at Age $55\frac{10}{12}$. |
|---|---|---|
| $H^M$ | 100· | 100· |
| $O^M$ | 100·14 | 100·21 |
| $O^M$ $O^{M(5)}$ | 100·60 | 100·59 |
| $H^M$ $H^{M(5)}$ | 101·39 | 101·39 |

The similarity of these figures seemed to him quite remarkable and worthy of consideration. Another point that was of interest to members of the Faculty, and which had a close relation to the values of vested bonus already given, was the relative cost of 1 per cent per annum of new bonus by the four tables: it was as follows:—

$$H^M = 100\,;\ O^M = 99{\cdot}22\,;\ O^M\ O^{M(5)} = 99{\cdot}40\,;\ H^M\ H^{M(5)} = 100{\cdot}55.$$

So that the same cash surplus would enable an office to declare a larger bonus by the new tables than by the old ones. It had to be kept in view that these figures were applicable to an old office with very large existing bonuses, and that of course different offices would bring out different results, depending to a great extent on the nature of the business. He might say that the reserves were made on the $H^M$ $H^{M(5)}$ basis, with interest at 3 per cent. He would only add that the new $O^M$ and $O^{M(5)}$ tables simply corroborated the results of the $H^M$ and $H^{M(5)}$ tables; and showed that in the latter they had very good tables for all practical purposes. The general conclusion to be drawn from Mr. Chatham's paper seemed to be that a valuation on the $H^M$ and $H^{M(5)}$ basis would put an office in the soundest possible position, as it gave reserves quite equal to what these should be under select tables founded on the most recent data.

Mr. Hewat said that, as a practical commentary upon Mr. Chatham's instructive figures, he would beg leave to submit some results from a trial valuation as at December last of an old office with a well-assorted variety of all classes of life business. Taking the principal classes—whole-life (ordinary and limited payments) and endowment assurances—as a whole, and comparing the $H^M$ with the $O^M$ results, he found the $O^M$ valuation brought out, for every £100 of reserve required by the $H^M$, an additional reserve of about 5s. 9d. When divided up into with- and without-profits, the without-profit class required about 3s. 8d. more, while the with-profit class required about 6s. 1d. more. Dividing up still further, the whole-life (ordinary) without-profit business required about 2s. 10d. more,

while the with-profit required about 4s. 9d. more. The limited-payment (whole-life) class, curiously enough, required, for the without-profits about 3s. 7d. *less* reserve, and for the with-profits about 14s. 7d. *less*. In the endowment-assurance class, the additional reserve required was, without-profits, about 9s. 3d., while in the with-profit class it was about 18s. 10d. These results referred to the totals, and included all ages of lives and all durations of policies. The endowment-assurance business, for all practical purposes, represented by itself the business of a young office rather than that of an old one, seeing that this class is a modern development of the past quarter of a century, and is therefore really a new experience, compared with that of the whole-life business. He observed in a pamphlet[1] he had received that day from New Zealand, the following proportions of whole-life and endowment-assurance business, as found from the Board of Trade and other returns :—

| | Whole Life. | Endowment Assurance. |
|---|---|---|
| United Kingdom, . . . | 71·5 per cent. | 24·8 per cent. |
| Australasian Colonies, . . | 57·4 " | 41·5 " |
| Three largest American Offices, | 76·5 " | 22·4 " |

With these figures they might be able to form some idea of the effect upon the finances of the life offices in relation to the reserves required for this rapidly growing endowment-assurance business. He joined with the other speakers in thanking Mr. Chatham for this paper on policy values, or reserves—the natural sequel to his former paper on premiums.

Mr. W. A. Robertson hardly understood Mr. Chatham's remark that "contrary to what has generally been supposed", selection endures for a shorter period in the case of young lives than in those of middle age. In a previous paper[2] Mr. Chatham had referred to the conclusion that in the *New York Mutual* experience selection was not so efficacious at the young ages as it was at the older, the same features being confirmed by the *Gotha* experience. Examination of the New Experience demonstrated the same principle, and he (Mr. Robertson) had this in mind when he stated, in the discussion on Mr. Chatham's paper on premiums, that, if the Select and Ultimate principle is to be adhered to, the select rates should be joined on to the ultimate after a varying term of years, somewhat after the manner in which Dr. Sprague formed his rates of remarriage among widowers. Mr. Chatham's Table 3, giving the ratios of the New Select to the $O^M$ values, was very interesting, and seemed to show, (1) that the loading in the office premium is not the only allowance made for expenses in a valuation, and (2) that a net premium for an age $t$ years older than the age at entry should not be employed without careful consideration. Mr. Chatham had pointed out that a valuation by the $H^M$ and $H^{M(5)}$ tables would closely approximate to a valuation based upon select tables. The figures deduced went to prove this, but the deficiency during the first five years must not be overlooked. The whole question was important, and might be considered from the following point of view. Dr. Sprague's Select Tables are known to give results approximating to the $H^M$ and $H^{M(5)}$. He (Mr. Robertson) had carefully examined the whole-life values on the New Select basis, and had found them to be invariably greater than those by Dr. Sprague's Select Tables, so that the $H^M$ and $H^{M(5)}$ would seem to be no longer a theoretical equivalent to the standard. He might further point out that it did not follow, simply because the $H^M$ and $H^{M(5)}$ exceeds the $O^M$ and $O^{M(5)}$, that they were justified in adhering to the former basis of valuation. For the purpose of comparing the reserves

[1] *Varieties of Life Insurance.* By Morris Fox. Wellington, N.Z., 1902.
[2] *J.I.A.*, xxix, 155.

by the different mortality tables, Mr. King's "model office" was not wholly satisfactory. It was constructed upon the basis of the old Institute Experience; but the incidence of the entrants at the various ages had altered considerably since that date, and Mr. Chatham had shown the importance of that point. It neglected bonuses, and was confined to the whole-life policies. The $H^M$ and $H^{M(5)}$ tables appeared, however, to yield greater whole-term reserves than the $O^M$ and $O^{M(5)}$; while the converse seemed to hold with endowment assurances, and in this connection they could not overlook the extent of the business and the largeness of the reserve. Mr. Chatham's summing up recalled a paper[1] read by Mr. Sorley before the late Actuarial Society in 1878, in which he stated his reasons for preferring the $H^M$ to the Select reserves. With regard to Mr. Chatham's remarks on the valuation of endowment assurances by a table based upon their own aggregate experience, he might point out that in Mr. Ackland's recent paper it had been shown that for endowment assurances payable at age 60, the $E^M$ table (that is, an aggregate table based on the endowment-assurance experience graduated by Woolhouse's formula) gives policy values lower at entry ages 20 and 25 for attained ages younger than 35, and higher for all other entry ages and durations. He had worked out the ratio per cent of the New Select values to the $E^M$ for endowment assurances payable at age 60, and compared them with the similar ratios of the New Select to the $O^M$, and would conclude that in the valuation of endowment assurances the $E^M$ is the better approach to the standard. On the other hand, the reserve for bonuses will be less by the $E^M$ than by the $O^M$. He had also worked out the ratios of the New Select to the $E^M$ and $E^{M(5)}$, which gave the strongest reserve for endowment assurances of any of the tables. The $E^{M(5)}$ reversion, however, would be still less than the $O^M$.

Mr. W. Young said, with reference to the statement that a particular table was the correct one to use, that he did not think this could be said of any table unless the premiums charged by the office were taken into account. Taking the case of a new office doing a large new business, the new tables might require a reserve larger than the office could afford to make; yet the office might, in the circumstances, be making an adequate reserve. Speaking in a general way, the new tables showed a lighter mortality than the old; and this simply meant that the claims might be expected to fall in at a later date than according to the tables at present in use. It seemed curious that, by changing the basis of valuation to the new tables, the fact that the claims were postponed should require higher reserves; and this arose from the premiums charged not being taken into account. Their office premiums had not been altered, and if the margin formerly allowed for expenses, etc., was adequate, there was no essential reason for altering the net premiums valued. Analysing the policy value, the reversion to value the sum assured would, speaking in a general way, be less; the annuity to value the future premiums greater; and, if the net premium were unaltered, the reserve would be less, as they would expect. The effect of using the new tables, where these brought out an increased reserve, would largely be to reserve a higher margin of premium for future expenses, etc.; and if this were not necessary, it could not be said that the new tables were the correct tables to use. He did not think the facts justified the conclusion drawn by Mr. Chatham on page 154, that the reserve made by offices for their without-profit policies is really stronger than for with-profit policies. This would only be the case if the premiums charged for without-profit policies were on a basis such as would justify a higher premium being valued than that valued in the case of with-profit policies; and this was not the case.

[1] *J.I.A.*, xxi, 192.

Mr. H. F. Falconer remarked that it occurred to him that, as the reserves by the $O^M$ table fell short of the Select reserves, and as the ratio of the select to the aggregate reserve decreased steadily with the duration of the assurance, the deficiency might be made up for practical purposes by valuing at a slightly lower rate of interest, the effect of which is to increase the reserves in a ratio decreasing with the duration.

Mr. Gordon Douglas said that, although Mr. Chatham had only touched upon the point in passing, one of the most interesting features was that which came out in his previous paper, as to selection continuing for a longer period than five years—in fact, as a rule, for fully ten years. He supposed it was an open secret that the new Select Tables, to be published by the Joint Committee, are to be based on a ten-year period of selection. Seeing that was found to be essential, it was recognised that for valuation purposes select tables would be altogether impracticable, and that accordingly the one purpose which select tables need be made to serve was to enable net or pure premiums to be calculated. It was well that they should be able to ascertain as definitely as possible the prime cost of the article which they sold, and leave the pure premiums thus ascertained to be loaded up for expenses and profit to whatever extent they thought prudent. As they would now have $O^M$, $O^{M(5)}$, and Select tables, they would require to keep in mind what the $O^{M(5)}$ table really is. When the $H^{M(5)}$ table was in vogue, it was believed to represent the mortality after selection had passed off; but that would not apply to the $O^{M(5)}$. It would only be a table useful for practical purposes, as showing the mortality when the greatest effect of selection had passed off, but not when it had disappeared. In the 70 per cent of offices valuing by the $H^M$ table, the first step would be to pass to the $O^M$ table; and it was curious to find that several offices had at once desired to see how much this would require in £ s. d. One old office with which he had something to do, made a trial valuation to see what the difference would be; and the result in the case of whole-life policies, on male and female lives, including paid-up policies, and both with- and without-profit assurances, came out practically identical with that mentioned by Mr. Stenhouse, namely, $O^M$ reserve 101, as compared with $H^M$ 100. When bonuses were included, that ratio was reduced to about 100·55 to 100. Of course endowment assurances would increase the difference considerably. He thought one thing had been clearly brought out—that the strain of initial expenses would be greatly increased under an $O^M$ valuation as compared with an $H^M$ valuation. In almost every case the reserve at the first valuation, while it might not be so much greater, in actual money, as Mr. Chatham's ratios might at first glance lead one to suppose, would still certainly be substantially larger than the $H^M$ reserve. It is, of course, on the early years of a policy that the initial expenses press heaviest; and if, notwithstanding this, a larger proportion of the premiums received would now have to be set aside in the shape of reserve, the strain would be all the greater. If the result of this should be to assist in putting a stop to excessive commissions, it would be a boon fervently to be desired. To the 30 per cent of offices valuing on the $H^M$ and $H^{M(5)}$ basis, it must be a source of great gratification to realise that their reserves leave nothing to be desired. It would produce the difficulty of not knowing whether to change their basis or to remain as they were; but as in the past they had so well kept the best interests of the policyholders in view, he did not doubt they would easily find a method of putting away the money so set free, did they prefer to change rather than to adhere to the old tables, which will doubtless gradually become less and less known as time goes on.

Mr. J. Nicoll thought some of the speakers had taken rather an erroneous view of the author's purpose as regards his paper. Mr. Chatham had given values of policies for different ages and durations, but he had not

brought these values together in any way so as to form a "model office". It had been pointed out that the "model office" founded upon the $H^M$ Experience is scarcely applicable to the New Experience. From the figures given in Mr. Chatham's paper, however, each actuary could form for himself a pretty accurate idea as to the effect upon his own particular office of a valuation by the New Experience. One very important lesson to be learned from the present paper was that it might be safely taken, as a general rule, that the more select the table being dealt with, the larger are the policy values brought out by it in the early years at least. This view was confirmed by reference to Dr. Sprague's tables, which show that the values of policies on select lives are larger than those on mixed lives. The same characteristic was also brought out in the present paper in the values of without-profit assurances, the lives under which are shown by the New Experience to be subject to a higher rate of mortality than those under with-profit policies. He was really not disposed to quarrel with anything in the paper; but on page 149, Mr. Chatham said that "when an office employs an aggregate table in its valuation, a considerable allowance is thereby made for the heavy initial expenses incurred in the acquisition of new business". He did not know whether Mr. Chatham meant to recommend such a course. It seemed to him (Mr. Nicoll) that, when a young office especially uses an aggregate table instead of a select one, and declares a large bonus in consequence, it is not acting too wisely in doing so. With reference to what Mr. Young had pointed out in regard to without-profit policy values, it would be remembered that Mr. Chatham in his former paper had shown that the rates of premium at present charged by many offices for without-profit assurances are too low. He assumed, therefore, that the values given for such policies in the paper take into account that the correct and larger premiums are charged. A clearer comparison between the Old and the New Experiences might be instituted, as regards some points at least, by the old-fashioned method of working out a fund based on each experience. He had assumed that 96,223 lives aged 20 effected endowment assurances of 100 each, for the term of 20 years, and he had worked out the results at $H^M$ $3\frac{1}{2}$ per cent and at $O^M$ $3\frac{1}{2}$ per cent. The premium by the former table was 3·826, and by the latter 3·708; and the following figures were some of the results:—

| Years elapsed. | Table. | Number of Survivors. | Years of Life. | Premiums received. | Interest received. | Death Claims paid. | Amount of Fund. | Value of each Policy. |
|---|---|---|---|---|---|---|---|---|
| 5 | $H^M$ | 93,061 | 474,812 | 1,816,628 | 177,977 | 316,200 | 1,678,405 | 18·04 |
| | $O^M$ | 94,163 | 477,119 | 1,769,158 | 180,758 | 206,000 | 1,743,916 | 18·52 |
| 10 | $H^M$ | 89,865 | 933,873 | 3,572,994 | 664,538 | 635,800 | 3,601,732 | 40·08 |
| | $O^M$ | 91,725 | 943,238 | 3,497,526 | 681,905 | 449,800 | 3,729,631 | 40·66 |
| 15 | $H^M$ | 86,281 | 1,376,143 | 5,265,176 | 1,502,200 | 994,200 | 5,773,176 | 66·91 |
| | $O^M$ | 88,787 | 1,396,201 | 5,177,113 | 1,547,016 | 743,600 | 5,980,529 | 67·39 |
| 20 | $H^M$ | 82,284 | 1,799,758 | 6,885,935 | 2,737,007 | 1,393,900 | 8,229,042 | 100· |
| | $O^M$ | 85,269 | 1,833,346 | 6,798,046 | 2,824,223 | 1,095,400 | 8,526,869 | 100· |

The following among other points might be noted in connection with the above figures:—(1) Although the number of entrants is the same in each case, the total years of life brought out by the New Experience is 1,833,346, as against 1,799,758 by the $H^M$ table. (2) Though the total

premiums received under the $O^M$ table are 87,800 less than under the $H^M$ table, that sum is almost entirely made up by the larger amount of interest received on the fund under the new table. (3) The total claims by death under the $O^M$ table are about 300,000 less than under the $H^M$ table, the death-rate under the former table being much less in the earlier years than under the latter table, and gradually approaching it as the term increases. The claims in the first year by the $O^M$ table are only 64 per cent of those by the $H^M$; in the sixth year, 73 per cent; in the eleventh, 79 per cent; in the sixteenth, 87 per cent; and finally in the twentieth year, 90 per cent. The *total* claims to the end of different periods show the following ratios:—five years, 65 per cent; ten years, 71 per cent; fifteen years, 75 per cent; and the whole twenty years, 79 per cent.

The President said he would have been glad if Mr. Chatham had been a little more explicit as to how he made his valuations upon the New Select basis—whether, for instance, they were made in a manner similar to an $H^M$ and $H^{M(5)}$ valuation, or whether he took the great labour of making the calculations for each year of duration separately. Of course they must bear in mind that this comparison—interesting as it was—was a comparison not with the polished and graduated tables that will be ultimately brought into use, but with values calculated by Mr. Chatham himself from the experience data. He could not say whether Mr. Chatham's figures gave a better view of the values indicated by the actual experience than if graduated tables had been used; but it appeared to him that the results might differ somewhat from those obtainable when the graduated tables were published. He had great sympathy himself with what Mr. Chatham had told them of the modern tendency towards simpler methods of calculation. He had been in hopes that some ingenious member of the profession would be able to devise, in connection with this New Experience, a shorthand method of valuation that would give them the effect of the influence of selection without the rather cumbrous device employed in using the $H^M$ and $H^{M(5)}$ tables. That apparently had not been found practicable hitherto. Mr. Meikle had favoured the late Actuarial Society with a paper[1] in which he described a method that seemed to give something like what he (the President) had indicated, but the method did not seem as yet to have taken hold of the profession. It was a matter of some regret to him that they had not attained something of the kind. It might be interesting if Mr. Chatham would give them a comparison of the ratios of the whole-life policy values for males and females; though he did not know if it would be of much practical importance. Certainly it concerned them to see how the values of endowment assurances by the new tables were likely to turn out. Whether they liked it or not, endowment assurance seemed likely to be the most attractive kind of policy in the future. He was rather surprised to see from the figures quoted by Mr. Hewat that in this country the percentage of endowment assurances to whole-life policies was still so small. He believed that if only the more recent figures were taken, the ratio would be found much higher, and as time went on endowment assurances would mount up and become the preponderating class. They should therefore be preparing for that time by making higher reserves, and should not forget that the provision for bonus would cost more in the future than in the past.

Mr. Chatham, in reply, said the present paper had been prepared because he was anxious to have information as to the exact effect of the new tables. This paper followed naturally upon the one that he had submitted last session; but he feared it had suffered a little by his having

[1] *T.A.S.E.*, iv, 599.

assumed a knowledge of the former paper, some of the points mentioned in which appeared to have been forgotten. Mr. Walton had referred to selection passing off in five years or ten years, and had said that the figures did not bear out the conclusions stated; but he (Mr. Chatham) saw no reason why they might not indicate them generally. It was true that the more correct way would have been to examine the rates of mortality; but it should be borne in mind that he had used the unadjusted rates, and he feared that was not practicable. Mr. Walton had referred to the $O^{[M]}$ select rates of mortality, but these, he believed, were graduated according to a mathematical law. In any event, another member had referred to a circumstance which he confessed he had forgotten, that in a former paper of his the same results were brought out in the adjusted rates of mortality of the *Gotha* office, also in the *New York Mutual* experience; so that he thought they might look upon the point as being practically settled now. With regard to the objection to using policy values, he thought Mr. Walton would admit that the total reserve which an office makes is composed of these individual policy values; and if they got out the ratio with regard to each policy value, and found that it was invariably greater than under the table they had been using—as, for instance, in the case of endowment assurances—they had got a good deal of information as to how the new tables would affect the position of the office. Mr. Nicoll had said what he (Mr. Chatham) intended to imply in the paper, namely, that they would be able, from the ratios given, to get a very good idea of how they stood with regard to the new tables. If Mr. Walton preferred the policy values themselves, they would be found at the end of the paper; but he thought the ratios could be more easily followed than the policy values themselves. As a matter of fact, Mr. Walton had arrived at the same conclusion as himself, namely, that the new tables will, as a rule, affect office valuations very little. He would have liked Mr. Walton to give some actual valuation results, taking them in the aggregate, as Mr. Stenhouse, Mr. Hewat, and Mr. Douglas had done; because such actual results were extremely interesting. In particular, Mr. Douglas had shown that the ratio obtained by him of the $O^M$ reserves to the $H^M$ was almost exactly the same as Mr. Stenhouse had found. With reference to Mr. Young's remarks, the two papers were to be taken together; and, as Mr. Nicoll pointed out, the office premiums for without-profit policies were given in his former paper, and he had assumed, perhaps too readily, that the offices would follow them and value accordingly. There was no doubt that higher policy values were got from select than from mixed tables. It was the light mortality at the beginning which made the reserve higher; and by an aggregate table that was not fully allowed for. He thought it was rather hard upon a young office that the $O^M$ table should bring out such large reserves—more particularly as the cost of getting business had a tendency to increase rather than diminish; but in any valuation he had made, no special allowance was made for expenses, although he believed some offices did so. It would be an advantage to show the ratios between policy values on males and females as suggested by the President, and he would do this. For each duration stated, the value of the policy was calculated separately for the New Select and for the other tables, so that he thought they could be relied on. After all, the general conclusion was that the new tables would not materially affect the valuations of an office; and that was a matter for congratulation. There was one crumb of comfort, which he had mentioned in the concluding paragraph of the paper—that a given cash surplus will yield a larger reversionary bonus, and that should not be overlooked.

On the motion of the President, a cordial vote of thanks was awarded to Mr. Chatham for his paper.

# CORRESPONDENCE.

## Mr. Moir's Paper—Maximum Mortality Periods.

*To the Editor of the Transactions of the Faculty of Actuaries.*

Sir,—I see that Mr. Moir in his paper on *First Year's Risk*, gives a table[1] entitled "Mortality amongst lives assured for different durations, as experi-" enced while passing through different ages", and states that the method he has used "shows that there is no maximum mortality period between the " 11th and 15th years, as was formerly contended by Dr. Sprague"; and I shall be glad if you will give me the opportunity of stating that, in my opinion, Mr. Moir's figures do not justify this conclusion. Before giving my reasons for this opinion, I have to remark, in the first place, that I object to Mr. Moir's statement, that I formerly "contended" that there was a maximum mortality period between the 11th and 15th years. This would appear to imply, that one of the objects I had in view in writing my paper of 1870, was to prove the existence of such a maximum mortality period; but I think that any person who reads that paper with an unbiassed mind, must admit that my object throughout was simply to ascertain what are the real facts. I have next to point out that, whereas my table, comparing the actual deaths *at all ages* with the computed, shows a maximum mortality at the years of insurance 11-15, the more extended tables, which give the figures for quinquennial groups of ages, show very different results. At ages attained 46-50, the maximum mortality occurs in the years 19-25; at ages 51-55 in the years 19-22; at ages 56-60 in the years 12-17; at ages 61-65 in the years 16-21; at ages 66-70 in the years 14-16; at ages 71-75 in the years 18-21; and at ages 76-80 in the years 27-36. It therefore gives an incorrect idea to say that I contended that there was a maximum mortality *between the 11th and 15th years*. For greater clearness it may be worth while to give the above figures in a tabular form; and, in order to give some idea of the weight to be attached to the figures, I give also the number of deaths in each group of ages.

| Ages attained. | Years of Insurance. | Actual deaths. |
|---|---|---|
| 46-50 | 19-25 | 231 |
| 51-55 | 19-22 | 354 |
| 56-60 | 12-17 | 565 |
| 61-65 | 16-21 | 463 |
| 66-70 | 14-16 | 213 |
| 71-75 | 18-21 | 229 |
| 76-80 | 27-36 | 268 |

Can it be possible that Mr. Moir read only the first half of my paper?

Mr. Moir refers to Mr. Macaulay's letter in the thirty-second volume of the *Journal of the Institute of Actuaries*; but he makes no reference to my letter in the same volume,[2] in which I prove, or at all events give reasons for believing, that Mr. Macaulay's explanation of the observed

[1] *See* page 28. [2] *J.I.A.*, xxxii, 197.

maximum (which is not merely an "apparent maximum", as Mr. Moir calls it) is not satisfactory. I think that, as Mr. Moir dealt with this point at all, it is a pity that he did not continue his investigations further, for ages above 60; especially as, in my results, those higher ages exhibit the phenomenon of the maximum mortality more clearly than most of the ages given in his table. This is the weak point in his argument. Instead of dealing, as I did, with all ages attained in each year of insurance (grouped together for greater regularity in the results), he simply gives figures for individual ages attained, 40, 45, 50, 55, 60, and for a few periods of insurance—1st, 2nd, 5th, 10th, 15th, 20th, 26th, and 30th; and I submit that such fragmentary observations cannot safely be relied on, compared with the much more extensive ones I dealt with.

I am, Sir,

Your obedient servant,

T. B. SPRAGUE.

EDINBURGH, 25 *June* 1902.

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*To the Editor of the Transactions of the Faculty of Actuaries.*

SIR,—I thank you for your courtesy in sending me a proof of the letter from Dr. Sprague, and am gratified to find so eminent an authority giving my paper such careful consideration. As to my use of the word "contended", it is for others to judge. In his introductory remarks in 1870 Dr. Sprague said :—

> "The effect of the withdrawals . . . is a question of fact. . . . I myself fully agree with those who attribute to these withdrawals a powerful influence in increasing the rate of mortality."—(*J.I.A.*, xv, 332.)

In the early part of his paper of 1870, Dr. Sprague carefully referred to the percentages and ratios of the Select figures to the $H^M$, but in the latter part and in his explanations of probable causes he assumed a maximum mortality and not merely a maximum ratio. For example :—

> "But no such argument can apply to explain the phenomena shown above, viz.—first the increase of the mortality and then its subsequent diminution."—(*J.I.A.*, xv, 351.)

He also advanced reasons (*J.I.A.*, xv, 349-351) to show how the maximum in the mortality was caused by withdrawals, adding (page 351)—

> "The foregoing observations not only demonstrate that the withdrawals produce a powerful effect in increasing the rate of mortality among the lives remaining under observation, but enable us to see clearly the way in which this effect is produced, and, to a certain extent, to measure its magnitude."

There is a great difference between a maximum mortality period, and a maximum ratio as between two tables, either or both of which may be distorted. Dr. Sprague certainly showed that a maximum ratio existed, but I do not think the method he adopted was capable of proving "a maximum mortality".

As I understand Mr. Macaulay's letter, he did not dispute the "fact" that a maximum appeared in the percentages, but he showed (and to me the demonstration is sufficient) that this maximum percentage was probably "due to an entirely different cause" from that put forward in Dr. Sprague's "Maximum Mortality Theory".

The most important point in Dr. Sprague's letter relates to the question of a maximum in the mortality, which Mr. Macaulay is more capable of discussing than I am.

I am, Sir,

Yours faithfully,

HENRY MOIR.

NEW YORK, 30 *October* 1902.

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## MR. CHATHAM'S PAPER—"DISCOUNTED-BONUS" MORTALITY.

*To the Editor of the Transactions of the Faculty of Actuaries.*

SIR,—I observe in the discussion which followed the reading of Mr. Chatham's paper[1] *On Premiums deduced from the New Mortality Experience*, that both Mr. Hewat and Mr. Chatham expressed the opinion that the mortality among policyholders assured under discounted-bonus schemes must be greater than the mortality under with-profit policies.

Some time ago the directors of the *Scottish Amicable* Office caused an investigation to be made into the mortality of their discounted-bonus class as compared with that under their ordinary with-profit class, in order to ascertain if the selection of those desiring a cheap premium scale had been detrimental to the Office. The following is a note of the result :—

*Statement of Sums Assured which became claims during the years* 1896 *to* 1900, *both inclusive, under With-Profit Policies and under Discounted-Bonus Policies respectively, and of the corresponding claims expected by the* H^M *Table.*

| Ages. | With-Profit Policies. | | | Discounted-Bonus Policies. | | |
|---|---|---|---|---|---|---|
| | Actual Claims. | Expected Claims. | Percentage of Actual to Expected. | Actual Claims. | Expected Claims. | Percentage of Actual to Expected. |
| Under 50 | £30,800 | £44,446 | 69·3 | £37,450 | £64,178 | 58·4 |
| 50 to 70 | 131,132 | 132,506 | 99·0 | 166,630 | 193,746 | 86·0 |
| Over 70 | 151,768 | 163,894 | 92·6 | 146,780 | 128,863 | 113·9 |
| Total, | £313,700 | £340,846 | 92·0 | £350,860 | £386,887 | 90·7 |

The above figures show that not only has the aggregate mortality been less among assurances under the discounted-bonus scheme, but also that at

[1] *See* page 109.

ages under 70 the superiority of the discounted-bonus business
marked.

In considering the foregoing figures the following points should
in view:

(1) The figures in both cases represent the mortality among *sums*
excluding bonuses. They are limited also to policies for the whole
by equal annual premiums.

(2) At the period selected (from 1896 to 1900), the office had
existence for about seventy years; and the discounted-bonus cla
adopted nearly fifty years ago: hence the experience covers policies
durations.

(3) As the rules which guided the selection *by the office* have be
same for both classes of assurance, any difference in the rate of mo
if proceeding from selection, must arise from selection *by the assured*.

I am, Sir,

Your obedient servant,

N. B. GU

35 St. Vincent Place,
Glasgow, 11 *July* 1902.

No. 7.

# TRANSACTIONS

OF THE

# FACULTY OF ACTUARIES

INSTITUTED 1856.

INCORPORATED 1868.

On the Book-keeping of a Life Office. By James John M'Lauchlan, F.F.A., Honorary Secretary of the Faculty of Actuaries, Secretary to the Scottish Equitable Life Assurance Society.

With Discussion thereon.

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1903.

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[Read before the Faculty, 15 December 1902.]

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## INTRODUCTORY.

THE Council of the Faculty of Actuaries lately proposed to reprint my paper 'On Life Assurance Book-keeping', read before the Actuarial Society of Edinburgh in 1886. In revising it for this purpose, I have introduced a set of specimens of the various books referred to, with relative entries, which will, I believe, make the paper easier to follow. Their introduction has made it necessary, in order to avoid repetition, to make considerable alterations in the text. I have also, without interfering with the general system described, made such improvements in the books

and entries described, as have been suggested to me by the experience of the last sixteen years. Having regard to those alterations, it would not be satisfactory to describe this paper as a second edition of my paper of 1886, and I have therefore given it a new title. The specimen books and entries have been prepared by Mr. John Notman, F.F.A., to whom I am indebted for valuable assistance in the work of revision.

1. Book-keeping by Double Entry is now universally admitted to be the only complete and satisfactory system, and its general principles have been shown to be adapted to all kinds of commercial or public accounts. The work of the accountant in arranging a special system adapted to the purposes of a Life Assurance Office, is therefore principally confined to fixing the number and forms of the subsidiary books, which serve either to receive the original entries of transactions, or to relieve the Principal Ledger of the particulars of individual accounts.

2. In Life Assurance Book-keeping, as in that of Mercantile Houses, Banks, or Government Departments, these subsidiary books must be specially adapted to the various classes of transactions or other financial facts which occur in the business. I have therefore found it convenient to deal with the subject of Life Assurance Book-keeping, by considering in succession the various classes of financial facts that fall to be recorded in conducting the business, and in the case of each class, describing the books required for receiving the original entries, and the subsequent progress of these entries through the books.

3. I shall confine myself to the case of a Mutual Office transacting Life Assurance business only, partly for the sake of simplicity, and partly because I am not practically acquainted with those transactions that are peculiar to proprietary companies. I have the less hesitation in taking this course, as the book-keeping of joint-stock companies has been already treated of by competent authorities. I assume the Office to be furnished with the usual organisation of Branches and Agents.

4. In considering the subject, it is necessary to keep steadily in view the objects to be attained by Life Assurance Book-keeping. These are—(1) To afford ready information as to the financial transactions of the Office with every individual with whom it does business, providing in the case of persons with whom there are continuous business transactions, and periodical settlements, a regular account; (2) To record fully the intromissions of the officials with the funds of the office; (3) To classify the various transactions of the Office so as to produce at the end of the year (*a*) a revenue account and (*b*) a balance sheet, in the forms required for the returns to Government, with such further details in the case of each as it may be desired to place on record.

5. The primary classification of the transactions of a Life Office is that into Receipts and Payments. Each of these classes again has more than one stage in its history. It will be convenient to take them in their natural order, and commence with that class of facts which forms the first stage in the history of the receipts.

## Receipts Falling Due.

6. The receipts of a Life Office are collected from the public by the intervention of its agents—a term which I shall use to include the Head Office Cashier and Branch Secretaries, as well as the ordinary agents. For the purpose of securing that all sums received by the agents on behalf of the Office are duly accounted for, it is the general rule to furnish them in advance with formal receipts for all sums to be collected by them, intimating to the policyholders that only such receipts are valid. The official receipt forms the agent's authority to receive the amount acknowledged therein, and as far as possible he should not receive any sum for which he does not hold a receipt.

7. For the purpose of keeping proper accounts with the agents, it is necessary to debit them with the amount of all receipts furnished to them for sums payable at their agencies. Strictly speaking, the first stage in the history of any item of receipt is its falling due,[1] but for practical purposes we may consider that the first stage in the collection of the sums receivable by a Life Office, is the transmission of the official receipts therefor to the agents; and such transmission must be recorded in the books of account, by entering the amount of receipts sent to each agent to his debit, and to the credit of some other account.

8. Another reason for making entries in the regular books of account in respect of all receipts sent to agents, is found in the fact that the weak point of every audit is on the receipts side of the accounts. It is a comparatively easy thing for an auditor to satisfy himself, by an examination of vouchers or otherwise, that the payments are in order; but in the case of the receipts, the same method of examination cannot be applied without an amount of labour which is generally quite impracticable. It is therefore desirable to provide an independent check within the office upon the correctness of all sums received, which is obtained by having all the sums to be collected by the agents, charged against them in advance in the Society's books.

9. The most obvious plan to adopt in entering the receipts

[1] Some interesting observations on the subject of the different stages in the history of money transactions, will be found in Sir John Bowring's second Report on the Public Accounts of France, published as a Parliamentary paper in 1831, pp. 7 and 8.

to the debit of the agents, would be to make a separate entry in the Journal for each agent every time receipts were sent to him, specifying the receipts so sent, and carrying them to the credit of the proper accounts. This plan, however, has the disadvantage of overloading the Journal and Principal Ledger, and a preferable course is (1) To provide subsidiary books for receiving the original entries to the debit of the agents, in which the principal classes of entries may be classified at the outset according to the ledger account to which they are to be credited, and from which they may be transferred to the Journal and Principal Ledger in mass; and (2) To debit the receipts in mass to a general account 'Agents', providing a subsidiary ledger to contain the accounts of individual agents, the entries in which are made directly from the primary records.

**10.** With these preliminary remarks, I shall now proceed to consider the transactions which form the first stage in the history of the principal classes of receipts, and the books required for receiving the original entries of them.

**11. First Premiums on New Policies.**—In the system of books which I am about to describe, a separate book, called the **New Business Day Book** (see below), is provided to contain a record of the first premiums on all new policies. It is understood that no policy is prepared until after the first premium has been received, but that an interim receipt for the first premium is sent to the agent to be given up on payment. This receipt is entered in the New Business Day Book, to be posted therefrom to the debit of the agent in his account in the Agents' Ledger (see § 82). The column 'Interim Receipts Returned' is for use in connection with the statement mentioned in § 24. In the case of offices issuing policies under the half-credit system from the outset, this book would require another money column headed 'Interest', to receive interest payable in advance on half premium loans.

**NEW BUSINESS DAY BOOK.**

*Interim Receipts for First Premiums*, 1901. 79

| Date. | Interim Receipt No. | Agency. | First Premium. | Fol. Agts. Led. | Policy No. | Interim Receipts Returned. | Remarks. |
|---|---|---|---|---|---|---|---|
| 1901 | | | | | | | |
| Jan. 1 | 3516 | 'A' Branch . . | 13 11 6 | | 54567 | .. | |
| " " | 3517 | Perth (J. Smith). | 25 10 0 | 621 | 54560 | .. | |
| " " | 3518 | Head Office . . | 16 6 0 | | 54568 | .. | |
| " " | 3519 | 'B' Branch . | 4 4 10 | | .. | 4 4 10 | |
| " " | 3520 | 'C' Branch . | 9 10 6 | | 54600 | .. | |
| " 2 | 3521 | Head Office . . | 75 6 8 | | 54578 | .. | |
| | | | &c. | | | &c. | |
| | | | 2666 5 1 | | | | |

**12. Renewal Premiums.**—The transmission of the renewal premium receipts to the agents each month, is recorded by making lists of the premiums on prepared sheets called the **Renewal Debit Sheets** (see p. 195). In making these lists, the premium receipts are sorted according to agency, the premiums collected by Head Office, each of the Branches, and each of the ordinary agents, being kept separate. The total of the receipts sent to each agent will thereafter be posted to his debit. A statement is made bringing out the total amount of the month's receipts, for the purpose of being carried to the Monthly Summaries Book mentioned in § 76. In the case of premiums payable at the Branches, or forwarded to agents through the Branches, the renewal debit sheets are prepared in duplicate, and one copy forwarded with the receipts to the Branch. These Renewal Debit Sheets contain extra renewal premiums, as well as ordinary renewal premiums, also interest as stated in § 14. The 'Commission paid' columns in the form are for the purpose of recording the payment of commission to agents or others. The Debit Sheets for Head Office, each of the Branches, and the agents in each District, are kept in separate cases, and ultimately bound together into volumes for preservation and convenience of reference, the Head Office section including the summary.

**13.** An alternative course in dealing with premiums payable to ordinary agents, is to enter the receipts individually in the Agents Ledger, and make a summary of the totals in the Renewal Debit Sheets.

**14. Periodical Interest.**—Under this heading is included, in addition to interest on ordinary loans, interest on bonds and debentures payable by coupon, interest or dividends on stocks or shares, and feu-duties or ground rents. The transmission of the interest receipts to the agents each month, is recorded by making lists of the amounts on the Renewal Debit Sheets (p. 195). When no receipt is required, a debit note is written instead, and the book entries made in the same way. The receipts for Head Office, each of the Branches, and each of the ordinary agents, are kept separate, as in the case of premiums. The total of the receipts sent to each agent will thereafter be posted to his debit. A statement is made bringing out the total amount of the month's receipts, as in the case of premiums. There is an exception to the general rule, in the case of periodical interest payable with an instalment of capital, which is entered, not in the Renewal Debit Sheets, but in the Repayments Day Book, as explained in § 18. The amount to be debited to each agent is the net amount of interest after deducting income tax. The Interest Book, which contains the information from which the receipts are written, is furnished with columns showing the amount of the income tax. This is dealt with as explained in § 23.

## RENEWAL DEBIT SHEETS.

*Premiums, Interest, and Rents due in* January 1901.

(Form 1—Head Office and Branches.)

60

| Policy No. | Due date. | Premium. | Interest *net.* | Date of Payment. | Commission paid. | Policy No. | Due date. | Premium. | Interest *net.* | Date of Payment. | Commission paid. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Head Office.*** | | | | | | B. f. | | 1882 7 10 | 97 3 2 | | |
| 9734 | 10 | 15 5 5 | 2 10 0 | | | | | &c. | &c. | | |
| 15721 | 13 | 6 12 0 | .. | | | | | | | | |
| 21538 | 5 | 37 12 6 | .. | | | | | | | | |
| 22716 | 26 | 3 3 4 | 0 10 6 | | | | | | | | |
| 22717 | ,, | 17 8 6 | .. | | | | | | | | |
| &c. | | &c. | &c. | | | | | 3265 0 1 | 564 6 11 | | |
| C. f. | | 1332 7 10 | 97 3 2 | | | | | | | | |

(Form 2—Agents.)

| Policy No. | Due Date. | Premium. | Interest *net.* | Date of Payment. | Commission paid. | Total Debits. Premiums. | Total Debits. Interest. | Fol. Agts. Led. | Remarks |
|---|---|---|---|---|---|---|---|---|---|
| ***Head Office Agents.*** | | | | | | | | | |
| Aberdeen, James Thomson. | | | | | | | | | |
| 35716 | 15 | 5 4 8 | .. | | | | | | |
| 35717 | 4 | 15 14 0 | 1 3 5 | | | | | | |
| 39324 | 27 | 14 18 2 | .. | | | | | | |
| | | 35 16 10 | 1 3 5 | | | 35 16 10 | 1 3 5 | | |
| | | &c. | &c. | | | &c. | &c. | | |
| | | | | | | 2725 4 10 | 42 8 5 | | |

(Form 1.)

| Policy No. | Due date. | Premium. | Interest *net.* | Date of payment. | Commission paid. | Policy No. | Due date. | Premium. | Interest *net.* | Date of Payment. | Commission paid. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | *Summary.* | | | | | | | |
| | | Head Office | | .. .. | .. .. | .. .. | .. | 3265 0 1 | 564 6 11 | | |
| | | do. | No. 2 | .. .. | .. .. | .. .. | .. | 334 15 5 | 53 11 10 | | |
| | | do. | Agents | .. .. | .. .. | .. .. | .. | 2725 4 10 | 42 8 5 | | |
| | | 'A' Branch | | .. .. | .. .. | .. .. | .. | 1541 3 11 | 175 9 6 | | |
| | | do. | Agents | .. .. | .. .. | .. .. | .. | 437 16 3 | 57 3 9 | | |
| | | 'B' Branch | | .. .. | .. .. | .. .. | .. | 962 7 8 | 41 5 6 | | |
| | | do. | Agents | .. .. | .. .. | .. .. | .. | 1310 2 0 | 11 12 5 | | |
| | | &c. | | | | | | &c. | &c. | | |
| | | | | | | | | 23131 18 9 | 2373 18 8 | | |

**15. Rents.**—Under this heading are included rents of property belonging to the Office, but not ground rents. Rents are dealt with in all respects like interest, but are kept separate in the Renewal Debit Sheets, so as to admit of their being carried to the credit of a separate account 'Rents'.

**16. Assignment Fees.**—The **Register of Assignments** (see below) contains columns for the amount of the fee charged for acknowledgment, and the date of payment. These fees are all sent to Head Office, and the total for each month is debited to the Head Office Cashier in his agency account (see § 82). The 'Fee' column thus forms a part of that account.

**REGISTER OF ASSIGNMENTS.**

*Notices of Assignment*, 1901. 82

| Notice No. | Date of Receipt. | Policy No. | Life. | Fee. | Date of Payment. | Entered. | Remarks. |
|---|---|---|---|---|---|---|---|
| | 1901 | | | | | | |
| 20856 | Jan. 1. | 31949 | J. W. Phillips | 0 5 0 | Jan. 1 | | |
| 20857 | ,, ,, | 50851 | W. Oliver | .. | .. | | No ackt. |
| 20858 | ,, 2. | 42593 | A. Macdonald | 0 5 0 | | | |
| &c. | | &c. | &c. | &c. | | | |
| | | | | 11 0 0 | | | |

**17. Miscellaneous Receipts of the Nature of Income.**—These will include premiums which do not fall to be entered in the Renewal Debit Sheets or New Business Day Book; namely, premiums on revived policies, first premiums under licenses for foreign residence or hazardous occupation issued in connection with subsisting policies, premiums altered, proportions of premium payable on change of the date of renewal; also interest on bank deposits, fines, and reassurance claims and surrenders. As the number of transactions under each of these classes is limited, they are classified, not in separate books, but in a single book provided with separate money columns, so as to admit, as far as possible, of the sums belonging to each ledger account being added up by themselves. The book for this purpose is called the **Debit Day Book** (see p. 198); and in it are entered all receipts sent to agents for items coming under this class. The officer of the company who signs or issues these receipts, at the same time places his initials in the 'Initials' column. Each entry is posted to the debit of the agent in his account in the Agents Ledger. Consistently with what has been said in the case of periodical interest, the interest entered in the Debit Day Book is the net amount after deduction of the income tax, which is entered in an inner column provided for the pur-

pose. Interest for fractional parts of a year payable on repayment of loans is, for the sake of convenience, entered not in the Debit Day Book but in the Repayments Day Book described in § 18.

**18. Receipts of the Nature of Capital.**—These include repayments of loans, and sums receivable on the sale of securities, or falling-in of reversions. The despatch to the agent of the formal receipt for the principal sum to be repaid, is recorded by entering the amount in the **Repayments Day Book** (see p. 198), the entry being initialed in the same way as similar entries in the Debit Day Book. All entries dealing with capital are finally made in the Head Office books, and all items of capital receivable must be accounted for by the Head Office Cashier. For the sake of convenience, the Repayments Day Book contains, together with the entries relating to capital, the entries debiting to the Cashier for collection all interest receivable at the same time as the capital; such as interest payable along with fixed instalments, interest forming part of an annuity payment, or fractional interest payable on repayment of a loan. The monthly total of the 'Interest' column is carried to the debit of the Cashier in his agency account, and the column thus forms part of that account. The entries in the 'Principal' column—and this is the peculiarity of the book—are not posted at all; but the date of payment of each item is entered in the adjoining date column. In this way the book provides a record showing at all times what receipts for items of capital are in the hands of the Cashier, or any other person, on behalf of the Office. The 'Income Tax' column is for income tax on fractional interest only. In the case of periodical interest, the income tax is entered in the Interest Book.

19. There is a good deal of difficulty in arranging a satisfactory system of dealing with the entries relating to items of capital, collected through agents. Three methods are, I believe, in use. One method follows the corresponding procedure in the case of items of income. The discharged bond or other formal acknowledgment for the sum receivable, is debited to the collecting agent, and credited to the proper investment account in the Principal Ledger. That account represents the aggregate of a number of accounts kept in the Investment Ledger, with individual borrowers or relating to particular purchases; and it will be seen that, while it is proper to debit the agent with the amount receivable, it would not be proper to credit the corresponding account in the Investment Ledger, until the amount has been actually collected. It is therefore necessary to make the entry in the Investment Ledger, by posting from the Cash Book. The effect of this is, to throw the subsidiary investment accounts out of correspondence with the principal accounts, so as generally to require some adjustment at the end of the financial year.

**DEBIT DAY**

54 *Miscellaneous Debits*

| Date. | Policy No. | Secy. No. | | Due Date. | Initials. |
|---|---|---|---|---|---|
| 1901 | | | | | |
| Jan. 1 | 49355 | .. | A. Fraser. Licence No. 5821, . . . . . | .. | A. B. |
| " " | .. | .. | Reassurance Claims. Reassce. No. 628, James Lee. 'X' Insce. Co., . . . . . | .. | A. B. |
| " " | 34862 | 697 | J. Elliot. Policy revived, . . . . . | Nov. 10 | A. B. |
| " 2 | 48826 | .. | J. G. Heron. Changed to Endowment, . . . | Dec. 25 | A. B. |
| " " | 47916 | .. | Richard Jones. Proportion of premium on change of date of Renewal, . . . . . | .. | A. B. |

Analysis of 'Sundries' Column.

| Date. | Reassurance Claims. | Fees. | &c. | Total. |
|---|---|---|---|---|
| 1 | 1000 0 0 | .. | .. | 1000 0 0 |
| 3 | .. | 0 10 6 | .. | 0 10 6 |
| | &c. | &c. | .. | &c. |
| | 1500 0 0 | 2 2 0 | &c. | 1641 13 4 |

**REPAYMENTS**

22 *Repayments of Capital*

| Date. | Policy No. | Secy. No. | Account to which Principal to be credited. | |
|---|---|---|---|---|
| 1901 | | | | |
| Jan. 1 | 40321 | 535 | Policy Loans, . . . . | James Anderson, . . . . . |
| " " | 25832 | 327 | Non-Forfeiture Advances, . | John Munro, . . . . . |
| " " | .. | 1282 | Reversions, . . . . | William Adams, . . . . . |
| " " | 44553 | 1075 | Policy Loans, . . . . | R. F. Harris, . . . . . |
| " " | .. | 786 | Loans on Rates, . . . | Burgh of——. Annuity due 1 Jan., |

20. A second method is, to debit the amount of the discharged bond or other document to the agent, and credit it to 'Agency Suspense Account'; and, when advice has been received of the completion of the transaction, then to make a cross entry in the Head Office Cash Book, debiting the amount to 'Agency Suspense Account' and crediting it to the proper investment account. The objections to this plan are, the cumbrous nature of the entries sometimes required, and the fact that it introduces into the Cash Book entries which have nothing to do with cash, and ought, according to strict principle, to be made in the Journal.

**BOOK.**

*to Agents*, 1901.

54

| Income Tax. | Interest *net*. | Premiums. | Fol. Regr. of Dedns. | Fines. | Sundries. | Total. | Agency Debited. | Fol. Agts. Led. |
|---|---|---|---|---|---|---|---|---|
| .. | .. | 10 0 0 | | .. | .. | 10 0 0 | 'A' Branch. | |
| .. | .. | .. | .. | .. | 1000 0 0 | 1000 0 0 | Head Office. | |
| 0 12 6 | 9 7 6 | 22 10 0 | | 2 0 3 | .. | 34 17 9 | Head Office. | |
| .. | .. | 13 12 11 | | .. | .. | 13 12 11 | Perth (J. Smith), | 621 |
| .. | .. | 3 2 10 | | .. | .. | 3 2 10 | 'B' Branch. | |
| &c. | &c. | &c. | | &c. | &c. | &c. | | |
| 7 18 6 | 137 4 10 | 457 9 11 | | 13 16 3 | 1641 13 4 | 2450 4 4 | | |

**DAY BOOK.**

*and Interest thereon*, 1901.

22

| Initials. | Income Tax on fractional interest. | Interest *net*. | Entered H. O. Cash Book. | Principal. | Entered H. O. Cash Book. | To be collected by |
|---|---|---|---|---|---|---|
| A.B. | 0 2 9 | 2 0 0 | Jan. 1 | 50 0 0 | Jan. 1 | Head Office. |
| A.B. | 0 0 8 | 0 3 10 | Jan. 1 | 7 8 7 | Jan. 1 | do. |
| A.B. | .. | .. | .. | 531 6 11 | Jan. 1 | do. |
| A.B. | 0 3 7 | 2 15 6 | Jan. 2 | 75 0 0 | Jan. 2 | 'A' Branch. |
| A.B. | .. | 553 2 6 | Jan. 1 | 433 0 0 | Jan. 1 | Head Office. |
| | &c. | &c. | | &c. | | |
| | 21 15 0 | 1831 2 9 | | | | |

21. A third plan is, to make no entries in the books of account, except in respect of cash received. Discharged bonds sent to agents are not debited to them, but are kept in view by means of a special memorandum book, in which they are marked off when accounted for. The cash, when received, is entered in the Cash Book to the credit of the proper investment account. It will be noticed that, as receipts of the nature of capital already stand debited in the books, they do not, so much as receipts of the nature of income, stand in need of the independent verification supplied by the entry to the debit of the agent collecting them.

**22.** The system described in § 18 may fairly be said to combine, as far as practicable, the advantages of the second and third methods described above. It agrees in principle with the latter. The Repayments Day Book, as regards the 'Interest' column, forms a section of the Head Office Cashier's agency account. As regards the 'Principal' column, it is supplementary to that account, and furnishes a record of the items of capital for which the Cashier has to account. This record is exactly the same in character, as if the various sums had been debited to the Cashier in the manner described in § 20, except that it is impossible to verify the outstanding items, by showing that the total of them is equal to the balance of the account. The outstanding items of *interest*, however, may be verified in that manner; and as each item of capital is entered alongside the corresponding interest, attention is necessarily drawn to any irregularity in marking off the former.

**23.** I have now described a set of books adapted to receive the original entries in respect of all receipts sent for collection to the agents of the Office, in which the principal classes of entries relating to income, are classified according to the ledger accounts to which they are to be credited, so as to admit of their being dealt with in mass in the Journal and Principal Ledger. The entries in these books, namely:—The New Business Day Book, Renewal Debit Sheets, Register of Assignments, Debit Day Book, and in the 'Interest' column of the Repayments Day Book, are added up at the end of each month, and the totals inserted. In the case of the Debit Day Book, there is also inserted a statement analysing the entries in the 'Sundries' column, and showing the amount to be credited to each ledger account. The proper totals are then carried to the Monthly Summaries Book as described in § 76. The 'Income Tax' columns in the Interest Book (§ 14) Debit Day Book, and Repayments Day Book, are added up, and the totals carried to the Income Tax Summary in the Monthly Summaries Book.

**24.** At the end of each year a statement is prepared showing that the premiums receivable in a year, as stated in the Register of Policies, have been all debited to the agents for collection or otherwise accounted for. This is in the form shown on p. 201.

**25.** Of the books mentioned in this statement, the Register of Policies is the permanent record of the particulars of all policies issued by the Office. The primary purpose of the Policy Guard Book is to record the policies signed by the Directors; but it is also a register of policies, containing such particulars as are useful for temporary purposes. The Register of Deductions contains particulars of all policies going off the books, in consequence of

***Reconcilement** of the Premium Income as stated in the Register of Policies, with the Total Premiums charged against Agents in* 1901 *per Renewal Debit Sheets and New Business Day Book.*

| | | | | | | |
|---|---|---|---|---|---|---|
| **I.** | | | | | | |
| 1. Premium Income at 31 December 1900, per Register of Policies, fol. , . . . . . | | | | 288,575 | 0 | 2 |
| 2. Premium Income on Policies effected 1901, per Register of Policies, fol. , . . . . . | | | | 24,018 | 1 | 8 |
| 3. Single Premiums on Policies effected 1901, per appended Statement, . . . . . | | | | 5,089 | 18 | 1 |
| 4. First Proportions of Premium on Policies effected 1901, per appended Statement, . . . . . | | | | 434 | 7 | 11 |
| 5. Premiums on Policies revived or changed 1901, chargeable in same year, per Register of Deductions, fol. , . . . . . | | | | 2,712 | 12 | 4 |
| | | | | 320,830 | 0 | 2 |
| **II.** | | | | | | |
| 1. Premiums charged 1901, per Renewal Debit Sheets :— | | | | | | |
| January, fol. , . . . . . | 10,167 | 14 | 0 | | | |
| February, ,, , . . . . . | 16,120 | 16 | 3 | | | |
| &c., | &c. | | | | | |
| | 279,462 | 15 | 6 | 279,462 | 15 | 6 |
| 2. First Premiums charged in 1901, less receipts returned not taken up, or unaccounted for on the closing of the Register,[1] per New Business Day Book, fol. , . | | | | 26,505 | 12 | 0 |
| 3. Half-Yearly Premiums, Quarterly Premiums, and Yearly Premiums where the first payment was a proportion, included under I. (2), not chargeable in 1901, per Policy Guard Book, fol. , . . . . | | | | 2,153 | 16 | 5 |
| 4. Premiums on Policies deducted from the Register or changed in 1901, not chargeable for that year, per Register of Deductions, fol. , . . . . . | | | | 12,707 | 16 | 3 |
| | | | | 320,830 | 0 | 2 |

*Statement referred to above.*

| Single Premiums. | | First Proportions of Premium. | |
|---|---|---|---|
| Policy No. | Amount. | Policy No. | Amount. |
| 54563 | 72 10 0 | 54589 | 19 18 0 |
| 54579 | 108 6 8 | 54603 | 8 2 6 |
| 54610 | 87 15 0 | 54605 | 4 10 0 |
| | &c. | | &c. |
| | 5089 18 1 | | 434 7 11 |

[1] The Register of Policies for each year is understood to be closed soon after 31 Dec., and to contain all assurances accepted in that year, which have been completed by payment of the premium.

their becoming claims or for other reasons, of policies revived after having been deducted as forfeited, and of all changes in premiums.

**26.** The Interest Book contains a column to receive the amount of each loan, or other interest-bearing investment, and a supplementary column, to contain the amount of loans upon which interest has been previously charged during the year, and loans repaid. By the help of these columns, the total amount upon which interest has been charged is reconciled with the amount of loans and investments appearing in the balance sheet. This serves to show that the interest receivable has been duly charged.

## Cash Receipts.

**27.** We now come to an essentially different class of financial facts, namely, Cash Receipts. In the case of the various classes of receipts mentioned above, the actual receipt of the cash forms the second stage in their history. The cash so received may be paid to any agent of the Society who has been authorised to receive it. In the case of ordinary agents, the Office does not require them to render a continuous account of sums which they

**HEAD OFFICE**

122 *Cash Received,*

| Date. | Policy No. | Secy. No. | | Due Date. | Head Office Collection. |
|---|---|---|---|---|---|
| 1901 | | | | | |
| Jan. 1 | 27316 | .. | David Baxter. Premium, | Dec. 10 | 67 3 4 |
| " " | 31949 | .. | J. W. Phillips. Assignment Fee, | .. | 0 5 0 |
| " " | 35978 | .. | John Dow. Premium, | Dec. 4 | 35 6 5 |
| " " | " | 600 | Do. Interest, | " 4 | 6 7 6 |
| " " | 40321 | 535 | James Anderson. Loan and Interest, | .. | 2 0 0 |
| " " | .. | 1726 | Alexander Morrison. Interest, | Dec. 31 | 436 12 6 |
| " " | 25832 | 327 | John Munro. Non-Forfeiture Advances. Advance and Interest, | .. | 0 8 10 |
| " " | 54568 | .. | Charles Young. Premium. Interim Receipt No. 3518, | .. | 16 6 0 |
| " " | .. | 786 | Burgh of —— Annuity, | Jan. 1 | 553 2 5 |
| " " | .. | .. | Royal Bank. Cheque No. 6559, | .. | .. |
| " " | .. | 1282 | William Adams. Reversions, | .. | .. |
| " " | .. | .. | Deposits. Royal Bank. Deposit Receipt, dated 11 Novem. 1900, and Interest, | .. | 8 1 7 |
| " " | .. | .. | A. Smith. Reassurance No. 749. 'X' Insurance Co., | .. | .. |
| " 2 | 42801 | 2007 | H. Allan. Policy loan settled at 'A' Branch per contra, | .. | 47 19 0 |
| " " | 44553 | 1075 | R. F. Harris. Loan and interest settled at 'A' Branch, | .. | 2 15 5 |
| " " | 34862 | .. | J. Elliot. Premium, | Nov. 10 | 22 10 0 |
| " " | " | 697 | " Interest, | " 10 | 9 7 6 |
| " " | " | .. | " Revival Fine, | .. | 2 0 3 |
| " " | .. | .. | Reassurance No. 628. J. Lee. Reassurance Claim. 'X' Insurance Co., | .. | 1000 0 0 |
| " " | .. | .. | Perth. John Smith, | .. | .. |
| | | | | | &c. |
| | | | 'Head Office Collection', | | 29438 15 11 |
| | | | Total 'Agents', | | |

may have received on its behalf, but only to account for the receipts which have been sent to them for collection at regular intervals. In the case, however, of salaried officers, a strict account ought, consistently with what is stated in § 4, to be kept of all cash received and paid on behalf of the Office.

**28.** The Head Office Cashier has to deal with the following different kinds of cash receipts, namely, (1) sums which may be looked upon as received by him in his capacity of an agent, whether (*a*) income, or (*b*) capital; (2) remittances from ordinary agents; (3) deductions made from reassurance premiums and policy loans for commission and stamps; and (4) cheques on the Society's Bankers. I shall consider these in their order. The Head Office **Received Cash Book** is shown below.

**29. Receipts of the nature of Income.**—These include premiums, interest, rents, and the other items of receipt mentioned under § 17, as well as assignment fees. Sums received under this head have, as explained above, been previously debited to the Cashier in his account in the Agents Ledger. The amounts are entered in the column headed 'Head Office Collection'. This column is added up at the end of the month, and the total entered

**RECEIVED CASH BOOK.**

1901. **122**

| Agents' Remittances. | Fol. Agts. Led. | Reassurance Commission. | Stamps on Loan Agreements, &c. | Policy Loans. | Loans on Rates. | Sundries. | Fol Invt. &c., Led. | Cheques on Society's Bankers. | Fol. Bk. Led. | Total. |
|---|---|---|---|---|---|---|---|---|---|---|
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 67 8 4 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 0 5 0 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 85 6 8 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 6 7 6 |
| .. | .. | .. | .. | 50 0 0 | .. | .. | | .. | .. | 52 0 0 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 426 12 6 |
| .. | .. | .. | .. | .. | .. | 7 8 7 | | .. | .. | 7 12 5 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 16 6 0 |
| .. | .. | .. | .. | .. | 433 0 0 | .. | | .. | .. | 986 2 6 |
| .. | .. | .. | .. | .. | .. | .. | .. | 1700 0 0 | 20 | 1700 0 0 |
| .. | .. | .. | .. | .. | .. | 531 6 11 | | .. | .. | 531 6 11 |
| .. | .. | .. | .. | .. | .. | 2000 0 0 | | .. | .. | 2008 1 7 |
| .. | .. | 15 0 0 | 0 14 6 | .. | .. | .. | .. | .. | .. | 15 14 6 |
| .. | .. | .. | 0 1 0 | .. | .. | .. | .. | .. | .. | 48 0 0 |
| .. | .. | .. | .. | 75 0 0 | .. | .. | | .. | . | 77 15 6 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 22 10 0 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 9 7 6 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 2 0 8 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 1000 0 0 |
| 125 7 8 | 621 | .. | .. | .. | .. | .. | .. | .. | .. | 125 7 8 |
| &c. | | &c. | &c. | &c. | &c. | &c. | | &c. | | &c. |
| 1810 17 4 | | 76 9 5 | 9 13 6 | 639 16 7 | 1051 8 1 | 13915 17 10 | | 45677 2 4 | | 92610 1 0 |
| 29428 15 11 | | | | | | | | | | |
| 31239 13 3 | | | | | | | | | | |

to the credit of the Cashier in his agency account. When a premium is paid at Head Office, which stands at the debit of an agent, it will be convenient to enter the amount in the Received Cash Book to the credit of the agent.

30. **Receipts of the nature of Capital.**—Sums received under this head have not previously been debited to the Cashier in his agency account; but, as explained in § 18, they have been entered in the Repayments Day Book. The sums received are therefore entered in the Cash Book, directly to the credit of the proper investment account in the Principal Ledger. In the case of policy loans and loans on rates, the name of the ledger account is indicated by entering the amount in the proper column. In other cases, the amount is entered in the 'Sundries' column, and the name of the ledger account stated in the narration column.

31. **Remittances from Ordinary Agents.**—These are entered in a special column, headed 'Agents' Remittances'. The individual items are posted to the credit of the respective agents, in their accounts in the Agents Ledger.

32. **Commissions and Stamps deducted from Reassurance Premiums and Policy Loans.**—These premiums and loans are entered in full in the Paid Cash Book (p. 207). The commission is entered in the column headed 'Reassurance Commission'. Stamp duty on the documents of security for policy loans, payable by the borrowers, and stamp duty repaid to the Principal Office in the case of reassurances, are entered in the column headed 'Stamps on Loan Agreements, etc.'.

33. **Cheques on the Society's Bankers.**—It is convenient to treat all these cheques as received by the Cashier. The amounts are entered in a special column headed 'Cheques on Society's Bankers'. The entry states the name of the particular account operated on. Debits made by bankers in their accounts for interest or charges, are treated as if a cheque had been drawn for the amount, being entered in the 'Cheques' column of the Received Cash Book, and in the proper column of the Paid Cash Book.

34. The various columns of the Received Cash Book are added up at the end of each month; the totals of the 'Head Office Collection' and 'Agents' Remittances' columns are added together; and a statement is inserted analysing the 'Sundries' column, and showing the total to be credited to each ledger account. All the totals are carried to the Journal as described in § 76.

35. The arrangements for recording the receipt of cash at the **Branches** are analogous to those at the Head Office. The **Branch Cash Book** is in the form shown on page 206. Remittances from

ordinary agents and cheques on the Branch Secretary's banking account, are entered in the special columns for the purpose: this account is replenished by cheques sent specially from Head Office for the purpose. All other receipts are entered in the 'Branch Collection' column.

**36.** Head Office cheques received for the purpose of making particular payments, such as claims and surrenders or policy loans of substantial amount, are not entered in the Branch Cash Book at all; the entry carrying them to the debit of the proper ledger account having been made in the Head Office Paid Cash Book.

**37.** When the Branch Secretary receives any item of the nature of capital, such as a policy loan repaid, he at once sends particulars of the transaction to Head Office; and a cross entry is made in the Head Office books, the amount being entered in the Received Cash Book, to the credit of the proper loan or investment account, such as 'Policy Loans', and in the Paid Cash Book, to the debit of the Branch. The latter entry is, in due course, posted to the debit of the Branch Secretary in his agency account. In this way, these important classes of entries are passed through the Head Office Received Cash Book, and the book-keeping at the Branches is simplified. Fractional interest receivable along with the capital is similarly advised to Head Office, the amount entered in the Head Office Paid Cash Book to the debit of the Branch, and the entry made in the Received Cash Book, as if the money had been received at Head Office.

**38.** The Branch Secretary is required to send to Head Office, at the end of each month, a copy of his Cash Book for that month. The sheets containing these copies are bound together, and the entries are posted into the Head Office books. The monthly total of the 'Branch Collection' column, is posted to the credit of the Branch Secretary in his agency account; the entries in the 'Agents' Remittances' column are posted to the credit of the agents in their respective agency accounts; and the monthly total of the 'Cheques' column, is posted to the credit of the Branch Secretary's bank account in the Bank Ledger. The totals of the 'Branch Collection' and 'Agents' Remittances' columns are added together, and the sum of them carried to the Monthly Summaries Book at the end of each month, as described in § 76.

### Payments falling due.

**39.** We have seen that there are two stages to be recorded in the history of sums received by a Life Assurance Office from the public, namely (1) the transmission of the official receipt to an agent of the office for collection, and (2) the actual receipt of the

BRANCH

62 *Cash Received.* 'A'

| Date. | Policy No. | | Due Date. | Branch Collection. | Agents' Remittances. | Fol. Agts. Led. | Cheques on Branch Secretary's Bank Account. | Total. |
|---|---|---|---|---|---|---|---|---|
| 1901 | | | | | | | | |
| Jan. 1 | 36217 | J. C. Smith. Premium . . | Dec. 11 | 42 1 5 | .. | .. | .. | 42 1 5 |
| " " | " | Do. Interest . . | " | 11 3 0 | .. | .. | .. | 11 3 0 |
| " " | .. | Oxford (J. C. Clarke) . . | .. | .. | 156 3 7 | | .. | 156 3 7 |
| " " | .. | National Bank . . . | .. | .. | .. | .. | 47 19 0 | 47 19 0 |
| " " | 44553 | R. F. Harris. Policy Loan . | .. | 75 0 0 | .. | .. | .. | 75 0 0 |
| " " | " | Do. Interest . . | .. | 2 15 6 | .. | .. | .. | 2 15 6 |
| " 2 | 54567 | M. Hay. Interim Receipt 3516 | .. | 13 11 6 | .. | .. | .. | 13 11 6 |
| | | | | &c. | &c. | .. | &c. | &c. |
| | | | | 5960 8 7 | 827 5 9 | | 699 13 1 | 7487 7 [illegible] |
| | | 'Branch Collection' . | .. | | 5960 8 7 | | | |
| | | Total 'Agents' . . | .. | | 6787 14 4 | | | |

cash by an officer of the Society. Each payment also has two stages in its history, namely (1) that of its falling due or being admitted as due, and (2) that of its being actually paid; and in the case of payments made by cheque, an intermediate stage may be added, that of the issue of a cheque for the amount, the cash being in this case paid by the Bank upon which the cheque is drawn.

**40.** It would be too cumbrous to keep a continuous record of all those stages in the books of account. In the set of books which I am describing, only one stage is recorded so as to be carried into the Ledger, namely the fact of delivery of the cash or cheque. The effect of a system of this kind is that, in the case of the various classes of cash payments coming into the Revenue Account, the Ledger shows the amount actually paid in the year; whereas the sum that ought to be entered in the Revenue Account is the amount properly applicable to the year, whether paid in the year or not. In the case of the more important classes of payments—Claims, Reassurance Premiums, Commission, and Expenses—statements are made at the end of the year of the amount due but not paid, and the ledger accounts adjusted in the manner described in §§ 101, 102, 103.

**41.** In the case of claims and reassurance premiums, a formal record is made of the falling due of the payment. A separate book, the **Claims Intimated Book** (see p. 207), is provided to receive the particulars of all claims intimated, and the **Reassurance Renewal Book** (p. 208) shows the renewal reassurance premiums that fall due each month. The figures in these books are not carried into the Ledger, but are used in

**ASH BOOK.**

*anch*, 1901. *Cash Paid.* 62

| ate. | Policy No. | | Voucher No. | Commission. | Expenses. | Sundries. | Payments into Bank. | Folio. | Total. |
|---|---|---|---|---|---|---|---|---|---|
| 901 | | | | | | | | | |
| n. 1 | 36217 | J. C. Smith. Renewal Commission | 1 | 2 11 0 | .. | .. | .. | .. | 2 11 0 |
| " " | 54567 | M. Hay. Medical fee, Dr. Black . | 2 | .. | 1 1 0 | .. | .. | .. | 1 1 0 |
| " " | 43216 | T. Hope. Surrenders . . . | 3 | .. | .. | 36 7 6 | .. | .. | 36 7 6 |
| " " | .. | National Bank . . . . | .. | .. | .. | .. | 236 8 7 | | 236 8 7 |
| " " | 42801 | Head Office, Policy Loan. H. Allan—*Less* Stamp duty . . | .. | .. | .. | 47 19 0 | .. | | 47 19 0 |
| | | | | &c. | &c. | &c. | &c. | | &c. |
| | | | | 67 2 1 | 149 11 10 | 726 0 5 | 6544 18 1 | | 7487 7 5 |

preparing the statements of outstanding claims and reassurance premiums referred to in § 40.

**CLAIMS INTIMATED BOOK.**

*Claims intimated*, 1901. 202

| e. | Claim No. | Policy No. | Life Assured. | Date of Death (or Maturity). | Sum Assured. | Sum Payable. By Death. | Sum Payable. By Survivance. | Date Admd. | Date Paid. | Unpaid at end of year. |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | |
| . 1 | 13861 | 23141 | Hugh Sutton, . . | 1900 Dec. 22 | 100 0 0 | 137 6 6 | .. | | | |
| " | 13862 | 11016 | James R. Robertson, | " " 30 | 1000 0 0 | 1696 13 0 | .. | | | |
| " | 13863 | 27270 | Robert H. Jones, . | " " 28 | 500 0 0 | .. | 658 11 5 | | | |

## Cash Payments.

**42.** The cash or cheque may be handed over, either by an officer of the Society, or by one of its ordinary agents. In the former case, consistently with the principle stated in § 4, an entry of the transaction must be made by the officer in his Cash Book. In the latter case, the agent will be allowed credit for it in his account.

**43.** Cash payments made at Head Office come under the following heads, namely (1) Claims, surrenders, and cash bonuses, (2) Reassurance premiums, (3) Commission and expenses, (4) Loans and other investments, (5) Remittances to agents, (6) Payments into bank. The Head Office **Paid Cash Book** is shown on p. 208.

**REASSURANCE**

49 *Reassurance Renewal Premiums*

| [illegible] | [illegible] | [illegible] | [illegible] | Life. | Reassuring Office. | Reassuring Office Policy No. | Sum Assured. |
|---|---|---|---|---|---|---|---|
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 'X' | 11716 | 1500 0 0 |
| [illegible] | [illegible] | [illegible] | [illegible] 4 | Thomas G. F. Parker | 'Y' | 15335 | 6000 0 0 |
| [illegible] | [illegible] | [illegible] | [illegible] | James A. [illegible] | 'Z' | 22916 | 2500 0 0 |

**HEAD OFFICE**

320 *Cash Paid*

| [illegible] | [illegible] | [illegible] | | Voucher No. | Claims. |
|---|---|---|---|---|---|
| [illegible] | | | | | |
| [illegible] 1 | [illegible] | .. | John R. Hinson. Claim No. 13845, | 1 | 1700 0 0 |
| | [illegible] 2 | .. | James Mather, | 2 | .. |
| | .. | .. | 'X' Life Insurance Co. Reassurance No. 749. A. Smith, | 3 | . |
| | [illegible] | .. | Renewal Commission to Thomas Duncan, | 4 | . |
| | [illegible] | .. | E. B. [illegible]. Bonus in Cash, | 5 | .. |
| | | | Royal Bank, | .. | .. |
| | [illegible] | [illegible] | 'A' Branch. Policy loan and interest, per contra, | .. | .. |
| | [illegible] | [illegible] | H. Allan. Settled at 'A' Branch, | 6 | &c. |
| | | | | | 11397 7 0 |

**44. Claims, Surrenders, and Cash Bonuses.**—The Paid Cash Book has a separate column for claims under policies. Surrenders and cash bonuses are entered in the 'Sundries' column. The sum entered is the full sum payable in each case. When a claim or surrender is settled by payment of the balance of the policy moneys or surrender value, after deduction of a loan and interest, the latter are entered in the Received Cash Book, as if they had been received independently.

**45. Reassurance Premiums.**—There is a separate column for Reassurance Premiums. They are entered in full, the commissions being entered in the Received Cash Book (see § 32).

**46. Commission and Expenses.**—Separate columns are provided for these. Payments of less than 5s. for 'Bank Charges' or 'Petty Office Expenses', are entered in the **Petty Cash Book** (see [illegible]09). This forms a part of the Paid Cash Book, into which the [illegible]thly totals of bank charges and petty office expenses are carried.

## NEWAL BOOK.

*e in December.* 49

| Remarks. | 1901. Premium. | Date of Payt. | 1902. Premium. | Date of Payt. | 1903. Premium. | Date of Payt. | 1904. Premium. | Date of Payt. | Reassurance No. | Date and Cause of Deduction. |
|---|---|---|---|---|---|---|---|---|---|---|
| | 54 16 3 | | | | | | | | | |
| | 98 10 0 | | | | | | | | | |
| | 43 15 0 | | | | | | | | | |
| | &c. | | | | | | | | | |
| | 2596 4 1 | | | | | | | | | |

## AID CASH BOOK.

01. 320

| Reassurance Premiums. | Agents. | Fol. Agts. Led. | Commission. | Expenses. | Policy Loans. | Sundries. | Fol. Inv. &c. Led. | Paid into Bank. | Fol. Bk. Led. | Total. |
|---|---|---|---|---|---|---|---|---|---|---|
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 1700 0 0 |
| .. | .. | .. | .. | .. | 80 0 0 | .. | | .. | .. | 80 0 0 |
| 56 1 3 | .. | .. | .. | .. | .. | .. | .. | .. | .. | 56 1 3 |
| .. | .. | .. | 1 13 8 | .. | .. | .. | .. | .. | .. | 1 13 8 |
| .. | .. | .. | .. | .. | .. | 13 16 0 | .. | .. | .. | 13 16 0 |
| .. | .. | .. | .. | .. | .. | .. | .. | 3985 13 4 | 20 | 3985 13 4 |
| .. | 77 15 6 | | .. | .. | .. | .. | .. | .. | .. | 77 15 6 |
| .. | .. | .. | .. | .. | 48 0 0 | .. | | .. | .. | 48 0 0 |
| &c. | &c. | .. | &c. | &c. | &c. | &c. | .. | &c. | .. | &c. |
| 523 8 10 | 2450 8 8 | | 196 12 4 | 1170 3 10 | 1815 10 0 | 30460 0 5 | | 43596 9 11 | | 92610 1 0 |

## PETTY CASH BOOK.

*Petty Cash Payments,* 1901. 28

| Date. | | Bank Charges. | Petty Office Expenses. | Total. |
|---|---|---|---|---|
| 1901 | | | | |
| Jan. 1 | Cheque on Peebles. G. F. Chambers, . | 0 1 8 | .... | 0 1 8 |
| " " | Watson & Co. Carriage of Parcels. . | .... | 0 3 6 | 0 3 6 |
| " " | Telegram to James Menzies, . . . | .... | 0 1 0 | 0 1 0 |
| " " | Cheque on Inverness. H. Fraser & Co., | 0 0 9 | .... | 0 0 9 |
| " " | Postal Orders. Robert Kerr, . . | .... | 0 0 3 | 0 0 3 |
| | | &c. | &c. | &c. |
| | | 2 3 10 | 7 17 3 | 1[illegible] 1 1 |

This form of book is adapted to Scottish banking practice. A single money column would be sufficient in England.

**47. Loans and other Investments.**—There is a separate column for policy loans. Other loans and investments are entered in the 'Sundries' column, the title of the proper account in the Principal Ledger being given in the entry. These entries contain the exact title of the loan or investment, so that the sum may be duly posted to the Investment or Loan Ledger.

**48. Remittances to Agents.**—It is sometimes necessary to send a remittance to one of the ordinary agents to balance his account. These are entered in the 'Agents' column. This column also contains the debits to 'Branches' described in § 37. When a remittance is sent to any agent for a specified purpose, for instance, to pay a claim or surrender value or policy loan, the amount is debited in the Head Office Paid Cash Book, not to the agent, but to the 'Claims', 'Surrenders', or proper investment account. This course places on the Head Office Cashier the responsibility of obtaining the proper voucher in due course.

**49.** Either of two other methods is sometimes followed by accountants in dealing with remittances of this kind. In each of the two cases the remittance is debited to 'Agents', and the amount posted into the agent's account in the Agents Ledger. When the agent advises completion of the transaction, and returns the proper voucher, the amount is then entered to his credit and to the debit of the 'Claims', 'Surrenders', or proper investment account, according to the *first* method, by a cross entry in the Head Office Cash Book, or according to the *second* method, by an entry in the Credit Day Book (see § 62), or similar book. In the case of Branches the amount would be entered in the Branch Cash Book, to the debit of Head Office, as explained in §§ 55, 56.

**50.** Both of these methods perhaps follow the facts more closely than that described in § 48, but the cross entries required by the first method are cumbrous, and, according to strict principle, out of place in the Cash Book; and the second method is, when the payments are in connection with loans or other investments, subject to the disadvantage that, as the entries in the Investment Ledger must be posted from the Cash Book, the particular investment accounts are thrown out of correspondence with the accounts in the Principal Ledger (see § 19). The course described in § 48 seems to me to be the best, especially having regard to the fact that a large proportion of the payments made at Branches or agencies, are made with crossed cheques drawn at Head Office, in favour of the parties entitled to receive the money.

**51. Payments into Bank.**—These are entered in a separate column. The name of the particular bank account is stated in the entry, and each item is posted to the debit of that account in the Bank Ledger.

**52.** The various columns of the Paid Cash Book are summed up at the end of each month, and a statement is inserted analysing the 'Sundries' column. All the totals are carried to the Journal as described in § 76.

**53.** The arrangements for recording cash payments at the **Branches** are analogous to those at the Head Office. The form of the Branch Cash Book is given on page 206.

**54.** Payments made by means of Head Office Cheques sent specially for the purpose, are not entered in the Branch Cash Book (see § 36).

**55.** Payments of the nature of capital, such as policy loans, are entered in the 'Sundries' column to the debit of 'Head Office', and at once advised to Head Office; the amount is there entered in the Received Cash Book in the 'Head Office Collection' column, and in the Paid Cash Book to the debit of the proper loan or investment account, such as 'Policy Loans'. The entry in the Branch Cash Book is posted to the debit of the Head Office Cashier in his agency account. If there is a deduction from the policy loan for stamp duty or law charges, the amount entered in the Branch Cash Book will be the net amount.

**56.** Surrenders and cash bonuses, for which no remittances have been sent from Head Office, are entered in the 'Sundries' column. If they are settled by payment of the balance remaining after deduction of a policy loan, such balance is entered to the debit of Head Office, and the transaction at once advised to Head Office; where the net balance is entered in the Received Cash Book, 'Head Office Collection' column, and the other entries made as if the transaction had been settled at Head Office.

**57.** Payments into Bank, which are made into an account operated on from Head Office only, are at once advised to Head Office, when the amounts are entered in the Bank Ledger, so that that book may show at any time the available balance of the Bank account.

**58.** The various columns of the Branch Cash Books are added up at the end of each month, and in each a statement is inserted analysing the 'Sundries' columns on the Paid side. A summary of the cash transactions at all the Branches is made in the Monthly Summaries Book, as described in § 76.

**59.** The Cash Books described in this paper are not adapted to the system now common in large establishments, of keeping cash received and cash paid separate from each other. Under such a system, all cash received is paid into the Bank, and all payments are made either by separate cheques, or out of the proceeds of cheques specially drawn for the purpose of making

payments. In adapting the Cash Books to this system, the 'Paid into Bank' column is placed in the Received Cash Book to the extreme right of the folio; and, after each payment into Bank, the summation of this column should equal the total cash received. The 'Cheques' column is placed in the Paid Cash Book, to the right of the folio. Each cheque is entered therein in a line with the corresponding payment, and the monthly total of the 'Cheques' column should equal the total cash paid in the month. The Received and Paid Cash Books thus form complete and independent accounts. The Received Cash Book shows that all the cash received has been paid into the Bank; and the Paid Cash Book shows how the money drawn from the Bank for the purpose of making payments has been disposed of.

**60.** An objection to the system in its ordinary form, is that cross entries cannot be made in the Cash Books, without drawing unnecessary cheques on the Bank Account. I see no reason why this difficulty should not be got over, by inserting in the Received Cash Book before the 'Paid into Bank' column, a column headed 'Paid per contra', and entering the amount of any cross entries therein. The principle of the system would be sufficiently complied with, if the sum of the 'Paid per contra' column and 'Paid into Bank' column was always equal to the total cash received. The sum of the 'Paid per contra' column and the 'Cheques' column in the Paid Cash Book would of course be equal to the total cash paid.

**61. Payments at Agencies taken credit for by the Agents in their Accounts.**—Having thus considered the various classes of cash payments made by officers of the Society, we come to the payments made by, or sums allowed to, the ordinary agents, who are not required to keep a Cash Book, but merely to send, at regular intervals, a statement of account, with a remittance for the balance.

**CREDIT**

58 *Miscellaneous Credits*

| Date. | Policy No. | | Voucher No. |
|---|---|---|---|
| 1901 | | | |
| Jan. 2 | .. | Agency Account, | 1 |
| " " | 70294 | P. Anderson—Premiums—Return of portion of extra, | 2 |
| " " | .. | Agency Account—Expenses—Advertising, | 3 |
| " " | 41176 | Charles Jones—Bonus in Cash, | 4 |
| | | 'Medical Fees', | |
| | | 'Postages and Petty Expenses', | |
| | | Total 'Expenses', | |

**62.** Receipts sent to these agents for collection, are accompanied by an invoice, arranged so that the agent may convert it into a statement of his account, and return it with his remittance. It is understood that the accounts of agents remitting to Branch Offices are sent on to Head Office for examination. When the account is received at Head Office, the net amount of cash received (if any) is, as mentioned in § 31, entered in the 'Agents' Remittances' column of the Received Cash Book, the summation of the account is verified, and the items on the credit side, such as commission, postages, medical fees, or returned receipts, checked by comparison with the vouchers and otherwise. These credits, except the returned receipts, are then entered in the **Credit Day Book** (see below). Occasionally an agent will pay a cash bonus, without having received a remittance for the purpose, or something else not coming under one of the heads for which a separate column is appropriated; and any such payment is entered in the 'Sundries' column.

**63.** At the end of each month, the various columns of the Credit Day Book are added up. A statement is inserted analysing the 'Sundries' column, and another statement bringing out the total amount to be debited to 'Expenses'. The totals are then carried to the Monthly Summaries Book, as described in § 76.

**64. Vouchers.**—Each of the books for receiving the original entries of the payments has a column headed 'Voucher No.', and there is a similar column in the Returned Receipts Book (see § 66). All vouchers are numbered consecutively in the order of the entries, the number being placed in the top right-hand corner of the voucher, and in the voucher column. In the case of transactions at the Branches relating to a policy, or to a loan or investment, the voucher is forwarded without delay to Head Office. This applies to discharged or surrendered policies, receipts for cash bonuses, and documents of security for policy loans.

**DAY BOOK.**

*o Agents*, 1901. 58

| Commission. | Medical Fees. | Postages and Petty Expenses. | Other Expenses. | Sundries. | Total. | Agency Credited. | Fol. Agts. Led. |
|---|---|---|---|---|---|---|---|
| 7 7 4 | 1 1 0 | 0 3 6 | .. | .. | 8 11 10 | Perth (J. Smith). | 621 |
| .. | .. | .. | .. | 10 0 0 | 10 0 0 | Newcastle (F. Hill). | |
| 10 0 0 | 1 11 6 | 0 4 6 | 1 1 0 | .. | 12 17 0 | Glasgow (M. Macgregor). | |
| .. | .. | .. | .. | 17 3 6 | 17 3 6 | Oxford (J. C. Clarke). | |
| &c. | &c. | &c. | &c. | &c. | &c. | | |
| 485 3 8 | 53 12 6 | 27 19 6 | 54 9 4 | 197 11 2 | 768 16 2 | | |
| | | | 53 12 6 | | | | |
| | | | 27 19 6 | | | | |
| | | | 136 1 4 | | | | |

## Miscellaneous Transactions.

**65.** Having now dealt in succession with the principal classes of receipts and payments that occur in the business of Life Assurance, we come to transactions that do not fall under any of the foregoing heads. The more important of these are—receipts returned; premiums or interest capitalised; and transactions in connection with the unclaimed surrender values of lapsed policies.

**66. Receipts returned.**—It of course frequently happens that a premium or interest receipt sent to an agent for collection, is not taken up by the policyholder. This may happen from various causes, such as inability to pay the amount, the death of the life, the surrender of the policy, or a change in the amount of the premium or the place of payment. The receipt, having been returned by the agent to whom it was sent, must be entered to his credit. This is done in a book called the **Returned Receipts** Book (see below). Premiums and interest to be written off are

**RETURNED**

42 *Receipts returned,*

| Date. | Policy No. | Secy. No. | | Due Date. | Voucher No. | Income Tax. |
|---|---|---|---|---|---|---|
| 1901 | | | | | | |
| Jan. 1 | 51123 | .. | R. Mitchell. Surrendered, | Jan. 10 | 1 | .. |
| " " | .. | .. | Thomas Scott, Interim Rect. No. 3453, | | 2 | .. |
| " " | 42179 | .. | G. Brook. Premium receipt to 'A' Branch, | Dec. 30 | .. | .. |
| " 2 | 53645 | 197 | A. Long. Forfeited, | Oct. 1 | 3 | 0 1 9 |
| " " | 32376 | .. | R. Brown. Fines. Revival not carried out, | | 4 | .. |
| " " | 48826 | .. | J. G. Heron. Changed to endowment, | Dec. 5 | 5 | .. |
| | | | | | | &c. |
| | | | | | | 2 3 6 |

**CAPITALISATION**

39 *Premiums, Interest, &c.*

| Date. | Capitalised as at | Policy No. | Secy. No. | | Due Date. | Initials |
|---|---|---|---|---|---|---|
| 1901. | | | | | | |
| Jan. 1 | Jan. 1 | 37221 | 1327 | John Bell, Premium, | Jan. 1 | C. B. |
| " " | " " | ,, | ,, | Do., Interest, | " " | C. B. |
| " " | Dec. 5 | 53996 | 242 | Thomas Jones, Half Premium, | Dec. 5 | C. B. |
| " " | Jan. 1 | 47541 | 1572 | William Graham, Premium, | Oct. 1 | C. D. |
| " " | " " | ,, | ,, | Do. Fine, | .. | C. D. |
| " " | " " | 43927 | 2162 | Thomas Gibson, Premium, | Jan. 1 | C. B. |

entered in the 'Premiums' and 'Interest' columns respectively, the income tax on interest written off being entered in the 'Income Tax' column. Unpaid interest on policy loans on lapsed policies is written off on the receipt being returned; the interest from the due date of the last payment to the date of lapse being in due course entered in the Unclaimed Surrender Values Book (see § 69). Any returned receipts for fines or fees are entered in the 'Sundries' column. Receipts returned for the purpose of being transferred to the debit of another agent, are entered in the 'Transfers' column.

**67. Premiums and Interest capitalised.**—Premiums and interest which are, as they fall due, to be charged as a debt on the policy, or otherwise capitalised, are entered in the Renewal Debit Sheets in the usual course, and debited in an account opened in the Agents Ledger under the title of 'Head Office No. 2'. The amounts are thereafter entered in the **Capitalisation Book** (see below) to the credit of 'Agents', and to the debit of the

**ECEIPTS.**

01. 42

| nterest *net.* | Premiums. | Fol. Regr. of Dedns. | Sundries. | Total. | Agency credited. | Folio credited. | Transfers. | Folio debited. |
|---|---|---|---|---|---|---|---|---|
| .. | 35 17 3 | | .. | 35 17 3 | Head office, | | .. | .. |
| .. | 17 2 6 | | .. | 17 2 6 | 'B' Branch, | | .. | .. |
| .. | .. | .. | .. | .. | Head office, | | 17 15 5 | |
| 1 9 0 | 12 2 10 | | .. | 13 11 10 | 'A' Branch, | | .. | .. |
| .. | .. | .. | 0 10 0 | 0 10 0 | London (S. Hooper), | | .. | .. |
| .. | 8 17 10 | | .. | 8 17 10 | Perth (J. Smith), | 621 | .. | .. |
| &c. | &c. | | &c. | &c. | | | &c. | .. |
| 8 18 5 | 1257 13 5 | | 29 7 4 | 1320 19 2 | | | 327 15 0 | |

**OOK.**

*pitalised*, 1901. 39

| Half Premium Loans. | Non-Forfeiture Advances. | Life Annuities. | Reversions. | Sundries. | Fol. Invt. &c., Led. | Total. | Agency Credited. | Fol. Agts. Led. |
|---|---|---|---|---|---|---|---|---|
| .. | 19 6 8 | .. | .. | .. | | 19 6 8 | Head Office, No. 2. | |
| .. | 0 18 0 | .. | .. | .. | | 0 18 0 | Do. Do. | |
| 21 7 10 | .. | .. | .. | .. | | 21 7 10 | Do. Do. | |
| .. | 12 8 4 | .. | .. | .. | | 12 8 4 | Glasgow (J. Cameron), | |
| .. | 0 12 5 | .. | .. | .. | | 0 12 5 | Do. | |
| .. | .. | .. | 43 17 6 | .. | | 43 17 6 | Head Office, No. 2, | |
| &c. | &c. | &c. | &c. | &c. | | &c. | | |
| 73 14 9 | 224 17 1 | 27 8 10 | 113 12 4 | 96 16 4 | | 536 9 4 | | |

proper investment account. Receipts returned by agents for premiums or interest which fall to be charged as a debt on the policy, are also entered in this book. The entries in the Capitalisation Book are posted to the credit of the proper accounts in the Agents Ledger.

**68.** At the end of each month, the various columns of the Returned Receipts Book and Capitalisation Book are added up. Statements are inserted in each book, analysing the 'Sundries' columns. The totals are then carried to the Monthly Summaries Book, as described in § 76. The total of the 'Income Tax' column in the Returned Receipts Book is carried to the Income Tax Summary in the Monthly Summaries Book. The total of the 'Transfers' column of the Returned Receipts Book is, however, not carried to the Monthly Summaries Book, but is required in connection with the Agents' Balance Book, described in § 93.

**69. Entries dealing with Unclaimed Surrender Values of lapsed policies.**—The object of these entries is—(1) to extinguish all loans or debts on lapsed policies, with interest due thereon, by carrying the amount to the debit of 'Surrenders'; and (2) to place the net surrender value, or the full value where there is no loan or debt, to the debit of 'Surrenders' and credit of an account opened under the head of 'Unclaimed Surrender Values'. The entries are made in the **Unclaimed Surrender Values** Book (see below). The various columns are added up at the end of each month, and the totals, except income tax, carried to the Journal, as described in § 76. The total of the 'Income Tax' column is carried to the Income Tax Summary in the Monthly Summaries Book (§ 76).

**UNCLAIMED**

18 *Debits to Surrenders for Unclaimed Surrender Values of*

| Date. | Policy No. | | Initials. | Income Tax. |
|---|---|---|---|---|
| 1901. | | | | |
| Jan. 1. | 22772 | James M'Intosh, Surrender Value exhausted. | C. D. | 0 1 5 |
| " " | 41600 | Thomas Anderson, Paid-up policy granted. | C. D. | 0 2 11 |
| " " | 38067 | Henry Muirhead, Lapsed. | C. D. | 0 3 2 |
| " " | 20467 | John H. Mason, " | C. D. | .. |
| " " | 44001 | Joseph Carr, " | C. D. | 0 0 7 |
| | | | | &c. |
| | | | | 1 2 0 |

**70.** If the surrender value is subsequently claimed and paid, the amount is debited in the Cash Book to 'Unclaimed Surrender Values'. If the policy is revived, the entry in the Unclaimed Surrender Values Book is reversed, by making a direct entry in the Journal.

**71.** Entries writing the value of investments up or down, or writing off agents' balances as irrecoverable, are made in the Journal, the entry being posted both to the Principal and subsidiary Ledgers.

### JOURNAL.

**72.** I have now referred in succession to the principal classes of financial facts, that fall to be recorded in conducting the business of Life Assurance, and have described a variety of subsidiary books specially adapted for receiving the original entries of these transactions, in order to classify and condense them, so that they may be transferred in mass to the Journal and Principal Ledger.

**73.** The New Business Day Book, Renewal Debit Sheets, Register of Assignments, Debit Day Book, and Repayments Day Book, as described above, contain particulars of the total sums receivable by the Office in respect of premiums, interest, rents, fines, fees, and other receipts of the nature of income, the items under each head being contained in separate books or separate columns, or otherwise arranged so as to admit of the totals of each class of receipts being readily ascertained at the end of each month.

**74.** The particulars of the cash receipts and payments by officers of the Society are contained in the Head Office and Branch

**SURRENDER VALUES.**

*lapsed policies or Policy Loans thereon,* 1901. 18

| Policy Loans. | Half Premium Loans. | Non-Forfeiture Advances. | Fol. Loan Led. | Interest *net.* | Net Surrender Value. | Total Surrender Value. | Net Value paid or written off. Date. | Fol. Cash Book or Jl. |
|---|---|---|---|---|---|---|---|---|
| 10 0 0 | .. | 24 15 8 | | 1 5 6 | .. | 36 1 2 | | |
| 68 0 0 | .. | .. | | 2 11 6 | .. | 70 11 6 | | |
| 77 0 0 | .. | .. | | 2 16 2 | 5 1 10 | 84 18 0 | | |
| .. | .. | .. | | .. | 8 17 8 | 8 17 8 | | |
| .. | 21 4 6 | .. | | 0 11 10 | .. | 21 16 4 | | |
| &c. | &c. | &c. | | &c. | &c. | &c. | | |
| 388 0 0 | 28 6 0 | 42 6 11 | | 21 2 8 | 51 4 3 | 530 19 10 | | |

Cash Books: the payments by or sums allowed to ordinary agents, are contained in the Credit Day Book; the Cash Book and Credit Day Book being arranged with separate columns for each of the principal classes of transactions.

75. Entries writing off, or capitalising, premiums, or other classes of receipts, are made in the Returned Receipts Book and Capitalisation Book respectively, those books being arranged in columns, so as to classify the items according to the ledger account to be debited. Entries dealing with the unclaimed surrender values of lapsed policies are made in the Unclaimed Surrender Values Book.

76. The monthly totals standing to be debited or credited to the various ledger accounts, in all those subsidiary books except the Head Office Cash Books, and Unclaimed Surrender Values

**MONTHLY**

*Monthly Summaries,*

73 *Debits and Credit*

| Principal Ledger Account. | Debits. New Business Day Book. | Renewal Debit Sheets. | Register of Assignments. | Debit Day Book. | Repayments Day Book. | Total. | Fol. Pl. Led. |
|---|---|---|---|---|---|---|---|
| Premiums, | 2666 5 1 | 23,131 18 9 | .. | 457 9 11 | .. | 26,455 13 9 | |
| Interest, | .. | 2,373 13 8 | .. | 137 4 10 | 1831 2 9 | 4,342 1 3 | |
| Rents, | .. | .. | .. | .. | .. | .. | |
| Fines, | .. | .. | .. | 13 16 3 | .. | 13 16 3 | |
| Fees, | .. | .. | 11 0 0 | 2 2 0 | .. | 13 2 0 | |
| Commission, | .. | .. | .. | .. | .. | .. | |
| Expenses, | .. | .. | .. | .. | .. | .. | |
| Half Premium Loans, | .. | .. | .. | .. | .. | .. | |
| Non-Forfeiture Advances, | .. | .. | .. | .. | .. | .. | |
| &c. | .. | .. | .. | &c. | .. | &c. | |
| | 2666 5 1 | 25,505 12 5 | 11 0 0 | 2450 4 4 | 1831 2 9 | 32,464 4 7 | |
| Folio Day Book, &c., | 79 | 60 | 82 | 54 | 22 | | |

*Cash Transactions*

| Branch. | Folio Cash Book. | Receipts. Agents. | Banks. | Total. | Agents. | Banks. |
|---|---|---|---|---|---|---|
| 'A' Branch, | 62 | 6787 14 4 | 699 13 1 | 7487 7 5 | 450 9 2 | 6544 13 1 |
| 'B' Branch, | | 4390 6 6 | 535 9 2 | 4925 15 8 | 521 7 10 | 3951 13 9 |
| &c. | | &c. | &c. | &c. | &c. | &c. |
| | | 16,874 14 7 | 1795 1 2 | 18,169 15 9 | 1562 13 2 | 15,477 6 1 |
| Folio Principal Ledger, | | | | | | |

Book, are carried to the **Monthly Summaries** Book (see below). The monthly totals of the 'Income Tax' columns in the Interest Book, Debit Day Book, Repayments Day Book, Returned Receipts Book, and Unclaimed Surrender Values Book, are carried to the Income Tax Summary in the Monthly Summaries Book. The totals in the Monthly Summaries Book, Head Office Cash Books, and Unclaimed Surrender Values Book, are carried to the Journal by making therein, at the end of each month, entries of the form shown on page 220. With a somewhat complicated set of subsidiary books, such as those described above, I think the advantages of passing the totals through the Journal outweigh the additional labour involved. One effect of this is to make it easy, at the end of the year, to apply the check described in § 108.

**SUMMARIES.**

January 1901. 73

*to Agents.*

| Credits. | | | | |
|---|---|---|---|---|
| Credit Day Book. | Returned Receipts. | Capitalisation Book. | Total. | Fol. Pl. Led. |
| .. | 1257 13 5 | .. | 1257 13 5 | |
| .. | 33 18 5 | .. | 33 18 5 | |
| .. | .. | .. | .. | |
| .. | 2 5 0 | .. | 2 5 0 | |
| .. | 0 15 0 | .. | 0 15 0 | |
| 435 3 8 | .. | .. | 435 3 8 | |
| 136 1 4 | .. | .. | 136 1 4 | |
| .. | .. | 73 14 9 | 73 14 9 | |
| .. | .. | 224 17 1 | 224 17 1 | |
| &c. | &c. | &c. | &c. | |
| 768 16 2 | 1320 19 2 | 536 9 4 | 2626 4 8 | |
| 58 | 42 | 39 | | |

*Debits and Credits to Income Tax.*

| Debits. | Fol. | Income Tax. | Fol. Jl. |
|---|---|---|---|
| Interest Book :— | | | |
| Investments, . . | | 110 16 11 | |
| Policy Loans, . . | | 23 16 2 | |
| Half Premium Loans, | | 2 7 5 | |
| Non - Forfeiture Advances, . . | | 3 1 4 | |
| Debit Day Book, . . | 54 | 7 18 6 | |
| Repayments Day Book, . | 22 | 21 15 0 | |
| Unclaimed Surrender Values, . . . | 18 | 1 2 0 | |
| | | 170 17 4 | |
| **Credits.** | **Fol.** | **Income Tax.** | **Fol. Jl.** |
| Returned Receipts, . | 42 | 2 3 6 | |

*at Branches.*

| Payments. | | | | | |
|---|---|---|---|---|---|
| Commission. | Expenses. | Surrenders. | Bonus in Cash. | &c. | Total. |
| 67 2 1 | 149 11 10 | 118 3 0 | .. | &c. | 7487 7 5 |
| 52 13 8 | 182 15 9 | 46 12 6 | 17 9 6 | &c. | 4925 15 8 |
| &c. | &c. | &c. | &c. | &c. | &c. |
| 201 17 9 | 463 3 0 | 227 18 9 | 39 10 5 | &c. | 18,169 15 9 |

## ORDINARY MONTHLY ENTRIES IN JOURNAL.

*Journal*, 31 January 1901. 94

| | | | | |
|---|---|---|---|---|
| | Agents, Dr.<br>To Sundries,<br>For sums debited to Agents in January, per Monthly Summaries, folio 73. | 32,464 4 7 | | 32,464 4 7 |
| § 34 | Cash, Dr.<br>To Sundries,<br>For cash received in January, per Head Office Received Cash Book, folio 122. | 92,610 1 0 | | 92,610 1 0 |
| | Cash, Dr.<br>To Sundries,<br>For cash received at Branches in January, per Monthly Summaries, folio 73. | 18,169 15 9 | | 18,169 15 9 |
| § 52 | Sundries, Dr.<br>To Cash,<br>For cash paid in January, per Head Office Paid Cash Book, folio 320. | 92,610 1 0 | | 92,610 1 0 |
| | Sundries, Dr.<br>To Cash,<br>For cash paid at Branches in January, per Monthly Summaries, folio 73. | 18,169 15 9 | | 18,169 15 9 |
| | Sundries, Dr.<br>To Agents,<br>For sums credited to Agents in January, per Monthly Summaries, folio 73. | 2,626 4 8 | | 2,626 4 8 |
| § 69 | Surrenders, Dr. | 530 19 10 | | |
| | To Policy Loans, | | | 388 0 0 |
| | To Half Premium Loans, | | | 28 6 0 |
| | To Non-Forfeiture Advances, | | | 42 6 11 |
| | To Interest, | | | 21 2 8 |
| | To Unclaimed Surrender Values,<br>For surrender values of policies lapsed or reduced in January, per Unclaimed Surrender Values, fol. 18. | | | 51 4 3 |
| | Income Tax, Dr.<br>To Interest,<br>For tax deducted from interest charged in January, per Monthly Summaries, folio 73. | 170 17 4 | | 170 17 4 |
| | Interest, Dr.<br>To Income Tax,<br>For tax deducted from interest written off in January, per Monthly Summaries, folio 73. | 2 3 6 | | 2 3 6 |
| | | 257,354 3 5 | | 257,354 3 5 |

## Principal and Subsidiary Ledgers.

**77.** The entries in the Journal are in due course posted into the **Principal Ledger** (see p. 222), in which the aggregate transactions of the Office are collected under various heads. The classification in the Principal Ledger is arranged so as to furnish all the materials for preparing the revenue account and balance sheet in the prescribed forms, as well as to provide such additional information as will be useful in connection with the management of the Office.

**78.** There are two principal classes of accounts in the Principal Ledger, namely:—(1), accounts representing the different kinds of income or outgo, the balances of which go to make up the revenue account; and (2), accounts representing debts or property, the balances of which furnish material for the balance sheet. The first class of accounts have been called by writers on book-keeping, *nominal* accounts, and the second class, *real* accounts. Among the latter are included accounts with agents, borrowers, and other persons. These are called *personal* accounts.

**79.** Nominal Accounts are opened under the following heads, namely, *Income*—Premiums, Interest, Rents, Fines, Fees, Profit on Reversions, Profit and Loss, Reassurance Claims; *Outgo*—Claims, Surrenders, Commission, Expenses, Bonuses in Cash, Income Tax, Reassurance Premiums. It will be observed that one account only is opened for Premiums, instead of providing separate accounts for New Annual Premiums, Single Premiums, and Renewal Premiums. The separate amounts of the new annual and single premiums are easily obtained in the manner shown in § 112, and it therefore seems unnecessary to incur the labour of keeping the items continuously separate in the books of account.

**80.** The real accounts to be opened will depend on what kinds of investments are held. The following is a list of accounts that may sometimes be suitable:—Real Property Loans, Loans on Limited Interests, Miscellaneous Property Loans, Loans on Rates, Real Property Loans Abroad, Policy Loans, Half Premium Loans, Non-Forfeiture Advances, British Government Securities, Indian Government Securities, Colonial Government Securities, Foreign Government Securities, Municipal Securities, Miscellaneous Debentures, American Railroad Bonds, Railway Debenture Stocks, Miscellaneous Debenture Stocks, Railway Guaranteed Stocks, Railway Preference Stocks, Miscellaneous Preference Stocks, Railway Ordinary Stocks, Premises, Feu-Duties, Reversions, Life Annuities, Agents, Deposits, Banks, Cash, Unclaimed Surrender Values. The last-mentioned account, of course, always represents a liability.

**PRINCIPAL**

*Dr.* Agents.

| | | | |
|---|---|---|---|
| 1901 Jan. 1 | Balance, | ... | 42,413 13 9 |
| " " | To Outstanding Premiums, | | 35,223 12 3 |
| " " | ,, Outstanding Interest, | | 28,128 9 4 |
| " " | ,, Sundries, | 94 | 32,464 4 7 |
| " " | ,, Cash, | | 2,450 8 8 |
| " " | ,, Do., | | 1,562 13 2 |

81. Each of these real accounts contains the aggregate transactions with a number of individuals or relating to a number of investments. With a view to fulfil the first object of Life Assurance Bookkeeping, without overloading the Journal, subsidiary ledgers are provided to record the transactions of the office with individual agents, borrowers, or bankers, or in connection with particular purchases. These are the Agents Ledger, Investment Ledger, Policy Loan Ledger, Half Premium Loan Ledger, Non-Forfeiture Advances Ledger, and Bank Ledger; and they contain the particular accounts corresponding to the account 'Agents', the various loan and investment accounts, and the account 'Banks', in the Principal Ledger. The debits and credits in those subsidiary ledgers, are made directly from the books containing the original entries, without the intervention of the Journal.

82. A specimen of the **Agents Ledger** is given below. It is understood that the Society keeps accounts in the Head Office books for all its agents, including those that remit to the Branches. No agents' accounts are kept in the books at the Branch offices. It contains accounts for the Head Office Cashier and each of the Branch Secretaries, as well as for the ordinary agents, also the Head Office No. 2 account mentioned in § 67. The entries to the debit of the agents are made from the various subsidiary books, the items being entered individually or in mass

**AGENTS**

*Dr.* Perth—

| Date. | | Fol. | Policy No. | Due Date. | |
|---|---|---|---|---|---|
| 1901 | Balance brought forward, | | .. | .. | 104 4 5 |
| Jan. 1 | Premiums, | | .. | .. | 35 16 10 |
| " " | Interest, | | .. | .. | 0 10 6 |
| " " | Interim Receipt, No. 3517, | 79 | 54560 | .. | 25 10 0 |
| " 2 | Premium changed, | 54 | 48826 | Dec. 25 | 13 12 11 |

**LEDGER.**

*Contra.* *Cr.*

| | | | |
|---|---|---|---|
| 1901 | | | |
| Jan. 1 | By Cash, . . . . . | | 31,239 13 3 |
| " " | " Do., . . . . . | | 16,374 14 7 |
| " " | " Sundries, . . . . . | 94 | 2,626 4 8 |

in the manner described above; and the aggregate debits in the Agents Ledger for each month should, after allowing for transfers (see § 66), be equal to the corresponding total entered to the debit of the general account 'Agents' in the Principal Ledger. The 'Commission paid' column is for the purpose of recording the payment of commission on premiums or interest, entered individually in the Agents Ledger.

**83.** Each of the Branch Secretaries keeps a **Charge and Discharge Account Book** (see p. 224), which corresponds with his account in the Agents Ledger at Head Office. On the 'Charge' side he enters the total amount of receipts received from Head Office for collection at the commencement of each month, as stated in the accompanying invoice, and such other receipts as may be received from time to time during the month; and on the 'Discharge' side the amount of receipts returned to Head Office as not taken up, or sent to other agents. The 'Charge' and 'Discharge' sides are balanced at the end of each month, the total of the 'Branch Collection' column of the Cash Book being entered on the 'Discharge' side; and the balance, which ought to be equal to the total amount of receipts in hand (see § 85), brought down to the 'Charge' side for the succeeding month. Each Branch Secretary is required to send monthly to Head Office, with his copy Cash Book, a copy of his Charge and Discharge Account Book for the month, the entries in which should correspond with the Agents Ledger.

**LEDGER.** **621**

John Smith. *Cr.*

| Date acctd. for. | Commission Paid. | Date. | | Fol. | |
|---|---|---|---|---|---|
| | | 1901 | | | |
| .. | | Jan. 2 | Returned receipt, 48826, Dec. 25, . . | 42 | 8 17 10 |
| .. | | " " | Cash, . . . . . | 122 | 125 7 8 |
| Jan. 2 | | " " | Charges, . . . . . | 58 | 8 11 10 |

**BRANCH CHARGE AND**

*Charge and Discharge Account*

| Date. | Policy No. | Charge. | | | |
|---|---|---|---|---|---|
| 1901 Jan. 1 | ... | Amount of receipts in hand at 31 December 1900, per last account, . | 917 | 6 | 5 |
| " " | ... | Receipts due in January, received 26 December, per Invoice :— | | | |
| | | Premiums, . . . . . | 1541 | 3 | 11 |
| | | Interest, . . . . . | 175 | 9 | 6 |
| " " | 44553 | Policy Loan Agreement. R. F. Harris, | 75 | 0 | 0 |
| " " | ,, | Do. Interest, | 2 | 15 | 6 |
| " 2 | 54567 | Interim Receipt No. 3516. Maurice Hay, . . . . . | 13 | 11 | 6 |
| " " | 49355 | Licence No. 5821. Alexander Fraser, | 10 | 0 | 0 |
| " " | 42179 | Premium Receipt. George Brooks, . | 17 | 15 | 5 |
| | | &c. | &c. | | |
| | | | 3178 | 16 | 2 |

**84.** It will be observed, however, that the Branch Secretary enters in his Charge and Discharge Account Book, receipts for items of capital as well as those for items of income, although the former do not find their way to the debit of his agency account at Head Office, until they are posted into it from the Head Office Paid Cash Book (see § 37). If, therefore, there are any receipts for items of the nature of capital, in hand at the Branch at the end of the month, the balance of the Charge and Discharge Account will exceed the balance of the agency account by the amount of such receipts.

**85.** The particulars of the receipts in hand are given in a separate **List of Receipts in hand** (see p. 225), arranged so as to show the separate amounts of premium, interest, and other kinds of receipts, also to distinguish between sums in ordinary course of collection and sums overdue.

**86.** A list of receipts in hand is furnished by the Head Office Cashier at the end of each year, similar to the monthly list required from the Branch Secretaries. Similar lists are furnished by him at other times, in connection with the examination of his accounts.

**87.** It will be seen that the Head Office Cashier and Branch Secretaries have each two accounts—an agency account contained in the Agents Ledger, and a cash account in the respective Cash Books, the balances of each showing the amounts of receipts and of cash in their hands respectively. Receipts, however, for items

**DISCHARGE ACCOUNT BOOK.**

'A' *Branch*, 1901.

| Date. | Policy No. | Discharge. | |
|---|---|---|---|
| 1901 Jan. 1 | 53645 | Premium receipt returned to Head Office. Forfeited, . . . | 12 2 10 |
| " " | " | Interest receipt returned to Head Office. Forfeited, . . . | 1 9 0 |
| | | &c. | &c. |
| " 31 | ... | Premiums, Interest, &c., received, as per Cash Account, . . . | 1492 3 10 |
| " " | ... | Balance, being amount of receipts in hand, as per List, . . . | 1036 17 8 |
| | | | 3178 16 2 |

of the nature of capital, do not appear in these agency accounts, except in the particular case referred to in § 37. Those are all held to be at the debit of the Head Office Cashier, and will be found in the 'Principal' column of the Repayments Day Book, where they appear as the unpaid items.

**BRANCH LIST OF RECEIPTS IN HAND.**

*Receipts in hand at* 'A' *Branch on* 31 January 1901.

| Policy No. | Due Date. | Premium. | Interest. | Miscellaneous. | Policy No. | Due Date. | Premium. | Interest. | Miscellaneous. |
|---|---|---|---|---|---|---|---|---|---|
| (1) *In ordinary course of collection.* | | | | | | | | | |
| 13562 | Dec. 18 | 7 8 9 | .. | | | | | | |
| 17406 | ,, 14 | 28 7 6 | 2 11 7 | | | | | | |
| 17407 | ,, 6 | 5 12 11 | .. | | | | | | |
| 17973 | ,, 28 | 60 13 4 | .. | | | | | | |
| | | &c. | &c. | | | | | | |
| (2) *Overdue.* | | | | | | | | | |
| 14563 | Oct. 11 | 4 8 6 | 1 17 7 | | | | | | |
| 31960 | ,, 30 | 14 6 8 | .. | | | | | | |
| 34198 | Nov. 7 | 8 12 6 | 1 0 0 | | | | | | |
| | | &c. | &c. | &c. | | | | | |
| | | 1443 8 7 | 37 10 8 | 11 5 0 | | | | | |
| *Summary.* | | | | | | | | | |
| Premiums . . . . | | | | 1443 8 7 | | | | | |
| Interest . . . . | | | | 37 10 8 | | | | | |
| Miscellaneous . . . | | | | 11 5 0 | | | | | |
| | | | | 1492 3 10 | | | | | |

**88.** The question may perhaps be asked, whether the amount of labour involved in keeping accounts with all the ordinary agents, is not out of proportion to the advantages obtained. The answer to this, in the case of any particular office, will depend on its special circumstances. It might perhaps sometimes be sufficient to have a single account in the Agents Ledger for all the agents in the same district; but there would be the risk of trouble, in getting out the particulars of the balance of each of these accounts, at the end of the year.

**89.** The accounts with individual borrowers, or in connection with particular investments, are kept in the **Investment Ledger** (see below), **Policy Loan Ledger, Half Premium Loan Ledger**, and **Non-Forfeiture Advances Ledger.** The entries affecting capital are posted directly to these ledgers, from the Head Office Received and Paid Cash Books, Capitalisation Book, and Unclaimed Surrender Values Book. The date of payment inserted in the Investment Ledger should be the date when the payment was actually made to the borrower, although that may not be the date of the Cash Book entry.

**INVESTMENT LEDGER.** 116

*Security No.* 816. James T. Macdonald.
*Principal Account.* Real Property Loans.
*Interest at* 4% *payable on* 10 January and 10 July.

| Date. | | Fol. | Dr. | Cr. | Dr. Balance. |
|---|---|---|---|---|---|
| 1901 | | | | | |
| Jan. 1 | Balance, . | ... | 2000 0 0 | ... | 2000 0 0 |
| " 10 | By Cash, . | | ... | 400 0 0 | 1600 0 0 |

**90.** Instead of the Investment Ledger, some offices have a Register of Investments, arranged with columns to contain the amount of the loan or other investment, and such other particulars as it is desired to record, a column for repayments, and a series of columns to receive the balances remaining due at the end of successive years, three or four lines being allowed for each loan. The Register is not so perfect or satisfactory a record as the Ledger. Its principal advantage is, that it admits of the total amount of the various classes of investments at the end of the year, being ascertained without making separate lists of the particulars. Similar Registers may be used for recording the particulars of the Policy Loans, Half Premium Loans, and Non-Forfeiture Advances.

**91.** The **Bank Ledger** (see below) contains all the current accounts of the Office with its Bankers, and will, if continuously posted up, show at any time the available balance of each of the accounts operated on from Head Office. It also serves as a record of all cheques signed on these accounts. When a cheque is written out, the number and amount are entered in the proper account in the Bank Ledger, and the Manager or other official who signs the cheque places his initials in the 'Initials' column.

**BANK LEDGER.**

Royal Bank. **20**

| Date. | | Fol. | Dr. (sums paid in). | Cr. (cheques). | Initls | State of Balance. | Balance. |
|---|---|---|---|---|---|---|---|
| 1901 | | | | | | | |
| Jan. 1 | Balance, . . . . | .. | 2735 15 11 | .. | .. | Dr. | 2735 15 11 |
| " " | Cheque No. 6559, . . | 122 | .. | 1700 0 0 | A.B. | " | 1035 15 11 |
| " " | Paid in, . . . | 320 | 3985 13 4 | .. | .. | " | 5021 9 3 |

**92.** One of the most important matters to be attended to in keeping a set of books arranged on the principles which I have described, is to secure that there shall be no discrepancy between the postings to the subsidiary Ledgers, and the corresponding aggregate postings to the Principal Ledger; because, if any such discrepancy exists, it will be found, on balancing the books at the end of the year, that the sums of the balances of the accounts in the subsidiary Ledger do not agree, as they ought to do, with the balances of the corresponding general accounts in the Principal Ledger.

**93.** In the case of the Investment and Loan Accounts, there is no need to adopt any special precautions against this kind of error; and the same remark applies to the Bank Accounts, which should be periodically reconciled with accounts independently furnished by the Banks. In the case, however, of the Agents account, it is otherwise. That account contains a great number of transactions, and steps are periodically taken to ascertain that the postings to it agree with those to the general account 'Agents'. Each of the accounts in the Agents Ledger is summed up (in pencil) to the end of each month's entries, and the totals inserted in the **Agents' Balance Book** (see p. 228). After making any necessary adjustment for transfers, classes of entries in the Agents Ledger which do not correspond to anything in the Principal Ledger, and for credit balances brought forward in the Agents Ledger, the sums of the debit and credit entries in the Agents Ledger, up to any date, should agree respectively with

those in the 'Agents' account in the Principal Ledger. This will be found a much more satisfactory method of securing the correctness of the entries in the Agents Ledger, than merely to require an agreement to be shown, between the total balances in the Agents Ledger at the end of the year, and the corresponding balance in the Principal Ledger.

**AGENTS**

*Debits and Credits*

| Fol. Agts. Led. | Agency. | January. | |
|---|---|---|---|
| | | Dr. | Cr. |
| | Head Office, | 34,756 7 1 | 25,937 13 10 |
| | Do., No. 2, | 415 12 9 | 379 4 6 |
| | Aberdeen—J. Thomson, | 83 8 7 | 46 8 4 |
| | &c., | &c. | &c. |
| | | 121,763 18 10 | 82,366 13 5 |
| | Deduct Credit Balances at 31 December 1900, | 17 4 11 | 17 4 11 |
| | | 121,746 13 11 | 82,349 8 6 |
| | Principal Ledger, | 121,418 18 11 | 82,021 13 6 |
| | Add Transfers, per Returned Receipts Book:— | | |
| | January, folio 42, | 327 15 0 | 327 15 0 |
| | February, " | .. | .. |
| | &c. | | |
| | | 121,746 13 11 | 82,349 8 6 |
| | Deduct Credit Balances at 31 December 1901, | | |

## Closing of Ledgers.

**94.** The balances of the accounts mentioned in §§ 79, 80, taken down at the end of the financial year, show the Income and Outgo, Liabilities and Assets of the Office under the heads represented by the titles of the accounts. These accounts, however, do not correspond exactly to the items in the forms of revenue account and balance sheet prescribed by the Life Assurance Companies Act; and in order that these statements may be in the prescribed forms, and may show the true financial position of the Office, certain adjustments are made on the accounts at the end of the year.

**95.** These adjustments are of two kinds, namely (1) adjustments affecting both the revenue account and balance sheet; and (2) adjustments affecting the balance sheet only—being made for the purpose of including assets under one head instead of another. Entries in the Journal are also required to transfer to the proper accounts certain transactions which, for the sake of convenience,

**BALANCE BOOK.**

*in Agents Ledger*, 1901.

| February. | | &c. | | December. | | Balance at 31 December. |
|---|---|---|---|---|---|---|
| Dr. | Cr. | | | Dr. | Cr. | |
| 41,040 4 5 | 33,171 18 9 | | | | | 29,147 14 5 |
| 706 11 2 | 691 10 2 | | | | | .. |
| 117 8 8 | 79 1 5 | | | | | 41 9 2 |
| &c. | &c. | | | | | &c. |
| 154,949 6 4 | 129,534 7 7 | | | | | 83,499 6 3 |
| 17 4 11 | 17 4 11 | | | | | |
| 154,932 1 5 | 129,517 2 8 | | | | | |
| 154,814 8 9 | 128,899 10 0 | | | | | |
| 327 15 0 | 327 15 0 | | | | | |
| 289 17 8 | 289 17 8 | | | | | |
| 154,932 1 5 | 129,517 2 8 | | | | | |
| | | | | | | 83,499 6 3 |
| . . . . . | | | | | | 8 10 0 |
| | | | | | | 83,490 16 3 |
| Outstanding Premiums and Interest at Head Office and Branches, . | | | | | | 40,545 12 8 |
| 'Agents' Balances', per Balance Sheet, . . . . . | | | | | | 42,945 8 7 |
| | | | | | | 83,490 16 3 |

are allowed to remain during the year in some other account. I shall describe this last-mentioned class first.

**96.** Ground rents, rates, taxes, repairs, fire insurance, and other outgoings in connection with properties owned by the Office are, at the time of payment, treated like other expenses, and debited to 'Expenses' account. At the end of the year the separate amounts expended upon each property under each head are ascertained by means of the Expenses Book (§ 115), and transferred to the debit of 'Rents' account (p. 230). The income tax paid by the Society, or deducted by tenants from their rents, is also debited in the first instance to 'Expenses'; and at the end of the year the total is transferred to the debit of 'Income Tax'. An entry is made at the same time to debit 'Expenses' and credit

'Rents', with a sum representing the yearly value of the offices occupied by the Society in buildings belonging to itself (see below).

## CLOSING ENTRIES IN JOURNAL (§ 96).

*Journal*, 31 December 1901.

| | | | |
|---|---|---|---|
| Rents, . . . . . Dr. | 2,593 19 2 | | |
| Income Tax, . . . . . Dr. | 307 4 8 | | |
| To Expenses, . . . . . | | 2,901 3 10 | |
| For outgoings in connection with Premises under the following heads, per Expenses Book, these having been, for the sake of convenience, debited to Expenses. | | | |

| | 'A' Premises | 'B' Premises | &c. | Total. |
|---|---|---|---|---|
| *Rents*— | | | | |
| Ground Rents and Rates, | 380 9 0 | 256 17 10 | | 1961 5 2 |
| Caretaker's Wages, . . | 65 0 0 | .. | | 156 1 10 |
| Coals and Lighting, . . | 34 14 2 | 18 19 6 | | 95 19 7 |
| Fire Insurance, . . | 10 10 0 | 4 0 0 | | 56 19 5 |
| Repairs, . . . . . | 39 15 10 | 10 6 3 | | 278 10 8 |
| Law Charges, . . . . | 26 8 11 | 2 0 0 | | 45 2 6 |
| | 556 17 11 | 287 3 7 | &c. | 2593 19 2 |
| *Income Tax*— | | | | |
| Schedule A, . . . . | 92 9 7 | 58 0 5 | &c. | 307 4 8 |
| Total, . | 649 7 6 | 345 4 0 | &c. | 2901 3 10 |

| | | |
|---|---|---|
| Expenses, . . . . . Dr. | 2,468 0 0 | |
| To Rents, . . . . . | | 2,468 0 0 |
| For sums imputed to the credit of Premises Accounts in lieu of rents, taxes, &c. | | |
| 'A' Premises, . . . . 750 0 0 | | |
| 'B' Premises, . . . . 300 0 0 | | |
| &c. &c. | | |
| 2468 0 0 | | |

97. When the accounts are kept in this way, the balance of 'Rents' account will show the return yielded by the Society's properties as a whole. If it is desired that the books should show the return on each of the properties separately, the entries may be made at the close of the year in the following manner, instead of as described in § 96. (1) Analyse the balance of the 'Rents' account, ascertaining the amount applicable to each property, and carry the total to the credit of 'Premises' (or other proper account) in the Principal Ledger, posting the individual sums to the credit of the particular property accounts in the Investment Ledger. (2) Debit 'Premises' and credit 'Expenses' with the outgoings mentioned in § 96, posting the individual

sums under each heading to the debit of the particular property account. (3) Debit 'Income Tax' and credit 'Premises' with the income tax included among the above-mentioned outgoings, putting the individual sums to the credit of the particular property accounts. (4) Debit 'Expenses' and credit 'Premises' with a sum representing the yearly value of the offices occupied by the Society in buildings belonging to itself, posting the individual sums to the credit of the particular property accounts. (5) Ascertain, in the case of each of the property accounts in the Investment Ledger, the amount by which the balance exceeds that at the commencement of the year. These will represent the separate returns, and the total of them must be debited to 'Premises', and credited to 'Rents', the individual sums being debited to the particular property accounts.

**98.** Entries of the following kinds also come under this head, namely—(1) Entries dealing with the first half-year's interest on stocks purchased, credited to 'Interest' when received, transferring the interest accrued at the date of purchase to the credit of the capital account; (2) Entries dealing with the proceeds of stocks sold, credited when received to the capital account, transferring the accrued interest to the credit of 'Interest'; (3) Entries dealing with the interest on stocks standing in the books at a premium and redeemable at par, credited to 'Interest' when received, transferring the proper proportion to the credit of the capital account. Entries of these three kinds are sometimes made in the Journal at the end of the year; but, in my opinion, the better course is to apportion the sums received or receivable in the first instance, and debit or credit the proper accounts in the original entries.

**99.** The adjustments under the first head referred to in § 95 are made for the purpose of satisfying the condition, that the income and outgo of the year, as stated in the revenue account, shall be the income and outgo properly applicable to the year, whether actually received or paid in the year or not. The principal accounts requiring adjustment in this way are—Interest, Claims, Reassurance Premiums, Commission, and Expenses; and the Journal entries shown on page 233 are made for the purpose.

**100.** Assuming that, according to the usual practice, credit is taken for the amount of interest accrued but not due at end of year, the amount of this, having been calculated, is, after deduction of income tax, carried to the credit of 'Interest' and debit of an account opened under the head of 'Outstanding Interest'.

**101.** The amount of claims arising previous to the date of the balance, but not paid, is ascertained from the Claims Intimated Book, and carried to the debit of 'Claims' and credit of 'Outstanding Claims'. A corresponding entry must be made, if necessary,

to deal with any claims under reassurances that may be due and unpaid at the close of the year, by debiting them to 'Outstanding Claims', and crediting them to 'Reassurance Claims'. The Claims Intimated Book is kept open at the end of the financial year so that, as far as possible, claims may be entered in the year to which they belong, although not intimated until after it has closed.

**102.** The amount of Reassurance Premiums due and unpaid at the close of the year is ascertained from the Reassurance Renewal Book, and otherwise, and carried to the debit of 'Reassurance Premiums' and credit of an account opened under the head of 'Outstanding Premiums'.

**103.** As explained in § 40, statements of the outstanding commission (less reassurance commission) and expenses, are made at the end of the year, and the amount carried to the debit of these accounts respectively, and credit of an account opened under the head of 'Outstanding Charges'.

**104.** It is understood that the reversions are dealt with by revaluing them on the occasion of each investigation, and taking credit for the increase in value as profit. This is credited to 'Profit on Reversions' account. When a reversion falls in, the difference between the sum realised and the sum at debit of the purchase is also profit, and is credited to 'Profit on Reversions' account. That account will show only the amount that has been actually realised; and it may sometimes be necessary, in order that the accounts may show more correctly the position of the Office, to make an entry in the Journal, to debit 'Reversions', and credit 'Profit on Reversions', with further sums in respect of profit arising in consequence of the falling-in of reversions, but not realised.

**105.** The adjustments under the second head referred to in § 95 are shown in the Journal entry on page 234.

**106.** The balance of the Agents account in the Principal Ledger shows the total premiums, interest, etc., at the debit of Head Office, branches, or ordinary agents for collection. For the purpose of producing a balance sheet according to the prescribed form, it is necessary to separate this total into agents' balances, outstanding premiums, and outstanding interest. There is considerable difference of opinion as to what ought to be included under the head of outstanding premiums; but it seems a convenient and reasonable course to include in it, the premium receipts at the debit of the Head Office and Branch Offices for collection, but not receipts at the debit of ordinary agents. This subject is fully discussed in Dr. Sprague's *Treatise on Life Insurance Accounts.* The Lists of Receipts in hand furnished by the Branches and the Head Office Cashier at the end of the year, are, as mentioned in § 85, so arranged as to show the separate amounts of premium

## CLOSING ENTRIES IN JOURNAL (§§ 99, etc.).

*Journal*, 31 December 1901.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 00 | Outstanding Interest, . . . . . Dr. | | | | 26,042 | 11 | 5 | | |
| | To Interest, . . . . . | | | | | | | 26,042 | 11 5 |
| | For amount of interest and dividends accrued but not due at this date, per List of Investments, now credited to Interest for the purpose of the annual balance. | | | | | | | | |
| 01 | Claims, . . . . . Dr. | | | | 53,905 | 8 | 8 | | |
| | To Outstanding Claims, . . . . . | | | | | | | 53,905 | 8 8 |
| | For claims outstanding at this date, per Claims Intimated book, folio , now debited to Claims for the purpose of the annual balance. | | | | | | | | |
| 02 | Reassurance Premiums, . . . . . Dr. | | | | 2,596 | 4 | 1 | | |
| | To Outstanding Premiums, . . . . . | | | | | | | 2,596 | 4 1 |
| | For reassurance premiums due and unpaid at this date, being premiums due in December, per Reassurance Renewal Premium Book, folio , now credited to Outstanding Premiums for the purpose of the annual balance. | | | | | | | | |
| | *The amount to be adjusted if necessary by deducting any premiums paid before the end of the year.* | | | | | | | | |
| 03 | Commission, . . . . . Dr. | | | | 2,012 | 3 | 9 | | |
| | Expenses, . . . . . Dr. | | | | 586 | 10 | 4 | | |
| | To Outstanding Charges, . . . . . | | | | | | | 2,598 | 14 1 |
| | For the following sums due and unpaid at this date, now credited to Outstanding Charges for the purpose of the annual balance, namely : | | | | | | | | |
| | *Commission*— | | | | | | | | |
| | Estimated amount of Commission on Outstanding Premiums, £38,175 : 12 : 3, . . | | | 750 | 0 | 0 | | | |
| | Amount of Commission on balances remaining at debit of Agents. Balances, . . . | 42,945 | 3 7 | | | | | | |
| | *Less*—Premiums on new policies included therein, per separate list, . | 1,401 | 7 9 | | | | | | |
| | | 41,543 | 15 10 | | | | | | |
| | Renewal Commission at 2½% on £41,543 : 15 : 10, . . . . | | | 1038 | 11 | 10 | | | |
| | First Year's Commission on new policies, per above-mentioned list, . | | | 288 | 10 | 0 | | | |
| | | | | 2077 | 1 | 10 | | | |
| | *Less* — Renewal Commission on reassurance premiums outstanding, £2,596 : 4 : 1, . . . . . | | | 64 | 18 | 1 | | | |
| | | | | 2012 | 3 | 9 | | | |
| | *Expenses*— | | | | | | | | |
| | C. S. Thomson & Son — books and stationery, . . . . . | | | 56 | 8 | 3 | | | |
| | T. Brown & Co.—printing, &c. . . . | | | 237 | 14 | 11 | | | |
| | | | | 586 | 10 | 4 | | | |

and interest receipts; and the total amount of premiums at the debit of Head Office and each of the Branches, ascertained by the help of these lists, is credited to 'Agents', and debited to 'Outstanding Premiums'. Similarly the interest is carried to the debit of 'Outstanding Interest'.

**CLOSING ENTRIES IN JOURNAL (§§ 105, 106).**

*Journal*, 31 December 1901.

| | | | |
|---|---|---|---|
| Outstanding Premiums, . . . . . | Dr. | 38,175 12 3 | |
| Outstanding Interest, . . . . . | Dr. | 2,370 0 5 | |
| To Agents, . . . . . | | | 40,545 12 8 |
| For amount of premium and interest receipts in hand at Head Office and Branches, now transferred to the former accounts for the purpose of the annual balance, as follows :— | | | |

| | Premiums. | Interest. |
|---|---|---|
| Head Office, . . | 27,309 16 4 | 1837 18 1 |
| 'A' Branch, . . | 977 2 11 | 59 14 9 |
| 'B' Branch, . . | 1,153 17 9 | 42 6 8 |
| &c. | &c. | &c. |
| | 38,175 12 3 | 2,370 0 5 |

**107.** The accounts having been adjusted as described above, an entry is made in the Journal to close the various income accounts, namely: Premiums, Interest, Rents, Fines, Fees, Profit on Reversions, and Reassurance Claims, by carrying the balances to the credit of 'Assurance Fund'. Similarly, the balances of the accounts showing the outgo for Claims, Surrenders, Commission, Expenses, Cash Bonuses, Income Tax, and Reassurance Premiums, are carried to the debit of 'Assurance Fund' (see p. 235), an account representing the whole funds of the Society, or the excess of its assets over its liabilities. The balance of Profit and Loss Account must be carried to the credit or debit of 'Assurance Fund' as may be required.

**108.** For the purpose of guarding against a balance of errors in posting from the Journal to the Principal Ledger, there is inserted in the Journal, at the end of each year, a statement showing that the total of the year's entries in the Journal and the balances at the beginning and end of the year, is equal to the aggregate entries in the Principal Ledger.

**109.** The above-mentioned accounts, Outstanding Premiums, Outstanding Interest, Outstanding Charges, and Outstanding Claims, are all temporary accounts opened for the purpose of the annual balance only, and the Journal entries constituting them are reversed at the commencement of the succeeding financial year.

## CLOSING ENTRIES IN JOURNAL (§ 107).

*Journal*, 31 December 1901.

| | | | |
|---|---|---|---|
| Premiums, | Dr. | 301,607 14 2 | |
| Interest, | Dr. | 110,890 9 11 | |
| Rents, | Dr. | 5,110 0 0 | |
| Fines, | Dr. | 432 17 4 | |
| Fees, | Dr. | 138 2 0 | |
| Profit on Reversions, | Dr. | 5,703 15 11 | |
| Profit and Loss, | Dr. | 398 3 9 | |
| Reassurance Claims, | Dr. | 5,500 0 0 | |
| To Assurance Fund, | | | 429,781 3 1 |
| For balances of the above-mentioned accounts, being income of the year 1901, | | | |
| Assurance Fund, | Dr. | 341,471 18 4 | |
| To Claims, | | | 265,247 14 11 |
| To Surrenders, | | | 15,216 6 2 |
| To Commission, | | | 10,222 9 10 |
| To Expenses, | | | 29,125 19 0 |
| To Bonus in Cash, | | | 2,795 10 0 |
| To Income Tax, | | | 5,356 13 6 |
| To Reassurance Premiums, | | | 13,507 4 11 |
| For balances of the above-mentioned accounts, being outgo of the year 1901, | | | |

### Revenue Account and Balance Sheet.

**110.** The Journal entry described in § 107, forms the basis of the Revenue Account, which is prepared in the manner shown on p. 236.

**111.** This Revenue Account contains no particulars of income and outgo beyond what are required by the Life Assurance Companies Act, 1870; but details of some of the items are obtainable from sources independent of the Principal Ledger.

**112. Premiums.**—These may be divided into Renewal Premiums, New Annual Premiums, and Single Premiums, by the help of the information in the Reconcilement on page 201.

**113. Claims.**—The separate amounts of claims by death and by survivance respectively are given in the Claims Intimated Book (see p. 207).

**114. Commission.**—The separate amount of the first year's commission may be obtained by the help of the statements which it is usual for Offices to make, showing the amount of new business from each of their Branches or Districts, and the cost of it. These give the amount of the first year's commission, and the abstract of them shows the total amount of it.

**115. Expenses.**—An analysis of the expenses is made in the **Expenses Book** (see p. 236). This book is divided into sections, one for Head Office, one for each of the Branches, and one for the ordinary agents. It may also have a section to receive the

expenses in connection with any properties owned by the Society other than their Branch offices. The items are posted into it from the Head Office and Branch Cash Books and Credit Day Book. The 'Miscellaneous' column of the Expenses Book contains items coming under such heads as 'Fidelity Premiums', 'Fire Insurance Premiums', 'Books and Newspapers', or 'Law Expenses'. Payments at Head Office for expenses applicable to the Branches are classified in the Expenses Book under the head of the Branch. For instance, sums remitted to a Branch Secretary for salary or travelling expenses are posted to the

**REVENUE**

Revenue Account

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| AMOUNT OF FUNDS AT THE BEGINNING OF THE YEAR (Assurance Fund), | | | | | 3,454,428 | 5 | 3 |
| PREMIUMS (AFTER DEDUCTION OF REASSURANCES), | | | | | 288,100 | 9 | 3 |
| Premiums, | 301,607 | 14 | 2 | | | | |
| *Less* Reassurance Premiums, | 13,507 | 4 | 11 | | | | |
| | 288,100 | 9 | 3 | | | | |
| INTEREST AND DIVIDENDS, | | | | | 116,000 | 9 | 11 |
| Interest, | 110,890 | 9 | 11 | | | | |
| Rents, | 5,110 | 0 | 0 | | | | |
| | 116,000 | 9 | 11 | | | | |
| PROFIT FROM FALLING-IN OF REVERSIONS, &c., | | | | | 6,101 | 19 | 8 |
| Profit on Reversions, | 5,703 | 15 | 11 | | | | |
| Profit and Loss, | 398 | 3 | 9 | | | | |
| | 6,101 | 19 | 8 | | | | |
| FINES FOR DELAY IN PAYMENT OF PREMIUMS (Fines), | | | | | 432 | 17 | 4 |
| ASSIGNMENT AND OTHER FEES (Fees), | | | | | 138 | 2 | 0 |
| | | | | | 3,865,202 | 3 | 5 |

**EXPENSES**

*Expenses.*

| Date. | | Medical Fees. | Postages. | Bank Charges. | Coals and Lighting. | Caretaker, Cleaning, &c. | Petty Office Expenses. |
|---|---|---|---|---|---|---|---|
| 1901 | | | | | | | |
| Jan. 1 | Stamping 5000 premium receipts, | .. | .. | .. | .. | .. | .. |
| " " | Insurance Record, | .. | .. | .. | .. | .. | .. |
| " " | Postages, | .. | 2 0 0 | .. | .. | .. | .. |
| " " | Sinclair & Rodger, Rubber Stamps, | .. | .. | .. | .. | .. | 0 17 6 |
| " " | Dr. Ferguson on Robert Duff, | 1 1 0 | .. | .. | .. | .. | .. |
| " " | Thomas Robertson & Co., | .. | .. | .. | .. | .. | .. |

section applicable to his Branch. At the end of the year, a statement is inserted in each section of this book, showing any repayments of sums previously charged, or other credits to 'Expenses' account.

**116.** The Balance Sheet of the Principal Ledger contains, in addition to the accounts mentioned in § 80, accounts headed Outstanding Premiums, Outstanding Interest, Outstanding Claims, and Outstanding Charges, and with the transformation shown on page 238, furnishes a balance sheet in the form required.

**ACCOUNT.**

for the year 1901.

| | | |
|---|---|---|
| CLAIMS UNDER POLICIES (AFTER DEDUCTION OF SUMS REASSURED), | | 259,747 14 11 |
| Claims, | 265,247 14 11 | |
| *Less* Reassurance Claims, | 5,500 0 0 | |
| | 259,747 14 11 | |
| SURRENDERS, | | 15,216 6 2 |
| COMMISSION, | | 10,222 9 10 |
| EXPENSES OF MANAGEMENT (Expenses), | | 29,125 19 0 |
| BONUSES IN CASH, | | 2,795 10 0 |
| INCOME TAX, | | 5,356 13 6 |
| AMOUNT OF FUNDS AT THE END OF THE YEAR, AS PER BALANCE SHEET, | | 3,542,737 10 0 |
| | | 3,865,202 3 5 |

**BOOK.**

Head Office, 1901.

| Directors' Fees. | Salaries. | Travelling Expenses. | Advertising. | Printing. | Books and Stationery. | Revenue Stamps. | Rents and Rates. | Repairs. | Income Tax—Sch. A. | Miscellaneous. | Total. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| .. | .. | .. | .. | .. | .. | 20 16 8 | .. | .. | .. | .. | 20 16 8 |
| .. | .. | .. | 5 8 6 | .. | .. | .. | .. | .. | .. | .. | 5 8 6 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 2 0 0 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 0 17 6 |
| .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 1 1 0 |
| .. | .. | .. | .. | 54 9 6 | .. | .. | .. | .. | .. | .. | 54 9 6 |
| | | | | | | | | | | | &c. |
| | | | | | | | | | | | 1170 8 10 |

**BALANCE**

Balance Sheet on

**LIABILITIES.**

| | |
|---|---|
| Total Funds, as per Revenue Account (Assurance Fund), | 3,542,737 10 0 |
| Claims Admitted (or intimated) but not Paid (Outstanding Claims), | 53,905 8 8 |
| Surrender Values Outstanding (Unclaimed Surrender Values), | 327 19 4 |
| Commission and Expenses of Management not Paid (Outstanding Charges), | 2,598 14 1 |
| | 3,599,569 12 1 |

**SHEET.**

31 December 1901.

| ASSETS. | | | | | | |
|---|---|---|---|---|---|---|
| MORTGAGES ON PROPERTY WITHIN THE UNITED KINGDOM, NAMELY:— | | | | | | |
| MORTGAGES ON REAL PROPERTY (Real Property Loans), | | | | | 1,423,951 | 6 | 3 |
| MORTGAGES ON LIFE INTERESTS, REVERSIONS, &c. (Loans on Limited Interests), | | | | | 231,988 | 19 | 5 |
| MORTGAGES ON OTHER SECURITIES (Miscellaneous Property Loans), | | | | | 74,936 | 19 | 6 |
| LOANS ON STATUTORY ASSIGNMENTS OF COUNTY AND OTHER RATES (Loans on Rates), | | | | | 253,705 | 6 | 2 |
| | | | | | 1,984,582 | 11 | 4 |
| MORTGAGES ON PROPERTY OUT OF THE UNITED KINGDOM (Real Property Loans Abroad), | | | | | 23,858 | 5 | 4 |
| LOANS ON THE SOCIETY'S POLICIES, | | | | | 189,429 | 17 | 7 |
| Policy Loans, | 174,516 | 0 | 5 | | | | |
| Half Premium Loans, | 8,466 | 18 | 0 | | | | |
| Non-Forfeiture Advances, | 6,446 | 19 | 2 | | | | |
| | 189,429 | 17 | 7 | | | | |
| INVESTMENTS:— | | | | | | | |
| BRITISH GOVERNMENT SECURITIES, | | | | | 26,005 | 5 | 2 |
| INDIAN AND COLONIAL GOVERNMENT SECURITIES, | | | | | 291,189 | 16 | 5 |
| Indian Government Securities, | 59,804 | 9 | 2 | | | | |
| Colonial Government Securities, | 231,385 | 7 | 3 | | | | |
| | 291,189 | 16 | 5 | | | | |
| FOREIGN GOVERNMENT SECURITIES, | | | | | 44,144 | 11 | 3 |
| MUNICIPAL SECURITIES, | | | | | 73,277 | 1 | 0 |
| RAILWAY AND OTHER DEBENTURES AND DEBENTURE STOCKS, | | | | | 240,503 | 5 | 9 |
| Miscellaneous Debentures, | 55,360 | 0 | 0 | | | | |
| American Railroad Bonds, | 35,482 | 12 | 6 | | | | |
| Railway Debenture Stocks, | 91,956 | 9 | 9 | | | | |
| Miscellaneous Debenture Stocks, | 57,704 | 3 | 6 | | | | |
| | 240,503 | 5 | 9 | | | | |
| RAILWAY AND OTHER SHARES (PREFERENCE AND ORDINARY), | | | | | 151,966 | 5 | 0 |
| Railway Guaranteed Stocks, | 22,443 | 18 | 1 | | | | |
| Railway Preference Stocks, | 11,044 | 8 | 2 | | | | |
| Miscellaneous Preference Stocks, | 27,096 | 3 | 10 | | | | |
| Railway Ordinary Stocks, | 91,381 | 14 | 11 | | | | |
| | 151,966 | 5 | 0 | | | | |
| HOUSE PROPERTY (Premises), | | | | | 127,706 | 18 | 0 |
| FEU-DUTIES (OR GROUND RENTS), | | | | | 94,899 | 0 | 0 |
| REVERSIONS, | | | | | 210,336 | 15 | 3 |
| LIFE ANNUITIES, | | | | | 20,730 | 5 | 5 |
| AGENTS' BALANCES (Agents), | | | | | 42,945 | 3 | 7 |
| OUTSTANDING PREMIUMS, | | | | | 35,579 | 8 | 2 |
| OUTSTANDING INTEREST (INCLUDING £26,042 : 11 : 5 ACCRUED FROM LAST DATES OF PAYMENT), | | | | | 28,412 | 11 | 10 |
| CASH, | | | | | 14,002 | 11 | 0 |
| ON DEPOSIT (Deposits), | 10,000 | 0 | 0 | | | | |
| IN HAND (Cash), | ... | | | | | | |
| ON CURRENT ACCOUNTS (Banks), | 4,002 | 11 | 0 | | | | |
| | 14,002 | 11 | 0 | | | | |
| | | | | | 3,599,569 | 12 | 1 |

**117.** The particulars of the balances which appear on the 'Assets' side of the statement are furnished by the various subsidiary ledgers, namely, the Investment Ledger, Policy Loan Ledger, Half Premium Loan Ledger, Non-Forfeiture Advances Ledger, Agents Ledger, and Bank Ledger. Lists are made of the balances of the accounts in these subsidiary ledgers, corresponding to each of the accounts in the Principal Ledger. A List of the items making up the balance of the 'Unclaimed Surrender Values' account is made up from the Unclaimed Surrender Values Book.

**118.** In making up the list of balances mentioned in § 117, the opportunity may be taken of making a more detailed classification of the investments, than that given in the Balance Sheet. For instance, the list of loans in the 'Real Property Loans' account, may be made up in three divisions, containing Scottish, English, and Irish loans respectively; the Colonial investments may be divided into Canadian, Australian, and South African; the railway securities may be divided according to locality; and so on. This course has the advantage of furnishing very full details of the investments, without introducing too many accounts, with complicated names, in the Principal Ledger.

**119.** I have only to add that the books described above are arranged on the principles described by me in 1886; and that it has been no part of my plan, in revising my original paper, to deal with the suggestions made by Mr. Smith Nicol and Mr. Lindsay Turnbull, in their valuable papers on *Life Office Books* and *Life Office Accounting* respectively.[1] These papers, and especially Mr. Turnbull's arrangement of the American card system, should be studied by all who are interested in Life Assurance Book-keeping.

[1] 'Forms of Life Office Books.' By William Smith Nicol, F.F.A. *Transactions of the Actuarial Society of Edinburgh*, vol. iv, p. 149. 'Some Suggestions in regard to Life Office Accounting, including *inter alia* an Arrangement of the American Card System suitable for the requirements of British Offices.' By A. D. Lindsay Turnbull, A.I.A., F.F.A. *Transactions of the Actuarial Society of Edinburgh*, vol. iv, p. 185.

## DISCUSSION.

THE paper having, in the absence of the author through indisposition, been read by the Secretary, Mr. W. A. Sim,

The PRESIDENT (Mr. G. M. Low) called upon Mr. John Lamb, C.A., one of the referees, to open the discussion.

Mr. LAMB said—I have had much pleasure in renewing my acquaintance with Mr. M'Lauchlan's paper in its altered form. The subject has always been one of peculiar interest to me—first from an outsider's point of view as an auditor's assistant, and secondly as one who from inside the office has to do his best to satisfy an auditor's almost proverbial thirst for information—a double experience which has not been without advantage. The hearty reception given to Mr. M'Lauchlan's paper in 1886 showed that, though travelling out of the line of purely actuarial thought, he had touched on a theme which, though dry and uninteresting to many, nevertheless came home to a considerable number of those present. Of its importance to one and all connected with a life assurance company, there can be but one opinion; and for the younger men no better study can be set, whether they have still the fear of the examiners before them or not. The passing of the 1870 Act closed what may be called the "dark ages" so far as life assurance accounts are concerned; and as systematic audits by professional accountants became more common, there was a stirring of the dry bones, resulting in improved methods in order to meet the form of accounts required by the Act. If I remember aright, only two papers on this subject had been read before the late Actuarial Society prior to 1886—one in 1864, the other in 1881; but neither was published, and it was left to Mr. M'Lauchlan to give us the first contribution in print. Mr. Smith Nicol followed this up, and Mr. A. D. L. Turnbull gave an interesting contribution more particularly with reference to the American card system of book-keeping. One must not overlook the exhaustive work by Dr. Sprague on *Life Insurance Accounts*; but this had reference entirely to the Board of Trade returns, and did not touch on the question before us to-night. So far as the Institute of Actuaries is concerned, I cannot find that they have devoted any attention to the subject, although in the discussion on Mr. Chatham's Messenger Prize Essay of 1895 one finds a suggestion that Mr. Chatham should formulate a complete set of books which would work in and be reconcilable with the valuation states. This is, I think, a point to which attention might profitably be given; but we are, I fear, still looking with expectant eyes in that direction. These papers, therefore, with the ordinary text-books, such as Carter and Lisle, form practically the student's library on the subject. Turning now to the paper itself, one has, speaking generally, nothing but commendation for the way in which the various forms given in the paper have been prepared; and these forms have no doubt, to a great extent at least, stood the test of daily use. One chief objection I had to the original paper was that many of the forms were over-elaborate. Writing theoretically, there is no doubt a tendency to fall into this in the endeavour, not only to make each

book as complete as possible for its primary purpose, but also to make it do duty in other secondary ways. This is, I think, from a practical point of view a mistake, as, the simpler a book is in form, so much less likely are mistakes to occur, and so much more simple will the book-keeper's work be. The forms now given are in many respects in more practical shape, and the result generally is much more satisfactory, showing that the experience of the past sixteen years has not been without a beneficial influence. Although the mode adopted in approaching the subject is an entirely natural and logical one, I must confess I should have liked if Mr. M'Lauchlan had followed up his enunciation of the objects of book-keeping, and as a preface to entering on details, by a few sentences which would show in a general way that, divested of details—admittedly of a complicated character—the book-keeping of a life assurance office is of the simplest nature. It is the mass of detail which confuses a student, and I think it would conduce much to clearness and an intelligent grasp of the whole matter, were it to be shown at the outset that, inasmuch as a life assurance company's income consists almost entirely of Premiums and Interest, the charge for these is derived primarily from the Policy Register, and the Loan Ledger or Register respectively. Similarly, as the outgo consists mainly of Claims and Expenses, these will be obtained from the Claims Book and Cash Book respectively, leaving out of sight for the moment any adjustment at the end of the year for outstanding accounts. With the aid of the Journal and Ledger for classification, the items on both sides of the Revenue Account are practically covered by these principal books. All the rest of the books, however important, are merely subsidiary for the purpose of working out the details. But it seems to me that a student will better comprehend these, if he has in the first place a comprehensive idea of the general scheme which he wishes to evolve. As is very properly remarked, the greatest difficulty of an audit is in the effectual checking of the receipts; but this is less the case with an assurance office than in many an ordinary trader's business; as the premiums can readily be checked as well as the interest on loans and investments, these forming together practically the bulk of the revenue. The subject necessarily lends itself so much to detail that it is difficult to confine one's self to generalities, and I must ask you to bear with me while I shortly refer to a few points which occurred to me in looking over the paper.

*Book-keeping by Double Entry* (§ 1).—We are, I doubt not, entirely at one as to book-keeping by double entry being the only satisfactory system; but we are at the same time rather inconsistent in adopting it only partially in office practice, as referred to in § 40. For, while that system is fully applied to the revenue side of the account, it is practically ignored on the other or expenditure side—unless as regards Claims, the full amount of which is sometimes and properly passed through the books at the end of each month. As regards Surrenders, Commission, Expenses, etc., these are practically cash payments, made so far as possible within the year to which they are applicable; so that a very slight adjustment at the end of the year will convert the actual payments into the expenditure applicable to that year, whether paid or outstanding. Theoretically there is no reason whatever why the outgo should not be dealt with in a similar manner to the revenue, but the practice referred to is evidently a relic of the dark ages when the published accounts very often consisted simply of an abstract of the receipts and payments. But the theoretical has had to give way to the practical; and, while to all intents and purposes the object is satisfactorily attained, we can hardly hold up the practice as a model of book-keeping by double entry. In passing I would remark that the word "subsidiary" in the last sentence of § 1 might be dispensed with, as

any book which contains *original* entries may well be held to be a principal book. I would here instance the Policy Register—a book of prime importance—which really forms the foundation of the business records. No direct reference is made to it, nor is any form of it given. It might, I think, be well, even though going a stage further back, to make it the starting point rather than the transmission of the receipts to the agents as in § 7. The monthly Premium Books, and the mode of reconcilement with the Policy Register—a point of considerable interest—are not dealt with as was done in the original paper. This is an omission which might well be rectified.

*First Premiums on New Policies* (§ 11).—It would apparently be an advantage were the amounts of these, which are entered in one column in the New Business Day Book for Head Office, Branches, and Agents, without distinction, to be split up into three columns so as to get, without further trouble, a total debit for each of these respective accounts in the general Ledger. There is such a separation suggested in connection with Renewal Premiums and Interest in the two following sections. In this connection it is well, I think, to have dispensed with the somewhat complicated Policy Guard Book of the original paper, which seemed to be primarily a record of all policies signed by the directors, and in addition a temporary policy register and an agents' debit book for first premiums. One good feature in it which I do not see continued, is the record of all policies signed by directors—a most essential point, in order that the auditor may have a complete check on all policies signed. This is readily done by means of a book kept for this purpose alone, to be initialed by one of the signing directors.

*Periodical Interest* (§ 14).—I think that the most usual form of Interest Book (a specimen of which might with advantage be given) is fitted rather for a record of interest chargeable after it has been calculated, than as a means of making the calculation. Perhaps this might be better done by means of a "Daily due book" for each day in the month, showing sufficient particulars of all loans or investments for the calculation of interest. I gather that it is considered possible to check in slump the sum of the loans on which interest is charged, by means of the summation of one of the two proposed columns in the Interest Book with the amount of loans in the Balance Sheet. But I doubt whether this is practicable without much labour and adjustment, even were all fractional interest relegated to the Debit Day Book; and in any event it does not lend itself to any ready means of checking the interest calculations in slump, without a classification under the different rates of interest. I fear that in the checking of the interest there is no such royal road.

*Assignment Book* (§ 16).—Assuming this form to be given in full, it seems to be lacking in the date of assignment, the place and hour at which notice was received, and the name, etc., of the assignee. It may be that these details appear elsewhere; but whether or not, it seems desirable that the Assignment Register should contain such information.

*Capital Receipts* (§ 18).—Mr. M'Lauchlan in his original paper was evidently much exercised as to the propriety of debiting to the office's solicitors or agents the amount contained in any discharge which had been executed by directors or officials, and handed over to them for settlement; and has now suggested a much more workable method, which really comes to be simply a record of principal sums in all deeds which have been executed that fall to be received, these sums not being posted to any account. My own opinion is, that where Deeds Signed Books are kept (one for directors and another for officials), and initialed by one of the signing directors or official respectively, it seems not only a mere burden

ing of the books of account to debit the solicitor or agent, but wrong in principle, seeing that no other account can be properly credited without leading to confusion. Even the alternative method of an Agency Suspense Account now suggested in § 20 is not free from objections.

*Cash Book* (§ 28).—I observe that separate cash books are recommended for receipts and payments—I assume with the object of facilitating posting, limiting the size of the book, or possibly also to save paper, through the number of payments being much less than the number of receipts. But none of these objects seems of sufficient weight to counterbalance the obvious advantages of having all cash entries in one book. If a cash book must be split, I would prefer one for alternate days; but there does not seem to be the same urgent call for this in an assurance office as in a bank. An additional column might be advantageously added for loans and investments other than policy loans, rather than entering them in the 'sundries' column (§ 30); and another column for annuities where these are dealt in, with a corresponding income tax column on the debit side. A tendency is observed to use the cash book for what are more properly Journal entries, as in §§ 37 and 55, in connection with receipts and payments at Branches of the nature of Capital. The Cash Book is best limited strictly to recording cash transactions at Head Office.

*Payments falling due—Claims* (§ 41).—As previously indicated, a more satisfactory method than relying on the Death Register for materials for adjustment at the end of the year of the claims chargeable against any given year, is to transfer month by month the total claims intimated, to the debit of Claims account, and to the credit of a Discharged Policies or Claims Payable account, to the debit of which the claims paid through Cash Book or Journal will duly come. The credit balance of this account at the end of the year, will be the amount of the Outstanding Claims. As the claims form the most serious item of expenditure, it is desirable that this method rather than the other be followed—especially when the practice is to have a monthly balance.

*Agents Ledger* (§ 88).—The question is raised whether the labour involved in keeping detailed accounts for each agent is worth the trouble; but in the case of an office with even a limited number of agents, it would, I think, very soon be found that to do otherwise was quite unworkable; and in an office with many agents it would become perfectly hopeless. It is often troublesome enough, even with all details available, to effect a reconcilement of the Agents Ledger with the account in the general Ledger. It is absolutely essential that the Head Office should preserve a complete record of all transactions with each agent, and that in a readily accessible form. It is observed that the case dealt with is that where no agents' accounts are kept in the books at the Branches, the accounts received there being sent on to the Head Office for examination (§ 62), and the duties of the Branch being limited to receiving the account and balance. This, I take it, is the common practice; but it would be interesting to learn whether, in any case of an ordinary Branch, it is entrusted with wider functions, and if so, the extent of the book-keeping involved. If a Branch's duty be limited as stated, no very useful purpose seems to be served in passing the account and balance through it; and there are obvious disadvantages. It is, I think, for consideration whether, in order partly to relieve the Head Office, a Branch might not, in addition to seeing that the agents' accounts are duly rendered, undertake the checking of these, so that they may be sent on to Head Office in proper order. This means no doubt a certain amount of additional labour at the Branch, but it would relieve the Head Office of much detail correspondence, and would also tend to induce closer relations between a Branch and its agents—a much-

to-be-desired end. Were all agents in a district to remit to the local branch, it would be desirable that a modified set of books be kept at each Branch, so that a balance could be struck each fortnight or at other period of rendering the Branch statement, showing the receipts on hand at the Branch and also in hands of the ordinary agents, lists of which should be periodically obtained. All overdue receipts should be at once returned for credit of an agent. In all this there is no suggestion of any slackening of supervision at the Head Office, which would still retain effective control of all accounts and a complete record of transactions as heretofore. There is at least one case in point where this method is in course of being tried as an experiment, with the hearty concurrence of the Branches, and with every hope of a satisfactory result.

*Investment Ledger* (§ 90).—To avoid encumbering this book with more details than are necessary for the calculation of interest, it is desirable to have in addition a Register of Investments in some such form as is indicated, containing full details of each investment, and which can be reconciled at the end of each year with the balance of the Loans account in the general Ledger and the detailed list of balances as taken from the Investment Ledger. I think that the inner columns, which it was proposed in the original paper to add to the Dr. and Cr. sides of the Investment Ledger to show whether or not interest was in arrear on any account, have been wisely omitted. Theoretically it seemed complete, but "the game is not worth the candle", as fortunately it is the exception and not the rule for interest on loans to be in arrear, and such information should be readily available from other sources.

*Bank Ledger* (§ 91).—The suggestion that whenever a cheque is written the number and amount should be entered in the Bank Ledger, and initialed by the signing official, does not seem to lend itself to dispatch of business, and is apt to lead to confusion in posting. It seems a more practical course to have the counterfoil of the cheque initialed, and the Bank Book kept written closely up and verified by the Bank periodically, a note of the bank balance being furnished by the cashier each morning.

*Closing of Ledgers* (§ 96).—It would save complication were the payments in connection with properties owned by the office to be at once debited to the Rents account, and not passed through Expenses account, to be afterwards adjusted at the end of the year. It seems to serve no purpose to pass them through this account, and renders it impossible to ascertain at any intervening period the state of the Rents account; which is not satisfactory. An analysis of the Rents account kept as suggested, the balance on which would be the *net* rents, would then without further trouble show the return from each property.

The final schedules of the *Revenue Account* and *Balance Sheet* (§§ 110 and 116), being in the usual form, do not call for any special remarks. One unsatisfactory point is the dubiety existing as to what is properly to be placed under Agents' Balances, Outstanding Premiums, and Outstanding Interest. Each office is a law unto itself in this matter, and though the question has been raised by the Board of Trade in connection with the returns, and they have been requested to state for guidance what in their view is the proper method, they have not, so far as I am aware, ventured to express an opinion.

The forms appended to the paper will, I feel sure, amply repay a closer study than I have been able to give to them; and I regret it has been impossible to do more than glance at the more salient points. We must all feel indebted to Mr. M'Lauchlan for the time and trouble he has, from first to last, devoted to the preparation of this important paper, which cannot fail to be of the greatest use to all interested in the subject. The

President of the Institute of Actuaries, in his inaugural address the other day, urged that purely scientific subjects should not monopolise attention, to the exclusion of practical subjects; and I think our Council has done well in giving time tonight for the consideration of such a practical question as that of the book-keeping of a life assurance office.

Mr. W. Smith Nicol, Glasgow, the other referee, who could not be present, sent the following remarks—The profession is much indebted to Mr. M'Lauchlan for his paper on book-keeping, as well as for his other contributions both to the Institute of Actuaries and to the late Actuarial Society of Edinburgh. At the outset let me say that I was somewhat startled to find that sixteen years had elapsed since Mr. M'Lauchlan's first paper on this subject appeared. The present paper contains much new matter, and many more alterations than I had expected to see. While I heartily welcome the new, I am not sure that I am quite reconciled to the excision of the old. Possibly, if we had not had the benefit of the former paper, we should not have known our loss. It may be that the author, in his desire to keep his revision within moderate compass, has sacrificed some of the explanations which appeared in the original paper. He has perhaps considered the wisdom of the omissions now referred to; but if it is possible for him to have some passages replaced, he would, I think, confer a distinct benefit. Similarly in regard to some of the forms which are not now given. Otherwise students will be in danger of missing a considerable amount of instruction. It is quite true that the original will still remain for consultation, but as it will be out of print it will in many cases also, I fear, be out of mind. As an example of what I should like to see retained, I may instance the very clear description formerly given of difficulties connected with the treatment of a payment of capital.[1] In the somewhat hurried consideration which I have been able to give, the impression has been left on me that more importance has now been given to what may be described as the book-keeping or cash side of the transactions, and that the continuous record or actuarial aspect has to some extent suffered. It would, of course, be impossible for me to go much into detail, but a point of considerable importance arises on § 11. No reference is now made to a New Policy Guard Book, but a New Business Day Book is provided to contain a record of first premiums; and it is interesting, in passing, to note the evident influence of the decision in Roberts *v.* The Security Company, Limited, inasmuch as provision is now made for the debit of interim receipts, the policies themselves not now being issued until the premium is actually paid. I think, however, we ought to have a check afforded that interim receipts and policies are issued only for proposals which have been passed by the directors. An obvious and convenient method of doing this was furnished in the Policy Guard Book, and I am sorry that it has been left out. I note with much interest the new forms of Cash Book, and the appearance for the first time of the Repayments Day Book, together with the amplification of the remarks on interest and investments. This is a section of great importance, and one which, owing to the changes in amount from year to year (not to say from week to week), does not lend itself to a mechanical system to the same extent that premiums do. I should have liked Mr. M'Lauchlan to say a little more about Mr. Lindsay Turnbull's most interesting paper on the Card System. That paper is a conspicuous example of the way in which a subject generally looked upon as dry and almost repellent, may be dealt with in a most attractive way. Americans have taught us much, and they are daily teaching us more. To them we owe a special debt of gratitude for the development in the use of cards in a great variety of directions. That cards have been too little, rather than too

[1] The paper as read did not contain §§ 19, 21, 22, 49, 50.

much, used in this country hitherto is undoubted; but the increase in their use should be confined to purposes to which they are suitable. In the book-keeping of an insurance office convenience should, of course, be studied; but convenience can sometimes be too dearly bought, and I am decidedly of opinion that this would be so by the use of cards for many of the purposes suggested by Mr. Turnbull. To my mind, the main book-keeping records, which form the Policy, Renewal, Investment, and Agents Registers, the Agents Ledgers, and the ordinary Ledgers, should be in books, not on cards. This does not prevent these being supplemented by cards containing details. The statement in regard to assignments in paragraph 79 of Mr. Turnbull's paper (page 232), holds at least equally in regard to other permanent records; namely, that the principal object of a register of this kind is, or should be, to make it impossible for any dealings with the policy of which the office has had notice, either directly or indirectly, to be overlooked. I have taken the opportunity of discussing the card system with gentlemen who have had a wide experience in auditing life office books, not only in this country but in America; and they are unanimous in condemning it. It will doubtless fall to the lot of comparatively few, to have the benefit of such a desirable system of book-keeping as Mr. M'Lauchlan has elaborated. His paper bristles with suggestions, and shows clear traces of having been put to the test of actual practice. Why should we not all adopt it or something like it? One step in the direction of a general improvement will undoubtedly be, that actuarial students should make a much more serious study of the subject of book-keeping than most of them do. Many of them never have a chance of passing through the accounts department of an office, their great desire being to get at as early a stage as possible into the actuarial department. When actually preparing for their examinations, they not infrequently come to the subject of book-keeping at a stage of their preparation when they are more or less tired with the application to actuarial matters. The subject well deserves more respectful treatment than it often gets. It will richly repay any time spent on it. I thank Mr. M'Lauchlan for placing at our disposal the results of his further consideration of this important subject, and I venture to predict for this paper an even wider circulation than the former one enjoyed.

THE PRESIDENT.—We are honoured on this occasion with the presence of some leading members of the Society of Accountants. I doubt not that any remarks from these gentlemen on the subject of the paper will be specially welcome to you. We are proud to see among them the President of the Society, one of our own members, and perhaps he will now favour us with any observations he may have to make.

Mr. HUGH BLAIR, *P.C.A.*—I am sure we have all listened with pleasure to this important paper, which is a valuable addition to the existing works on the book-keeping of a life office; and I have no doubt that it will become a text-book of the Faculty. I observe that the author confines himself to the case of a mutual office. This is unnecessary, as the books of a proprietary office may be kept on precisely the same lines as those of a mutual office, with the addition of course of certain accounts at the Head Office which would not apply to the latter. These would be, (1) a proprietors' capital account, and (2) an account which would be credited with interest on the capital and the shareholders' proportion of profits, and debited with the dividends paid to them. That is practically the only difference between the two kinds of offices. There must also be a Share Register, a Transfer Book, etc.; but these have no real connection with the subject before us. Referring to §§ 8 and 9, in which the auditor's work is specially mentioned, I quite agree with the author that the examination

of the receipts side is the most difficult part of this work, and that it is necessary not to overload the Principal Journal and Ledger with details which should be relegated to sub-ledgers. With regard to the agents, I think they should render their accounts once a month, remitting the balance due, and returning any receipts not taken up with an explanation of the cause. From these accounts it will be found when a premium has been paid, and the date so ascertained should be entered in the sub-ledger opposite the individual premium. When this has been done, it is easy to see what premiums or interests have not been paid at an agency. The receipts returned should be compared with the blanks; and where a blank is found without a corresponding receipt, enquiries should be made to ascertain whether, for instance, the receipt is still held by the agent, or has been transferred to another agent. Then, with regard to premiums, interest, etc., debited to the Head Office cashier, he is much in the position of an agent, with this difference—that his transactions can be posted up more frequently in the sub-ledger. In order to ascertain the correctness of the cashier's entries, it is a good plan for the auditor to take possession of his receipts at indefinite intervals, and compare them with the amounts standing at his debit in the sub-ledger. This would show what receipts were unaccounted for, and a list of them would be made up by the auditor. It would probably be found that most of the differences were caused by certain sums having been received and debited in the cash book, but which had not been posted to the sub-ledger. The posting of the latter should be immediately brought up to date; and if any differences still remained, special enquiry in regard to them would require to be made. Passing to § 14, the cashier may be debited with many items of periodical interest for which he holds no receipt, such as interest on inscribed stocks.[1] With regard to § 27, it usually happens that ordinary agents have no transactions except with reference to receipts sent to them for collection; and of course if there were any other transactions they should be required to account as strictly as a salaried official. Turning to § 36, the proposal that Head Office cheques for meeting claims, surrenders, and policy loans of substantial amount, should not pass through the branch accounts, I consider to be a good suggestion, as it is desirable to make the book-keeping at a branch as simple as possible, leaving the Branch Secretary and his staff to devote their energies as much as possible to getting new business, and not troubling them with any book-keeping that can be avoided. When payments of the nature of capital (§ 55) are received at a branch, they must necessarily appear in the Branch Cash Book, and should of course be notified to the Head Office. In my opinion they should be entered in the Debit Day Book to the debit of the branch, and to the credit of the account in respect of which the repayment is made. I think it is not desirable that these entries should be made in the Head Office Cash Book, because that is the Cash Book of the Head Office cashier, and no entries should be made in it for which he is not primarily responsible. The Cash Books of the office consist of Cash Summaries Books, which are prepared from the Head Office Cash Books and the Branch Cash Books, and, if possible, there should be no repetition of facts in them.

Mr. JAMES HALDANE, C.A.—I have not had time to look over the paper except in the most cursory way; but there is one matter to which I should like to draw attention in connection with § 82. The particular sentence is, "No agents' accounts are kept in the books at the Branch Offices." After a considerable experience of insurance accounts, I am inclined to doubt the propriety of such a direction. It seems to me that the branch

[1] A reference to those cases has been inserted.

should have full charge of its agencies. I do not think there is any mercantile business in which such matters are not left in the charge of the branch manager. I think the Head Office should send all receipts to the Branch Secretary to be distributed by him among his own agents, and he should be responsible that they are duly accounted for. He knows the circumstances of the agents, and can keep his eye on them. At every Branch Office there must be one or two clerks, and they could easily keep the Agents Ledger without any addition to the staff.

Mr. RICHARD BROWN, C.A.—I should not have ventured to make any observations on this paper had it not been that your President gave me a copy of it two days ago, with a pressing invitation to read it and to make any observations that might occur to me. I have read it with the greatest interest. The paper is a most elaborate one, and lends itself to very careful study—longer study than I have been able to give. The views I have formed are much on the lines of Mr. Lamb's remarks. I have found the plan of life assurance book-keeping one of the most difficult things to grasp. The first thing that an auditor of a large concern ought to do, is to form a clear idea of the scheme of book-keeping as a whole; and then he will be in a position to understand the subsidiary books. But with the insurance offices whose system I have seen in operation, it has been difficult to find a general scheme which would be a sort of chart in examining the books. Mr. Lamb desiderated in the paper something more of a clear outline of the general system, and this would certainly have been of assistance to us. The perusal of the paper has, however, tended to clear one's ideas as to the uses of the various books. There are still some things which are rather upsetting to an auditor, such as are referred to in § 18, where certain entries in a book are to be posted and others are not. Another peculiarity is entering cheques in the Bank Ledger immediately they are signed, instead of posting the entries from the Cash Book. It is bound to produce difficulties if the entries are originally made in the Ledger. There is one general principle with regard to the system of book-keeping here described which is of great importance. It is intimately connected with the number of adjustments (§§ 94-106) that require to be made upon the books before a balance can be struck. The primary object of books is to show, as nearly as possible, the actual position of the office at any time; and the adjustments necessary in order to strike a balance, should be reduced to a minimum. A considerable number of the adjustments described in this paper are necessary; but some might be dispensed with. They arise from the fact that the principle of double-entry is not followed out consistently. It is followed with regard to premiums falling due, but not in respect to claims falling due. The case appears to me much stronger for the claims and other liabilities being dealt with in that way; because, while they can be enforced against the office in a court of law, it is doubtful whether an office can sue for a premium. The latter point is indeed of little moment, because the premium is merely a sum paid for the purpose of keeping up a liability on the part of the office. If the premium is not paid, the office is not liable, and it would not trouble to sue for a premium. To debit a premium about to fall due, because the receipt has been forwarded to the agent for collection, is not consistent with the omission to put through as liabilities the claims which have emerged; and it has the greater disadvantage of treating as an asset what is not in reality an asset, because it is quite open to the assured to allow the policy to lapse. It should be possible to devise a system by which the amount of matured liabilities should appear in the books without any adjustment being necessary, and the actual amount of premiums unpaid be accurately ascertained. Mr. M'Lauchlan has referred

(§ 106) to the difficulty of deciding what is outstanding at the end of the year. Of course there cannot be any difficulty in separating premiums from interest, etc., assuming the office to have complete returns from its branches and agents. The difficulty lies in the fact that agents have been debited with these premiums, whether they are to be paid or not, and at the end of the year crediting entries are necessary for premiums not to be paid or to be cancelled. With regard to putting premiums falling due to the debit of agents and credit of premiums, I think it is not sound in principle, for the reason that it is merely an internal transaction of the office. It is not really a transaction of the company, and does not become such until the money is handed to the company or its agent. I consider that in the books of the company, transactions with officials or agents should be distinguished from transactions between the company and the public. They are essentially different, and the books would be clearer if it were possible to maintain the distinction. It is obviously necessary that a proper record should be kept of the premiums which agents are authorised to collect; but, if possible, that record should not be a part of the books of the company in the sense of the books necessary to a balance sheet. I agree with the view that it might simplify matters if agents' accounts were kept at the branch offices; but this would necessitate more frequent returns from the branches. I think a monthly statement is not often enough. In banks it is the rule to send a full copy of the cash book once a week, and I think it would be an improvement if insurance companies had the same rule. In § 92 Mr. M'Lauchlan refers to the books being brought to a balance at the end of the year. I do not know whether it is meant to balance oftener; but I think in a large concern like an assurance office, the books should be agreed at least once a month. These are minor points, and I am bound to say the paper shows a very full and thorough knowledge of the subject, and has been drawn up with much care. To me personally it has been of great assistance, having helped to make some things clear about which I had experienced difficulty.

Mr. J. CHATHAM.—Since Mr. M'Lauchlan prepared his former paper a number of changes in practice have taken place, and that no doubt accounts for some of the alterations now introduced. No change has been of greater importance to assurance offices than the case of Roberts *v.* The Security Company, Limited, as it has entirely altered office practice. Formerly policies were generally issued before the first premiums were paid. Now it is the custom to issue interim receipts for collection, and it is only after the premium is paid that the policy is written. Mr. M'Lauchlan has done right in making that interim receipt the foundation of his system. On one point I think it would be desirable to have fuller information. The Policy Register has been referred to. If examination candidates were asked for the names of the principal books in use in a life office, they would be pardoned for mentioning the Policy Register as one of them; and yet no form for it is given in the paper, although it forms part of the book-keeping system in many offices. Again, the Renewal Premium Book is not in any way described; although the corresponding reassurance book is described, and its form given in the appendix. The paper is rightly regarded as a classic, and largely used both by accountants and students; and although it is easily followed by the former, yet for the latter it might be fuller in other respects also. For instance, Mr. M'Lauchlan goes from new premiums to renewal premiums; but there are various steps to follow before that, which it would be better to describe. I can endorse Mr. Blair's remarks with regard to the very slight difference in the book-keeping of mutual and of proprietary offices. In the system of accounts in operation in the office with which I am connected two great principles are followed. The

first is that the new premiums are dealt with entirely by themselves, being entered in a book very similar to Mr. M'Lauchlan's New Business Day Book and called the "New Premiums Written-on Book". It differs in two respects, inasmuch as the premiums are divided into ordinary, single (including one-year term policies), extra, and miscellaneous, and there is a column for commission. The interim receipt is posted from this book to the debit of the agent in the Agents Ledger, and despatched to him with a counterfoil attached, which the agent returns along with a cheque for the amount. The cheque is entered in the Cash Book, and carried to his credit in the Agents Ledger, and there is an end of the matter. If he does not send a cheque, but returns the interim receipt, it is entered in a book called the New Premiums Written-off Book, exactly corresponding to the New Premiums Written-on Book. That is the other great principle, namely, that what is written on shall be written off in the same way. The difference between the totals of the two books must of course agree with the Policy Register; and the comparison may be made as often during the year as necessary. The renewal premiums are written into Monthly Premium Books from the Policy Register; and there is a book called the Old Premium Book, with divisions for writing on and writing off, corresponding to the books for new premiums written on and off. If, for instance, a policyholder reduces the sum assured, the original premium would be written off, the new premium would be written on in the same line, and in a third division the effect of the two on the premium income would be shown. Formerly we had Renewal Debit Sheets corresponding to those described by Mr. M'Lauchlan, but in their place we introduced an Agency Renewal Premium Book, arranged according to months, subdivided according to agencies, and serving for five years, so that it is not necessary to rewrite it every year as in the case of the Renewal Debit Sheets, but only to extend the amount of the premium. The renewal receipts are written from the Monthly Premium Book. The only reason for continuing the Monthly Premium Book was that it had several years still to run; but, as it is of assistance in sending out the renewal receipts, it is a question whether it will not be continued as at present. Another advantage of the Agency Renewal Premium Book, is that it shows the quality of the business, and whether or where the business is running off quickly. If an agent returns a receipt unpaid, it is written off in the Old Premiums Written-off Book that I have described. With reference to repayments of capital, referred to by Mr. Blair and Mr. Brown, I think the better plan is the old one, of debiting the agent or bank which collects the money. With regard to the American card system—while always ready to adopt anything calculated to save labour, as soon as we ascertained that the effect was to collect renewal premiums without any check upon them, the idea was abandoned.

Mr. A. Hewat.—There is one small point to which I should like to refer—so small that it is apt to escape the eye of the auditor. It is with reference to Mr. M'Lauchlan's "Register of Assignments". In the form given, that is not a Register of *Assignments*, but—a very different thing—a Register of Assignment *Fees*. I think all the assignment notices should be entered, and where there is no fee, that this should be noted; because, as is well known, the fee is not payable for recording the notice but for giving an acknowledgment of it. In such cases the auditor should be satisfied that no acknowledgment has been given, and consequently no fee is receivable. When there is a fee, its amount need not be stated, as it is fixed by statute. The total in pounds and shillings can be got by merely adding up the number of acknowledgments and dividing by four. The number of the policy being given, it should not be necessary to enter the

names of the lives assured: even their initials might be dispensed with. In regard to the Policy Guard Book, I think it was useful for checking the sums spent on policy stamps; indeed it is probably the only book that contains a direct record of them.

Mr. D. Deuchar.—I had noted down a few points in the paper with the view of saying something upon them, but all that I intended saying has been anticipated by previous speakers, and therefore I shall say nothing on these points. I should like, however, to allude to one marked feature of Mr. M'Lauchlan's paper, namely, the extremely clear manner in which he has expressed what he has to say. I also venture to express the hope that the valuable remarks made by Mr. Lamb will be printed in full along with the paper, and not merely in a condensed form.

The President.—I can only repeat the regret that Mr. M'Lauchlan is not here to speak for himself, because many interesting points have been raised in the course of the discussion. However, before this paper is published Mr. M'Lauchlan will have the opportunity of putting in writing, as a supplement to the discussion, any observations he may have to make. It only remains for me to ask you to express your cordial thanks to Mr. M'Lauchlan for this valuable paper. It is not too high praise to say that it will be in the future a most valuable text-book for actuarial students, and that they will derive from it much instruction in regard to the book-keeping of a life assurance office, that is obtainable from no other source.

### Mr. M'Lauchlan's Reply.

In dealing with the various criticisms made in the course of the discussion on my paper, I shall begin with those which involve general principles.

Mr. Lamb agrees with me in thinking that book-keeping by double entry is the only satisfactory system, but thinks it inconsistent of me to apply it fully on the revenue side of the accounts, and practically ignore it on the expenditure side. I do not see, however, that the system described by me is lacking in the essentials of a double-entry system. I understand the principle of double entry to be, that each entry in the primary records is carried to two accounts in the ledger, so that the ledger furnishes, in addition to the balance sheet, a revenue account, or profit and loss account, showing how the funds at the beginning of the year have become converted into the funds at the end of the year. What Mr. Lamb objects to—and Mr. Brown agrees with him—is my not recording and carrying into the ledger the claims and other outgoings *as they fall due.* I submit that the question involved here is not that of double-entry book-keeping against some other system, but a question of what stages in the history of a particular class of payments are to be recorded. The answer to a question of this kind always depends more or less on practical considerations. The same question arises when an office enters into a binding contract to purchase a reversion or some other property. The property passes on the contract being made, and the agreed price becomes a liability. Strictly speaking, this should be shown by the accounts, but the ordinary practice would be to defer entering the transaction in the accounts until the price was paid.

To carry into the Principal Ledger, in the regular way, the claims and other outgoings *as they fall due,* would certainly make the system described more complete and symmetrical, but it would involve additional labour without, in my opinion, securing any counterbalancing advantage. In the case of the claims, it would be necessary to keep a Claims Admitted Book, and make an entry in the Journal at the end of each month to debit

'Claims' and credit 'Claims Payable' with the amount of claims admitted by the directors during the month. Claims when actually paid would be debited to 'Claims Payable'. On the Principal Ledger being balanced at any time, the 'Claims' account would show the amount of claims admitted since the last balance, and the 'Claims Payable' account would show the amount of claims admitted and not paid. But these are not the figures required for the Revenue Account and Balance Sheet. The usual practice is to enter in the Revenue Account the amount of claims *arising* in the financial year, whether admitted in the same year or not; and, consistently with this, the outstanding claims in the Balance Sheet include claims arising in the year but not admitted until after it had closed. It is therefore necessary, when the accounts are kept in this way, to make an adjustment at the end of the year, to debit 'Claims' and credit 'Claims Payable' with the amount of claims intimated as having arisen in the year but not admitted until after its close. To debit the claims in the Ledger immediately on their being admitted, does not therefore get rid of the necessity of an adjustment at the end of the year; and it seems to me that the most practically convenient course is to make the adjusting entry deal with the whole of the outstanding claims, and get rid of the labour of making continuously two sets of entries relating to the claims, instead of one.

Reassurance premiums may be dealt with in the same way as claims, that is to say, the total reassurance premiums payable in each month may be carried, at the commencement of the month, to the debit of 'Reassurance Premiums' and credit of 'Reassurance Premiums Payable', and the reassurance premiums debited to the latter account when paid; but it does not appear to me that, in this case either, the advantage obtained is worth the extra labour.

Mr. Brown thinks that the number of adjustments required in preparing the final Balance Sheet should be reduced to a minimum, so that a Balance Sheet of the Ledger, made at any time, may show as nearly as possible the true financial position of the office. But it must be remembered that it would take very much less labour to make the adjustments which I have described whenever a Balance Sheet is required, than to keep up continuously accounts which would render the adjustments unnecessary. Again, in a mutual life assurance society, there is no great advantage in producing frequent balance sheets, as the balance sheet is of little use without a valuation.

Mr. Lamb and Mr. Brown object to making the transmission of the official receipts to the agents the first stage of account in dealing with the receipts, and Mr. Brown points out that such transmission is merely an internal transaction of the society, and not a transaction between the society and the public. It must be of course admitted that, as I have stated in § 7, the first stage in the history of any item of receipt, is its falling due. I knew of one case in which an insurance company opened a ledger account for each policyholder, debited him with his renewal premiums as they became due, and credited him with the cash when they were paid; but none of us would, I think, consider it necessary to proceed in this way. In making the transmission of the receipts to the agents, the first stage of account in the collection of renewal premiums, I hold that I am not treating transactions between officers and agents of the society as if they were transactions of the society itself; but rather that I am, for the sake of convenience, treating as identical, two financial facts or events which are always associated, and always closely follow each other, namely, the transmission of the official receipt to the agent, and the actual

falling due of the corresponding premium. The reconcilement mentioned in § 21 will be found to show, each year, that the premiums, as stated in the Policy Register, have been all charged against the agents; so that the Policy Register is, to all intents and purposes, made the starting point. The course adopted has the advantage of making the entries in the books coincide with the first step taken to collect the premiums.

But Mr. Brown goes a step further than Mr. Lamb, and objects to treating premiums that fall due, as assets, on the ground that it is quite open to the assured to allow the policy to lapse. It is quite true that, as there is no contract on the part of the assured to pay the renewal premiums, these are not in the strict sense assets. In the case, however, of all except recently-effected policies, the assured has a very strong inducement to pay the renewal premium. A very large proportion of them are, as a matter of fact, duly paid. Indeed, I have little doubt that the percentage of renewal premiums not duly paid is less on the average than the percentage of loss on the book debts of many traders. Looking at the matter from another point of view, and supposing that all the renewal premiums or even any considerable proportion of them, were left unpaid, the society would be in a stronger and not a weaker financial position than before.

I submit then, that in order to give a proper view of the financial position of a life office, the renewal premiums that have fallen due *must* be included among the assets. This is obviously necessary on the occasion of an investigation and division of profits, bonuses being allotted to all the policies, on the assumption that all premiums due up to the *date* of valuation are paid. If it happens, in any particular case, that the last premium has not been paid, the policy is treated as lapsed, and the assured does not get the bonus which has been allotted to him. Finally, it seems quite clear from the form of balance sheet prescribed by the Life Assurance Companies Act 1870, that the legislature contemplated that premiums due and unpaid would be included among the assets of a life office.

Mr. Haldane doubts the propriety of keeping no agents' accounts in the books at the Branch Offices, and thinks that the Head Office should send all receipts to the Branch Secretary to be distributed by him among his own agents, the Branch Secretary to be responsible that the receipts are duly accounted for. Mr. Lamb thinks it absolutely essential that the Head Office should preserve a complete record of all transactions with each agent, and that in a readily accessible form, but suggests a modified set of books at the Branches. In reply to these observations, I have to explain that I have not attempted to discuss the general question of how the work connected with the collection of premiums through agents should be divided between Head Office and Branches. The best distribution of the book-keeping and other work will, in the case of each company, necessarily depend upon its specal circumstances. The plan of keeping all the agents' accounts in the Head Office books while forwarding the receipts and collecting the balances through the Branch Offices, has the advantage of reducing and simplifying the office work at the branches, while giving the Branch Secretary, to a large extent, that full charge of his agencies which Mr. Haldane refers to as desirable. I may add the present paper is primarily a description of the specimen books given. Those specimens are of necessity adapted to the circumstances and methods of a particular business, and I have not, in most cases, attempted to explain how the forms might be modified so as to suit different business arrangements.

It would no doubt be a simple and in some respects a convenient arrangement if, as Mr. Brown suggests, the Cash Books at the Branches were on

the same footing as the Head Office Cash Books, so that the cash receipts or payments at the Branches affecting capital, might be entered once for all in the Branch Cash Book. It must be kept in mind, however, that the Repayments Day Book described in § 18 is arranged on the footing, that all items of capital receivable must be accounted for by the Head Office Cashier, and entered in the Head Office Cash Book; and that it is desirable to simplify the book-keeping at the Branches as much as possible, so that the entries in the Branch books may be made without any special skill or knowledge; also that it is very convenient to be able to find all capital transactions in one book, such as the Head Office Cash Book, instead of having to search through all the Branch Cash Books for the small number of capital transactions that are settled at the Branches. One way of dealing with the matter would be to require that the Branches should neither receive nor pay sums of the nature of capital, but that all such business should be transacted directly from Head Office. It would, however, frequently be very inconvenient, if a Branch Secretary were precluded from paying or receiving repayment of a policy loan, and the course described in the paper, though open to objection on theoretical grounds, appears to me, having regard to the various considerations stated above, to be the best that is open to us. It will be observed that the paper is written from the point of view of a life assurance official; and I have acted on the principle that the requirements of the man of business may sometimes be a sufficient reason for modifying the theoretical rules of account-keeping.

Mr. Lamb thinks that the ground rents and other outgoings mentioned in § 96, should not pass through 'Expenses' account, but should be debited at once to 'Rents'. The course described in the paper has two advantages. A large proportion of the payments for expenses of all kinds are made at the Branches. It is desirable, in order to simplify the work at the Branches, that the Branch cashier should be able to debit them all to one account, and not have to keep in mind that certain kinds of expenses must be debited to 'Rents'. Again, the Expenses Book affords a convenient means of analysing the outgoings in connection with properties. If the various items were debited directly to 'Rents' account, it would be much more difficult to make the analysis referred to by Mr. Lamb.

Mr. Lamb and Mr. Brown object to entering cheques in the Bank Ledger before they are signed. This objection, is, I assume, based on the principle that the original entry of a cheque is supposed to be made in the Cash Book, and that the regular course is to post from the primary records into the subsidiary ledgers. I submit that there is an alternative course which is quite legitimate and sometimes convenient, namely, to make entries independently in the Cash Book and subsidiary ledger, and then mark off the two sets of entries against each other. By following this course in the case of cheques, we obtain the advantage that the entries of cheques in the Bank Ledger all pass under the eye of the signing officer and are authenticated by his initials.

With reference to the remarks made by Mr. Lamb and Mr. Hewat on the Register of Assignments, I have to explain that this book is meant to be merely a register of notices of assignment. It contains all notices whether an acknowledgment is given or not, but as it is used in combination with a system of recording the particulars of notices on the backs of copy policies, it does not include information as to the date and purport of the assignment.

Mr. Smith Nicol complains of the omission of some of the matter contained in my original paper. In reply to this, I have to explain that I was unexpectedly asked to get my paper ready for reading at the second

meeting of the Faculty for the session, instead of the third. I was therefore obliged to put the paper into shape very hurriedly, and omitted some portions of my former paper, from want of time to consider how they ought to be fitted into the paper in its new shape. In preparing the paper for the *Transactions of the Faculty*, I have included some additional matter from the old paper.

Some of the speakers think that I should have described the Policy Guard Book, Policy Register, Renewal Premium Book, and Interest Book. In order to keep my paper within the necessary limits of space, I have confined myself to dealing with books which are essentially books of *account*, and have left out the four books mentioned above, which, although in close relation to the book-keeping system, are primarily books of *record*.

In conclusion I desire to thank the gentlemen who listened to my paper, and especially those who took part in the discussion, for their favourable reception of it, and for their kind vote of thanks. While I was exceedingly sorry to be prevented from reading the paper in person, I am glad that I have had the opportunity of dealing with the various interesting questions raised in the discussion, in a more adequate way than I could possibly have done in an oral reply.

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# TRANSACTIONS

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## On an Investigation to show the Rates of Mortality and Marriage among Daughters of Peers and of Heirs Apparent; with values of certain Benefits, including Reversions. By M. Mackenzie Lees, F.F.A.

With Abstract of Discussion thereon.

Presentation to Mr. James Meikle.

LONDON:
CHARLES W. SUTTON LAYTON,
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[illegible] JAMES PULLAR

[illegible] WILLIAM ROBERTSON
[illegible] EDWARD JOHN ROBERTS
[illegible] SAMUEL SACKSON
[illegible] CHARLES W. H. JAMESON
[illegible] WM. RICHARD SCHENKS

[illegible] WILLIAM HENRY WALTO

[illegible] WILLIAM ADAMSON
[illegible] WM. LITTLETON ARMSTRONG
[illegible] DAVID LESLIE BINNIE, C.A.
[illegible] JAMES JOHN F. BINNIE
[illegible] THOMAS F. CONNELLY
[illegible] JOHN ROBERT SOMERVILLE
[illegible] WILLIAM BANNATYNE
[illegible] HUGH BLAIR, C.A.
[illegible] FERGUS C. A. BLAIR
[illegible] WILLIAM HIBBON WATSON
[illegible] ROBERT BOYD, C.A.
[illegible] JOHN BOYTE
[illegible] THOMAS WILLIAM CRAMMEN
[illegible] HUGH WILLIE BROWN
[illegible] JAMES BUCHANAN, M.A., LL.B.
[illegible] GEORGE CAMPBELL
[illegible] WILL CAMPBELL
[illegible] DAVID BARCLAY
[illegible] DAVID BARCLAY CALDWELL
[illegible] JAMES CHRISTIE
[illegible] DAVID S. CLAPERTON, C.A.
[illegible] PETER COCHRANE
[illegible] HENRY HOBERTSON COCHRANE
[illegible] JOHN WILLIAM COLSON
[illegible] STUART F. S. DUNCANS
[illegible] WALTER DUNCAN
[illegible] JOHN JAS. WALLACE DEUCHAR
[illegible] THOMAS COLDIE DICKSON, C.A.
[illegible] ALEX. GRAHAM DONALD, M.A.
[illegible] WILLIAM HOBBLEY DOTTEY
[illegible] CAMPBELL KERR DUFF, C.A.
[illegible] ALEXANDER DUNCAN
[illegible] JOHN EDGER
[illegible] WILLIAM EVANS
[illegible] HARDWIN FRANCIS FALCONER
[illegible] FRANCIS KERR FENTICK
[illegible] JAMES FENTON
[illegible] JOHN FERGUSON
[illegible] CHARLES EDWARD FOX
[illegible] ALEXANDER FRASER, JUN.
[illegible] ROBERTSON GAFF, C.A.
[illegible] JAMES BURNETT GIBB
[illegible] WILLIAM GIBSON
[illegible] JOHN BROWN GILLESPIE
[illegible] NEIMAN GING, C.A., C.A.
[illegible] CHARLES GORDON

[illegible] WILLIAM S. GUTHRIE
[illegible] PETER D. HARRIS, C.A.
[illegible] ALBERT J. [illegible]
[illegible] WILLIAM [illegible]
[illegible] GEORGE HILLAND

[illegible] HONORARY ACTUARY.
[illegible] JOHN WINTERLIST

[illegible] Clerk of the Examinations.
[illegible] JOHN WARDEN WARDEN

[illegible] SECRETARY.
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# TRANSACTIONS

OF THE

# FACULTY OF ACTUARIES.

No. 8.—Vol. I.

**On an Investigation to show the Rates of Mortality and Marriage among Daughters of Peers and of Heirs-Apparent; with values of certain Benefits depending on Marriage and Death. By M. Mackenzie Lees, F.F.A.**

**With Abstract of Discussion thereon.**

**Presentation to Mr. James Meikle.**

LONDON:
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TRANSACTIONS

OF THE

FACULTY OF ACTUARIES.

*On an Investigation to show the Rates of Mortality and Marriage among Daughters of Peers and of Heirs-Apparent; with values of certain Benefits depending on Marriage and Death.* By M. MACKENZIE LEES, F.F.A., *of the Scottish Equitable Life Assurance Society.*

[Read before the Faculty, 16 February 1903.]

EVERY one who has had to deal with the valuation of life interests, reversions, and entails, or with investigations involving Marriage Rates, must have been impressed with the carefully compiled and very complete tables which have been furnished by Dr. Sprague, dealing with the combined probabilities of marriage and mortality. These relate entirely to the marriage and mortality probabilities of bachelors and widowers of the Peerage; and, so far as I am aware, no similar tables have been published relating to females. I find that in the discussion on Dr. Sprague's paper of 1879 (*J.I.A.*, xxi, 448), Mr. C. J. Bunyon stated that about fifteen or sixteen years previously he had formed a table precisely similar to Dr. Sprague's, showing the probabilities of death and marriage among 10,000 unmarried people, up to the age of 60; only his table was not a table of bachelors, but of spinsters. The marriage-rate was very low at the ages of 15 or 16, culminated at 22, and then

rapidly reduced. I cannot find, however, that the table referred to has ever been published. It will be readily admitted that tables showing the values of benefits depending upon the combined contingencies of marriage and mortality among spinsters would be useful. I have, therefore, made an investigation in order to form such tables.

As no statistics were, so far as could be ascertained, readily available for this purpose, it was necessary to look about for a source from which these could be drawn. After careful consideration, I came to the conclusion that the most satisfactory results would be obtained by basing the investigation entirely upon the data contained in the works relating to the Peerage Families; and the best work for the purpose appeared to be Lodge's Peerage, which gives very full information as to the dates of births, deaths, and marriages. It has been observed by other writers who have made use of Lodge's Peerage, that these dates cannot always be relied on, especially for the collateral branches. I therefore decided to include in the data only the daughters of peers and of heirs-apparent; as in their case the information given is almost invariably correct.

Starting with Lodge's Peerage of 1863, a card was written for each female coming under the above description, and these females were then traced throughout subsequent volumes, additional cards being filled up for those born after 1863. The last volume from which information was taken was that for 1898. In passing from volume to volume, each card that had already been written was carefully scrutinised, in order to bring the particulars down to date. It was frequently found that in the later volumes the figures of the year of birth had been transposed in reprinting; but a reference to the source from which the data was originally taken, and also to some other Peerage work, such as Burke or Foster, enabled me readily to decide which was the correct date. In all, 4,485 cards were written; but of these, after careful search had been made in other Peerage publications, 45 had to be thrown out for want of sufficient particulars.

The number of lives dealt with is thus 4,440, of whom 2,010 died, and 2,430 were existing at the close of the observations. Of the number existing, however, 5 were under six months of

age, and are held as not having been at risk. The following is a specimen of the card used :—

Spinster { .....
.....

| PAGE. | | D. — M. | YEAR. | | |
|---|---|---|---|---|---|
| ..... | Born | ..... | 1..... | | |
| ..... | Married (1) | ..... | 18..... | ..... | ..... |
| ..... | 1st Child | ..... | 18..... | ..... | ..... |
| ..... | Widowed | ..... | 18..... | ..... | ..... |
| ..... | Married (2) | ..... | 18..... | ..... | ..... |
| ..... | 1st Child | ..... | 18..... | ..... | ..... |
| ..... | Died | ..... | ..... | ..... | ..... |

..... Father's Title.....

..... 1st Husband.....

..... 2nd Husband.....

It will be observed that the card provides for the date of birth of the first child, and for the dates of subsequent marriages, if

TABLE 1.—*Females—Mortality Data.*

| Age. $x$ | Females living at age $x$. | Of whom | | Age. $x$ | Females living at age $x$. | Of whom | |
|---|---|---|---|---|---|---|---|
| | | Died between $x$ and $x+1$. | Existing at age $x+1$. | | | Died between $x$ and $x+1$. | Existing at age $x+1$. |
| | | | | B. f. | 153,943 | 1,174 | 1,600 |
| 0 | 4435 | 229 | 9 | 50 | 1661 | 12 | 42 |
| 1 | 4197 | 63 | 7 | 51 | 1607 | 18 | 37 |
| 2 | 4127 | 34 | 10 | 52 | 1552 | 18 | 36 |
| 3 | 4083 | 25 | 10 | 53 | 1498 | 19 | 37 |
| 4 | 4048 | 24 | 14 | 54 | 1442 | 15 | 33 |
| 5 | 4010 | 15 | 16 | 55 | 1394 | 24 | 37 |
| 6 | 3979 | 10 | 18 | 56 | 1333 | 20 | 24 |
| 7 | 3951 | 23 | 18 | 57 | 1289 | 12 | 35 |
| 8 | 3910 | 15 | 17 | 58 | 1242 | 21 | 25 |
| 9 | 3878 | 14 | 17 | 59 | 1196 | 18 | 46 |
| 10 | 3847 | 13 | 15 | 60 | 1132 | 25 | 33 |
| 11 | 3819 | 17 | 19 | 61 | 1074 | 17 | 22 |
| 12 | 3783 | 18 | 24 | 62 | 1035 | 17 | 29 |
| 13 | 3741 | 11 | 25 | 63 | 989 | 19 | 20 |
| 14 | 3705 | 14 | 24 | 64 | 950 | 24 | 24 |
| 15 | 3667 | 15 | 35 | 65 | 902 | 23 | 30 |
| 16 | 3617 | 14 | 29 | 66 | 849 | 27 | 29 |
| 17 | 3574 | 20 | 37 | 67 | 793 | 16 | 21 |
| 18 | 3517 | 16 | 42 | 68 | 756 | 27 | 19 |
| 19 | 3459 | 27 | 26 | 69 | 710 | 29 | 24 |
| 20 | 3406 | 15 | 43 | 70 | 657 | 33 | 22 |
| 21 | 3348 | 18 | 39 | 71 | 602 | 29 | 20 |
| 22 | 3291 | 15 | 47 | 72 | 553 | 25 | 31 |
| 23 | 3229 | 23 | 28 | 73 | 497 | 22 | 16 |
| 24 | 3178 | 17 | 29 | 74 | 459 | 22 | 11 |
| 25 | 3132 | 20 | 41 | 75 | 426 | 27 | 12 |
| 26 | 3071 | 19 | 44 | 76 | 387 | 29 | 13 |
| 27 | 3008 | 17 | 46 | 77 | 345 | 36 | 11 |
| 28 | 2945 | 21 | 54 | 78 | 298 | 25 | 14 |
| 29 | 2870 | 19 | 31 | 79 | 259 | 15 | 8 |
| 30 | 2820 | 17 | 39 | 80 | 236 | 22 | 6 |
| 31 | 2764 | 25 | 44 | 81 | 208 | 23 | 13 |
| 32 | 2695 | 24 | 38 | 82 | 172 | 25 | 9 |
| 33 | 2633 | 21 | 34 | 83 | 138 | 21 | 6 |
| 34 | 2578 | 17 | 47 | 84 | 111 | 12 | 8 |
| 35 | 2514 | 17 | 42 | 85 | 91 | 15 | 3 |
| 36 | 2455 | 29 | 41 | 86 | 73 | 13 | 6 |
| 37 | 2385 | 18 | 47 | 87 | 54 | 9 | 3 |
| 38 | 2320 | 15 | 33 | 88 | 42 | 9 | 1 |
| 39 | 2272 | 16 | 35 | 89 | 32 | 5 | 2 |
| 40 | 2221 | 25 | 22 | 90 | 25 | 7 | ... |
| 41 | 2174 | 18 | 44 | 91 | 18 | 4 | 1 |
| 42 | 2112 | 14 | 43 | 92 | 13 | 2 | 1 |
| 43 | 2055 | 19 | 39 | 93 | 10 | 1 | 1 |
| 44 | 1997 | 18 | 39 | 94 | 8 | 1 | ... |
| 45 | 1940 | 15 | 45 | 95 | 7 | ... | 2 |
| 46 | 1880 | 16 | 34 | 96 | 5 | 1 | 2 |
| 47 | 1830 | 14 | 50 | 97 | 2 | ... | ... |
| 48 | 1766 | 23 | 36 | 98 | 2 | 1 | ... |
| 49 | 1707 | 12 | 34 | 99 | 1 | 1 | ... |
| C. f. | 153,943 | 1,174 | 1,600 | Total | 188,078 | 2,010 | 2,425 |

any, but I do not on this occasion deal with probabilities of issue and remarriage, for which these facts would be required. Table 1 (p. 260) shows the tabulation of the data for females, so far as relates to mortality only.

From this table we can deduce the probability of a female aged $x$ dying in a year. For example, we see that at age 15, 3667 females were alive, of whom 15 died between the ages of 15 and 16, and 35 were existing at age 16. The ages of the existing in all cases are taken as age nearest birthday at 31 December 1898, or, what is the same thing, are treated as under observation for the nearest integral number of years, seeing that all the lives start from age 0. The probability of a female of 15 dying within a year is therefore $15 \div 3667$, or ·0041. Having obtained the probabilities of dying in a year for all ages, I graduated these by the graphic method. The following Table 2 shows the unadjusted and adjusted $q_x$, and the comparison of the expected and actual deaths.

Having obtained the adjusted probability of dying for females, it will be interesting to compare it with the probabilities by some well-known existing tables. This is done in Table 3 (p. 264). It will be observed that from ages 5 to 77 inclusive the rate of mortality as deduced by me is lighter than that shown by any of the other tables. From ages 0 to 4 inclusive it closely resembles the mortality of Dr. Thiele's table, and is much lighter than that shown by the Carlisle and the English No. 3 Females tables. From age 77 onwards the rate is higher than that of Dr. Thiele's table, but continues lighter than the mortality shown by any of the other tables until age 93 is reached, where it is approximately equal to the Carlisle, and thereafter it is greater.

The next step was to construct a mortality table for females; and in order that the number living at age 15 (being the earliest marrying age) may be 1,000,000, I worked backwards from $l_{15} = 1{,}000{,}000$ to $l_0 = 1{,}135{,}588$; and the result is shown in Table 4 (p. 265):—

TABLE 2.—*Females—Probability of Dying in a Year, and Comparison of Expected and Actual Deaths.*

| Age. $x$ | At Risk. | $q_x$. Un-adjusted. | $q_x$. Adjusted. | Deaths. Actual. | Deaths. Expected. | Difference. (6)-(5). | Difference. Accumulated. | Age. $x$ |
|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 0 | 4485 | ·0516 | ·0516 | 229 | 228·85 | − ·15 | − ·15 | 0 |
| 1 | 4197 | ·0150 | ·0151 | 63 | 63·38 | + ·38 | + ·23 | 1 |
| 2 | 4127 | ·0082 | ·0082 | 34 | 33·84 | − ·16 | + ·07 | 2 |
| 3 | 4083 | ·0061 | ·0062 | 25 | 25·32 | + ·32 | + ·39 | 3 |
| 4 | 4048 | ·0059 | ·0053 | 24 | 21·45 | −2·55 | −2·16 | 4 |
| 5 | 4010 | ·0037 | ·0046 | 15 | 18·45 | +3·45 | +1·29 | 5 |
| 6 | 3979 | ·0025 | ·0041 | 10 | 16·31 | +6·31 | +7·60 | 6 |
| 7 | 3951 | ·0058 | ·0040 | 23 | 15·80 | −7·20 | + ·40 | 7 |
| 8 | 3910 | ·0038 | ·0039 | 15 | 15·25 | + ·25 | + ·65 | 8 |
| 9 | 3878 | ·0036 | ·0038 | 14 | 14·74 | + ·74 | +1·39 | 9 |
| 10 | 3847 | ·0034 | ·0037 | 13 | 14·23 | +1·23 | +2·62 | 10 |
| 11 | 3819 | ·0045 | ·0036 | 17 | 13·75 | −3·25 | − ·63 | 11 |
| 12 | 3783 | ·0048 | ·0037 | 18 | 14·00 | −4·00 | −4·63 | 12 |
| 13 | 3741 | ·0029 | ·0038 | 11 | 14·22 | +3·22 | −1·41 | 13 |
| 14 | 3705 | ·0038 | ·0039 | 14 | 14·45 | + ·45 | − ·96 | 14 |
| 15 | 3667 | ·0041 | ·0041 | 15 | 15·40 | + ·40 | − ·56 | 15 |
| 16 | 3617 | ·0039 | ·0046 | 14 | 16·64 | +2·64 | +2·08 | 16 |
| 17 | 3574 | ·0056 | ·0050 | 20 | 17·87 | −2·13 | − ·05 | 17 |
| 18 | 3517 | ·0046 | ·0053 | 16 | 18·64 | +2·64 | +2·59 | 18 |
| 19 | 3459 | ·0078 | ·0056 | 27 | 19·47 | −7·53 | −4·94 | 19 |
| 20 | 3406 | ·0044 | ·0058 | 15 | 19·74 | +4·74 | − ·20 | 20 |
| 21 | 3348 | ·0054 | ·0059 | 18 | 19·75 | +1·75 | +1·55 | 21 |
| 22 | 3291 | ·0046 | ·0059 | 15 | 19·42 | +4·42 | +5·97 | 22 |
| 23 | 3229 | ·0071 | ·0060 | 23 | 19·37 | −3·63 | +2·34 | 23 |
| 24 | 3178 | ·0054 | ·0060 | 17 | 19·07 | +2·07 | +4·41 | 24 |
| 25 | 3132 | ·0064 | ·0061 | 20 | 19·11 | − ·89 | +3·52 | 25 |
| 26 | 3071 | ·0062 | ·0061 | 19 | 18·73 | − ·27 | +3·25 | 26 |
| 27 | 3008 | ·0057 | ·0062 | 17 | 18·65 | +1·65 | +4·90 | 27 |
| 28 | 2945 | ·0071 | ·0063 | 21 | 18·55 | −2·45 | +2·45 | 28 |
| 29 | 2870 | ·0066 | ·0065 | 19 | 18·66 | − ·34 | +2·11 | 29 |
| 30 | 2820 | ·0060 | ·0068 | 17 | 19·18 | +2·18 | +4·29 | 30 |
| 31 | 2764 | ·0090 | ·0072 | 25 | 19·90 | −5·10 | − ·81 | 31 |
| 32 | 2695 | ·0089 | ·0075 | 24 | 20·21 | −3·79 | −4·60 | 32 |
| 33 | 2633 | ·0080 | ·0077 | 21 | 20·27 | − ·73 | −5·33 | 33 |
| 34 | 2578 | ·0066 | ·0078 | 17 | 20·11 | +3·11 | −2·22 | 34 |
| 35 | 2514 | ·0068 | ·0079 | 17 | 19·86 | +2·86 | + ·64 | 35 |
| 36 | 2455 | ·0118 | ·0080 | 29 | 19·64 | −9·36 | −8·72 | 36 |
| 37 | 2385 | ·0076 | ·0081 | 18 | 19·32 | +1·32 | −7·40 | 37 |
| 38 | 2320 | ·0065 | ·0082 | 15 | 19·02 | +4·02 | −3·38 | 38 |
| 39 | 2272 | ·0070 | ·0083 | 16 | 18·86 | +2·86 | − ·52 | 39 |
| 40 | 2221 | ·0113 | ·0084 | 25 | 18·66 | −6·34 | −6·86 | 40 |
| 41 | 2174 | ·0083 | ·0085 | 18 | 18·48 | + ·48 | −6·38 | 41 |
| 42 | 2112 | ·0066 | ·0086 | 14 | 18·16 | +4·16 | −2·22 | 42 |
| 43 | 2055 | ·0092 | ·0087 | 19 | 17·88 | −1·12 | −3·34 | 43 |
| 44 | 1997 | ·0090 | ·0088 | 18 | 17·57 | − ·43 | −3·77 | 44 |
| 45 | 1940 | ·0077 | ·0088 | 15 | 17·07 | +2·07 | −1·70 | 45 |
| 46 | 1880 | ·0085 | ·0089 | 16 | 16·73 | + ·73 | − ·97 | 46 |
| 47 | 1830 | ·0077 | ·0090 | 14 | 16·47 | +2·47 | +1·50 | 47 |
| 48 | 1766 | ·0130 | ·0092 | 23 | 16·25 | −6·75 | −5·25 | 48 |
| 49 | 1707 | ·0070 | ·0095 | 12 | 16·22 | +4·22 | −1·03 | 49 |
| 50 | 1661 | ·0072 | ·0099 | 12 | 16·44 | +4·44 | +3·41 | 50 |
| C. f. | 155,604 | ..... | ..... | 1,186 | 1,189·41 | +3·41 | ..... | C. f. |

TABLE 2.—*Continued.*

| Age. $x$ | At Risk. | $q_x$. Unadjusted. | $q_x$. Adjusted. | Deaths. Actual. | Deaths. Expected. | Difference. (6) - (5). | Difference. Accumulated. | Age. $x$ |
|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| B. f. | 155,604 | ..... | ..... | 1,186 | 1,189·41 | ..... | ..... | B. f. |
| 51 | 1607 | ·0112 | ·0105 | 18 | 16·87 | − 1·13 | +2·28 | 51 |
| 52 | 1552 | ·0116 | ·0112 | 18 | 17·38 | − ·62 | +1·66 | 52 |
| 53 | 1498 | ·0127 | ·0119 | 19 | 17·83 | − 1·17 | + ·49 | 53 |
| 54 | 1442 | ·0104 | ·0127 | 15 | 18·31 | + 3·31 | +3·80 | 54 |
| 55 | 1394 | ·0172 | ·0135 | 24 | 18·82 | − 5·18 | −1·38 | 55 |
| 56 | 1333 | ·0150 | ·0143 | 20 | 19·06 | − ·94 | −2·32 | 56 |
| 57 | 1289 | ·0093 | ·0151 | 12 | 19·46 | + 7·46 | +5·14 | 57 |
| 58 | 1242 | ·0169 | ·0159 | 21 | 19·75 | − 1·25 | +3·89 | 58 |
| 59 | 1196 | ·0151 | ·0167 | 18 | 19·97 | + 1·97 | +5·86 | 59 |
| 60 | 1132 | ·0221 | ·0175 | 25 | 19·81 | − 5·19 | + ·67 | 60 |
| 61 | 1074 | ·0158 | ·0184 | 17 | 19·76 | + 2·76 | +3·43 | 61 |
| 62 | 1035 | ·0164 | ·0194 | 17 | 20·08 | + 3·08 | +6·51 | 62 |
| 63 | 989 | ·0192 | ·0206 | 19 | 20·37 | + 1·37 | +7·88 | 63 |
| 64 | 950 | ·0253 | ·0221 | 24 | 21·00 | − 3·00 | +4·88 | 64 |
| 65 | 902 | ·0255 | ·0241 | 23 | 21·74 | − 1·26 | +3·62 | 65 |
| 66 | 849 | ·0318 | ·0266 | 27 | 22·58 | − 4·42 | − ·80 | 66 |
| 67 | 793 | ·0202 | ·0296 | 16 | 23·47 | + 7·47 | +6·67 | 67 |
| 68 | 756 | ·0357 | ·0331 | 27 | 25·02 | − 1·98 | +4·69 | 68 |
| 69 | 710 | ·0408 | ·0369 | 29 | 26·20 | − 2·80 | +1·89 | 69 |
| 70 | 657 | ·0502 | ·0409 | 33 | 26·87 | − 6·13 | −4·24 | 70 |
| 71 | 602 | ·0482 | ·0451 | 29 | 27·15 | − 1·85 | −6·09 | 71 |
| 72 | 553 | ·0452 | ·0495 | 25 | 27·37 | + 2·37 | −3·72 | 72 |
| 73 | 497 | ·0443 | ·0541 | 22 | 26·89 | + 4·89 | +1·17 | 73 |
| 74 | 459 | ·0479 | ·0589 | 22 | 26·03 | + 4·03 | +5·20 | 74 |
| 75 | 426 | ·0634 | ·0639 | 27 | 27·22 | + ·22 | +5·42 | 75 |
| 76 | 387 | ·0749 | ·0692 | 29 | 26·78 | − 2·22 | +3·20 | 76 |
| 77 | 345 | ·1044 | ·0748 | 36 | 25·81 | −10·19 | −6·99 | 77 |
| 78 | 298 | ·0839 | ·0808 | 25 | 24·08 | − ·92 | −7·91 | 78 |
| 79 | 259 | ·0579 | ·0880 | 15 | 22·79 | + 7·79 | − ·12 | 79 |
| 80 | 236 | ·0932 | ·0970 | 22 | 22·89 | + ·89 | + ·77 | 80 |
| 81 | 208 | ·1106 | ·1070 | 23 | 22·26 | − ·74 | + ·03 | 81 |
| 82 | 172 | ·1454 | ·1170 | 25 | 20·12 | − 4·88 | −4·85 | 82 |
| 83 | 138 | ·1522 | ·1275 | 21 | 17·60 | − 3·40 | −8·25 | 83 |
| 84 | 111 | ·1081 | ·1383 | 12 | 15·35 | + 3·35 | −4·90 | 84 |
| 85 | 91 | ·1648 | ·1492 | 15 | 13·58 | − 1·42 | −6·32 | 85 |
| 86 | 73 | ·1781 | ·1611 | 13 | 11·76 | − 1·24 | −7·56 | 86 |
| 87 | 54 | ·1667 | ·1750 | 9 | 9·45 | + ·45 | −7·11 | 87 |
| 88 | 42 | ·2143 | ·1905 | 9 | 8·00 | − 1·00 | −8·11 | 88 |
| 89 | 32 | ·1563 | ·2060 | 5 | 6·59 | + 1·59 | −6·52 | 89 |
| 90 | 25 | ·2800 | ·2220 | 7 | 5·55 | − 1·45 | −7·97 | 90 |
| 91 | 18 | ·2222 | ·2360 | 4 | 4·25 | + ·25 | −7·72 | 91 |
| 92 | 13 | ·1539 | ·2490 | 2 | 3·23 | + 1·23 | −6·49 | 92 |
| 93 | 10 | ·1000 | ·2610 | 1 | 2·61 | + 1·61 | −4·88 | 93 |
| 94 | 8 | ·1250 | ·2730 | 1 | 2·18 | + 1·18 | −3·70 | 94 |
| 95 | 7 | ·0000 | ·2860 | ... | 2·00 | + 2·00 | −1·70 | 95 |
| 96 | 5 | ·2000 | ·3060 | 1 | 1·53 | + ·53 | −1·17 | 96 |
| 97 | 2 | ·0000 | ·3700 | ... | ·74 | + ·74 | − ·43 | 97 |
| 98 | 2 | ·5000 | ·5400 | 1 | 1·08 | + ·08 | − ·35 | 98 |
| 99 | 1 | 1·0000 | 1·0000 | 1 | 1·00 | ·00 | − ·35 | 99 |
| Total | 183,078 | ..... | ..... | 2,010 | 2,009·65 | − ·35 | ..... | Total |

TABLE 3.—*Females—Comparison of $q_x$ with other Tables.*

| Age. $x$ | Carlisle. | English No. 3, Females | Government Annuitants, Females | British Peerage, Females. Thiele. | British Peerage, Females. Lees. | Age. $x$ | Carlisle. | English No. 3, Females | Government Annuitants, Females | British Peerage, Females. Thiele. | British Peerage, Females. Lees. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ·15390 | ·13471 | Not available under Age 50. | ·0597 | ·0516 | 50 | ·01342 | ·01622 | ·01123 | ·0141 | ·0099 |
| 1 | ·08061 | ·06192 | | ·0164 | ·0151 | 51 | ·01429 | ·01667 | ·01211 | ·0147 | ·0105 |
| 2 | ·06492 | ·03538 | | ·0070 | ·0082 | 52 | ·01520 | ·01714 | ·01273 | ·0154 | ·0112 |
| 3 | ·03794 | ·02418 | | ·0052 | ·0062 | 53 | ·01615 | ·01764 | ·01379 | ·0162 | ·0119 |
| 4 | ·02872 | ·01769 | | ·0048 | ·0053 | 54 | ·01690 | ·01968 | ·01495 | ·0172 | ·0127 |
| 5 | ·01780 | ·01328 | | ·0048 | ·0046 | 55 | ·01792 | ·02098 | ·01585 | ·0184 | ·0135 |
| 6 | ·01228 | ·01056 | | ·0049 | ·0041 | 56 | ·01900 | ·02235 | ·01668 | ·0196 | ·0143 |
| 7 | ·00880 | ·00908 | | ·0049 | ·0040 | 57 | ·02090 | ·02380 | ·01791 | ·0209 | ·0151 |
| 8 | ·00658 | ·00768 | | ·0050 | ·0039 | 58 | ·02421 | ·02535 | ·01882 | ·0223 | ·0159 |
| 9 | ·00508 | ·00662 | | ·0052 | ·0038 | 59 | ·02827 | ·02703 | ·01953 | ·0237 | ·0167 |
| 10 | ·00449 | ·00585 | | ·0054 | ·0037 | 60 | ·03349 | ·02883 | ·02050 | ·0252 | ·0175 |
| 11 | ·00482 | ·00536 | | ·0056 | ·0036 | 61 | ·03579 | ·03086 | ·02166 | ·0268 | ·0184 |
| 12 | ·00500 | ·00511 | | ·0059 | ·0037 | 62 | ·03741 | ·03307 | ·02283 | ·0287 | ·0194 |
| 13 | ·00518 | ·00508 | | ·0062 | ·0038 | 63 | ·03825 | ·03550 | ·02471 | ·0308 | ·0206 |
| 14 | ·00553 | ·00524 | | ·0065 | ·0039 | 64 | ·03977 | ·03818 | ·02695 | ·0331 | ·0221 |
| 15 | ·00619 | ·00556 | | ·0069 | ·0041 | 65 | ·04109 | ·04113 | ·02944 | ·0358 | ·0241 |
| 16 | ·00671 | ·00600 | | ·0072 | ·0046 | 66 | ·04250 | ·04436 | ·03212 | ·0386 | ·0266 |
| 17 | ·00691 | ·00656 | | ·0076 | ·0050 | 67 | ·04439 | ·04791 | ·03489 | ·0414 | ·0296 |
| 18 | ·00696 | ·00718 | | ·0078 | ·0053 | 68 | ·04645 | ·05178 | ·03753 | ·0443 | ·0331 |
| 19 | ·00701 | ·00787 | | ·0080 | ·0056 | 69 | ·04911 | ·05600 | ·04086 | ·0471 | ·0369 |
| 20 | ·00706 | ·00857 | | ·0081 | ·0058 | 70 | ·05165 | ·06058 | ·04457 | ·0498 | ·0409 |
| 21 | ·00695 | ·00879 | | ·0081 | ·0059 | 71 | ·05885 | ·06556 | ·04931 | ·0525 | ·0451 |
| 22 | ·00699 | ·00901 | | ·0082 | ·0059 | 72 | ·06813 | ·07095 | ·05492 | ·0553 | ·0495 |
| 23 | ·00704 | ·00921 | | ·0082 | ·0060 | 73 | ·07812 | ·07675 | ·06154 | ·0584 | ·0541 |
| 24 | ·00709 | ·00942 | | ·0082 | ·0060 | 74 | ·09017 | ·08299 | ·06825 | ·0619 | ·0589 |
| 25 | ·00731 | ·00961 | | ·0083 | ·0061 | 75 | ·09552 | ·08967 | ·07548 | ·0659 | ·0639 |
| 26 | ·00737 | ·00981 | | ·0084 | ·0061 | 76 | ·10297 | ·09682 | ·08239 | ·0704 | ·0692 |
| 27 | ·00777 | ·01000 | | ·0085 | ·0062 | 77 | ·10743 | ·10443 | ·08994 | ·0753 | ·0748 |
| 28 | ·00870 | ·01019 | | ·0086 | ·0063 | 78 | ·10882 | ·11253 | ·09743 | ·0806 | ·0808 |
| 29 | ·00983 | ·01039 | | ·0088 | ·0065 | 79 | ·11841 | ·12112 | ·10618 | ·0859 | ·0880 |
| 30 | ·01010 | ·01058 | | ·0089 | ·0068 | 80 | ·12172 | ·13020 | ·11534 | ·0912 | ·0970 |
| 31 | ·01021 | ·01076 | | ·0090 | ·0072 | 81 | ·13381 | ·13978 | ·12651 | ·0965 | ·1070 |
| 32 | ·01013 | ·01097 | | ·0093 | ·0075 | 82 | ·14069 | ·14986 | ·13743 | ·1020 | ·1170 |
| 33 | ·01005 | ·01116 | | ·0095 | ·0077 | 83 | ·15088 | ·16044 | ·15043 | ·1078 | ·1275 |
| 34 | ·01015 | ·01137 | | ·0097 | ·0078 | 84 | ·15879 | ·17153 | ·16376 | ·1145 | ·1383 |
| 35 | ·01026 | ·01158 | | ·0101 | ·0079 | 85 | ·17528 | ·18312 | ·17983 | ·1223 | ·1492 |
| 36 | ·01055 | ·01179 | | ·0104 | ·0080 | 86 | ·19346 | ·19520 | ·19391 | ·1316 | ·1611 |
| 37 | ·01086 | ·01202 | | ·0107 | ·0081 | 87 | ·21622 | ·20777 | ·21294 | ·1426 | ·1750 |
| 38 | ·01117 | ·01227 | | ·0110 | ·0082 | 88 | ·21983 | ·22082 | ·23112 | ·1557 | ·1905 |
| 39 | ·01188 | ·01252 | | ·0112 | ·0083 | 89 | ·21547 | ·23433 | ·25082 | ·1705 | ·2060 |
| 40 | ·01301 | ·01277 | | ·0114 | ·0084 | 90 | ·26056 | ·24831 | ·26632 | ·1869 | ·2220 |
| 41 | ·01378 | ·01304 | | ·0115 | ·0085 | 91 | ·28571 | ·26271 | ·29102 | ·2049 | ·2360 |
| 42 | ·01437 | ·01332 | | ·0116 | ·0086 | 92 | ·28000 | ·27755 | ·30731 | ·2239 | ·2490 |
| 43 | ·01458 | ·01363 | | ·0117 | ·0087 | 93 | ·25926 | ·29278 | ·32689 | ·2436 | ·2610 |
| 44 | ·01480 | ·01373 | | ·0118 | ·0088 | 94 | ·25000 | ·30840 | ·34211 | ·2642 | ·2730 |
| 45 | ·01481 | ·01427 | | ·0121 | ·0088 | 95 | ·23333 | ·32437 | ·37818 | ·2850 | ·2860 |
| 46 | ·01482 | ·01462 | | ·0124 | ·0089 | 96 | ·21739 | ·34069 | ·40058 | ·3060 | ·3060 |
| 47 | ·01460 | ·01499 | | ·0127 | ·0090 | 97 | ·22222 | ·35731 | ... | ·3263 | ·3700 |
| 48 | ·01394 | ·01538 | | ·0131 | ·0092 | 98 | ·21429 | ·37421 | ... | ·3458 | ·5400 |
| 49 | ·01368 | ·01578 | | ·0135 | ·0095 | 99 | ·18182 | ·39136 | ... | ... | 1·0000 |

TABLE 4.—*Females—Mortality Table.*

| $x$ | $l_x$ | $log\ l_x$ | $d_x$ | $x$ | $l_x$ | $log\ l_x$ | $d_x$ |
|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| 0 | 1,135,588 | 6·05522 | 58,599 | 50 | 776,895 | 5·89036 | 7,692 |
| 1 | 1,076,989 | 6·03221 | 16,257 | 51 | 769,203 | 5·88604 | 8,080 |
| 2 | 1,060,732 | 6·02561 | 8,694 | 52 | 761,123 | 5·88145 | 8,522 |
| 3 | 1,052,038 | 6·02203 | 6,523 | 53 | 752,601 | 5·87656 | 8,955 |
| 4 | 1,045,515 | 6·01933 | 5,537 | 54 | 743,646 | 5·87137 | 9,444 |
| 5 | 1,039,978 | 6·01702 | 4,783 | 55 | 734,202 | 5·86582 | 9,910 |
| 6 | 1,035,195 | 6·01502 | 4,246 | 56 | 724,292 | 5·85991 | 10,355 |
| 7 | 1,030,949 | 6·01324 | 4,126 | 57 | 713,937 | 5·85366 | 10,777 |
| 8 | 1,026,823 | 6·01150 | 4,007 | 58 | 703,160 | 5·84705 | 11,182 |
| 9 | 1,022,816 | 6·00980 | 3,889 | 59 | 691,978 | 5·84009 | 11,556 |
| 10 | 1,018,927 | 6·00814 | 3,773 | 60 | 680,422 | 5·83278 | 11,904 |
| 11 | 1,015,154 | 6·00653 | 3,658 | 61 | 668,518 | 5·82511 | 12,300 |
| 12 | 1,011,496 | 6·00496 | 3,746 | 62 | 656,218 | 5·81705 | 12,730 |
| 13 | 1,007,750 | 6·00335 | 3,832 | 63 | 643,488 | 5·80854 | 13,253 |
| 14 | 1,003,918 | 6·00170 | 3,918 | 64 | 630,235 | 5·79950 | 13,930 |
| 15 | 1,000,000 | 6·00000 | 4,102 | 65 | 616,305 | 5·78980 | 14,850 |
| 16 | 995,898 | 5·99821 | 4,580 | 66 | 601,455 | 5·77920 | 16,003 |
| 17 | 991,318 | 5·99621 | 4,921 | 67 | 585,452 | 5·76749 | 17,327 |
| 18 | 986,397 | 5·99405 | 5,257 | 68 | 568,125 | 5·75444 | 18,807 |
| 19 | 981,140 | 5·99173 | 5,498 | 69 | 549,318 | 5·73982 | 20,268 |
| 20 | 975,642 | 5·98929 | 5,662 | 70 | 529,050 | 5·72350 | 21,637 |
| 21 | 969,980 | 5·98676 | 5,726 | 71 | 507,413 | 5·70536 | 22,884 |
| 22 | 964,254 | 5·98419 | 5,739 | 72 | 484,529 | 5·68532 | 23,986 |
| 23 | 958,515 | 5·98160 | 5,753 | 73 | 460,543 | 5·66327 | 24,918 |
| 24 | 952,762 | 5·97898 | 5,765 | 74 | 435,625 | 5·63911 | 25,660 |
| 25 | 946,997 | 5·97635 | 5,778 | 75 | 409,965 | 5·61275 | 26,198 |
| 26 | 941,219 | 5·97369 | 5,790 | 76 | 383,767 | 5·58407 | 26,557 |
| 27 | 935,429 | 5·97101 | 5,800 | 77 | 357,210 | 5·55292 | 26,721 |
| 28 | 929,629 | 5·96831 | 5,857 | 78 | 330,489 | 5·51916 | 26,703 |
| 29 | 923,772 | 5·96557 | 6,003 | 79 | 303,786 | 5·48257 | 26,733 |
| 30 | 917,769 | 5·96273 | 6,238 | 80 | 277,053 | 5·44256 | 26,876 |
| 31 | 911,531 | 5·95977 | 6,567 | 81 | 250,177 | 5·39825 | 26,769 |
| 32 | 904,964 | 5·95663 | 6,789 | 82 | 223,408 | 5·34910 | 26,139 |
| 33 | 898,175 | 5·95336 | 6,916 | 83 | 197,269 | 5·29506 | 25,151 |
| 34 | 891,259 | 5·95000 | 6,951 | 84 | 172,118 | 5·23583 | 23,803 |
| 35 | 884,308 | 5·94660 | 6,985 | 85 | 148,315 | 5·17119 | 22,129 |
| 36 | 877,323 | 5·94316 | 7,017 | 86 | 126,186 | 5·10101 | 20,329 |
| 37 | 870,306 | 5·93967 | 7,047 | 87 | 105,857 | 5·02472 | 18,526 |
| 38 | 863,259 | 5·93614 | 7,075 | 88 | 87,331 | 4·94117 | 16,638 |
| 39 | 856,184 | 5·93257 | 7,103 | 89 | 70,693 | 4·84938 | 14,564 |
| 40 | 849,081 | 5·92895 | 7,136 | 90 | 56,129 | 4·74919 | 12,459 |
| 41 | 841,945 | 5·92528 | 7,160 | 91 | 43,670 | 4·64018 | 10,307 |
| 42 | 834,785 | 5·92157 | 7,181 | 92 | 33,363 | 4·52327 | 8,307 |
| 43 | 827,604 | 5·91782 | 7,202 | 93 | 25,056 | 4·39891 | 6,540 |
| 44 | 820,402 | 5·91403 | 7,220 | 94 | 18,516 | 4·26755 | 5,055 |
| 45 | 813,182 | 5·91019 | 7,197 | 95 | 13,461 | 4·12908 | 3,849 |
| 46 | 805,985 | 5·90633 | 7,174 | 96 | 9,612 | 3·98281 | 2,941 |
| 47 | 798,811 | 5·90244 | 7,188 | 97 | 6,671 | 3·82419 | 2,468 |
| 48 | 791,623 | 5·89852 | 7,281 | 98 | 4,203 | 3·62356 | 2,269 |
| 49 | 784,342 | 5·89451 | 7,447 | 99 | 1,934 | 3·28646 | 1,934 |

In dealing with the rate of mortality among life-tenants, Mr. Campbell in his recent paper (*T.F.A.*, i. 91) states that "the "importance of having some knowledge of the rate of mortality "among life-tenants is evident, for, without such knowledge, the "most suitable table by which to value reversions can only be a "matter of conjecture". He gave the result of an investigation which he had made into the rate of mortality among life-tenants, and showed that the rates deduced are at the high ages in close agreement with the rate of mortality among peerage females as given by Dr. Thiele. It will therefore be interesting to compare Mr. Campbell's results with those brought out in Table 2. The probabilities of dying in a year at ages 58 to 86 inclusive, being the ages dealt with by Mr. Campbell, are as follows:—

TABLE 5.

| Age. $x$ | $q_x$. Life-tenants. Campbell. | $q_x$. Peerage females. Lees. | Age. $x$ | $q_x$. Life-tenants. Campbell. | $q_x$. Peerage females. Lees. | Age. $x$ | $q_x$. Life-tenants. Campbell. | $q_x$. Peerage females. Lees. |
|---|---|---|---|---|---|---|---|---|
| 58 | ·016 | ·0159 | 68 | ·037 | ·0331 | 78 | ·078 | ·0808 |
| 59 | ·017 | ·0167 | 69 | ·040 | ·0369 | 79 | ·083 | ·0880 |
| 60 | ·018 | ·0175 | 70 | ·043 | ·0409 | 80 | ·088 | ·0970 |
| 61 | ·019 | ·0184 | 71 | ·046 | ·0451 | 81 | ·093 | ·1070 |
| 62 | ·021 | ·0194 | 72 | ·050 | ·0495 | 82 | ·100 | ·1170 |
| 63 | ·023 | ·0206 | 73 | ·054 | ·0541 | 83 | ·107 | ·1275 |
| 64 | ·025 | ·0221 | 74 | ·059 | ·0589 | 84 | ·116 | ·1383 |
| 65 | ·028 | ·0241 | 75 | ·063 | ·0639 | 85 | ·126 | ·1492 |
| 66 | ·031 | ·0266 | 76 | ·068 | ·0692 | 86 | ·138 | ·1611 |
| 67 | ·034 | ·0296 | 77 | ·073 | ·0748 | | | |

The close agreement of the rates here shown is remarkable, and has led me to undertake the calculation of the values of benefits depending on the rate of mortality as deduced by me, in the hope that these may prove useful in connection with the valuation of reversions depending upon female lives. Commutation columns, and the values of annuities and assurances, at 3, 4, and 5 per cent interest, are given in Tables 16 to 19 (see pages 288-292). Taking ages 60, 70, and 80 as examples, the values of reversions to 1000 at 5 per cent interest are as follows:—

| Age. | Carlisle. | Life-tenants. | Peerage females. |
|---|---|---|---|
| 60 | 527 | 458 | 455 |
| 70 | 651 | 593 | 601 |
| 80 | 761 | 713 | 736 |

TABLE 6.—*Spinsters—Marriage and Mortality Data.*

| Age. $x$ | Spinsters living at age $x$. | Of whom: Died unmarried between $x$ and $x+1$. | Married between $x$ and $x+1$. | Existing unmarried at age $x+1$. | Age. $x$ | Spinsters living at age $x$. | Of whom: Died unmarried between $x$ and $x+1$. | Married between $x$ and $x+1$. | Existing unmarried at age $x+1$. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | B. f. | 53,007 | 284 | 2,346 | 742 |
| 15 | 3667 | 15 | 3 | 35 | 56 | 295 | 5 | 3 | 5 |
| 16 | 3614 | 14 | 8 | 29 | 57 | 282 | 6 | 1 | 11 |
| 17 | 3563 | 20 | 49 | 36 | 58 | 264 | 6 | ... | 8 |
| 18 | 3458 | 15 | 114 | 42 | 59 | 250 | 3 | ... | 9 |
| 19 | 3287 | 24 | 176 | 26 | 60 | 238 | 7 | ... | 7 |
| 20 | 3061 | 13 | 219 | 38 | 61 | 224 | 8 | 1 | 4 |
| 21 | 2791 | 13 | 192 | 33 | 62 | 211 | 3 | ... | 5 |
| 22 | 2553 | 9 | 211 | 34 | 63 | 208 | 3 | ... | 5 |
| 23 | 2299 | 12 | 212 | 18 | 64 | 195 | 8 | ... | 8 |
| 24 | 2057 | 8 | 194 | 20 | 65 | 179 | 4 | ... | 6 |
| 25 | 1835 | 8 | 146 | 26 | 66 | 169 | 7 | ... | 7 |
| 26 | 1655 | 6 | 137 | 31 | 67 | 155 | 2 | ... | 2 |
| 27 | 1481 | 7 | 121 | 27 | 68 | 151 | 7 | ... | 4 |
| 28 | 1326 | 6 | 105 | 27 | 69 | 140 | 9 | ... | 3 |
| 29 | 1188 | 4 | 75 | 11 | 70 | 128 | 7 | ... | 4 |
| 30 | 1098 | ... | 60 | 18 | 71 | 117 | 8 | 1 | 1 |
| 31 | 1020 | 8 | 64 | 20 | 72 | 107 | 3 | ... | 6 |
| 32 | 928 | 6 | 41 | 14 | 73 | 98 | 6 | ... | 6 |
| 33 | 867 | 7 | 30 | 6 | 74 | 86 | 3 | ... | 2 |
| 34 | 824 | 5 | 39 | 17 | 75 | 81 | 4 | ... | 1 |
| 35 | 768 | 2 | 20 | 15 | 76 | 76 | 6 | ... | 2 |
| 36 | 726 | 5 | 25 | 13 | 77 | 68 | 6 | ... | 4 |
| 37 | 683 | 3 | 18 | 15 | 78 | 58 | 3 | ... | 1 |
| 38 | 647 | 7 | 11 | 7 | 79 | 54 | 1 | ... | 1 |
| 39 | 622 | 5 | 14 | 7 | 80 | 52 | 6 | ... | ... |
| 40 | 596 | 4 | 15 | 3 | 81 | 46 | 9 | ... | 3 |
| 41 | 574 | 2 | 9 | 17 | 82 | 34 | 4 | ... | 4 |
| 42 | 546 | 3 | 6 | 15 | 83 | 26 | 4 | ... | 2 |
| 43 | 522 | 6 | 8 | 12 | 84 | 20 | 3 | ... | 1 |
| 44 | 496 | 7 | 2 | 7 | 85 | 16 | 1 | ... | ... |
| 45 | 480 | 3 | 5 | 14 | 86 | 15 | 1 | ... | 1 |
| 46 | 458 | 3 | 5 | 7 | 87 | 13 | 5 | ... | ... |
| 47 | 443 | 3 | 2 | 15 | 88 | 8 | 2 | ... | 1 |
| 48 | 423 | 9 | 5 | 11 | 89 | 5 | 1 | ... | ... |
| 49 | 398 | 4 | 2 | 11 | 90 | 4 | 2 | ... | ... |
| 50 | 381 | 5 | ... | 11 | 91 | 2 | .. | ... | ... |
| 51 | 365 | 3 | ... | 11 | 92 | 2 | ... | ... | ... |
| 52 | 351 | 4 | 1 | 8 | 93 | 2 | 1 | ... | ... |
| 53 | 338 | 3 | 2 | 15 | 94 | 1 | ... | ... | ... |
| 54 | 318 | 1 | ... | 12 | 95 | 1 | ... | ... | ... |
| 55 | 305 | 2 | ... | 8 | 96 | 1 | ... | ... | 1 |
| C. f. | 53,007 | 284 | 2,346 | 742 | Total | 57,084 | 448 | 2,352 | 867 |

Having ascertained the number of females living at each age, and the corresponding deaths, the next step was to construct a table showing the relative number of spinsters living, marrying, and dying. It follows, of course, that the difference between the number of females and the number of spinsters living at a given

age, is the number of married women living, it being understood that the term 'married women' includes those who have married but have become widows. The table on p. 267 shows the tabulation of the marriage and mortality data for spinsters.

We have now to ascertain the numbers of spinsters exposed to the risk of dying unmarried in a year at each age. This is obtained by deducting from the numbers who entered upon a year of life, half the number who married during that year. For example, we find from Table 6 that 3061 spinsters entered upon a year of life at age 20, of whom 13 died unmarried, and 219 married, between the ages of 20 and 21; therefore the number at risk of dying unmarried is $3061 - 109{\cdot}5 = 2951{\cdot}5$, and $13 \div 2951{\cdot}5 = {\cdot}0044$, is the annual death-rate among the spinsters who do not marry in the year. Summing in quinquennial groups the numbers at risk obtained in this way and the deaths, I have placed these alongside the corresponding figures deduced from those given on pages 196 and 197 of the volume containing the unadjusted data in the Combined Experience of Life Annuitants (1863-1893), as shown in the following Table 7:—

TABLE 7.—*Unadjusted Rates of Mortality—Comparison, by Quinquennial Groups, with the Combined Experience of Life Annuitants* (1863-1893).

| Ages | Peerage. | | | | | | | | | Life Annuitants, "Old" and "New" Aggregate Table, Females. | | | Ages |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Females. | | | Spinsters. | | | Married. | | | | | | |
| | At Risk. | Deaths. | Rate of Mortality. | At Risk. | Deaths. | Rate of Mortality. | At Risk. | Deaths. | Rate of Mortality. | At Risk. | Deaths. | Rate of Mortality. | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) |
| 15-19 | 17834 | 92 | ·0052 | 17414 | 88 | ·0051 | 420 | 4 | ·0095 | ... | ... | ... | 15-19 |
| 20-24 | 16452 | 88 | ·0054 | 12247 | 55 | ·0045 | 4205 | 33 | ·0079 | 215 | 1 | ·0047 | 20-24 |
| 25-29 | 15026 | 96 | ·0064 | 7193 | 31 | ·0043 | 7833 | 65 | ·0083 | 440 | 4 | ·0091 | 25-29 |
| 30-34 | 13490 | 104 | ·0077 | 4620 | 26 | ·0056 | 8870 | 78 | ·0088 | 792 | 6 | ·0076 | 30-34 |
| 35-39 | 11946 | 95 | ·0080 | 3397 | 22 | ·0065 | 8549 | 73 | ·0085 | 1438 | 17 | ·0118 | 35-39 |
| 40-44 | 10559 | 94 | ·0089 | 2714 | 22 | ·0081 | 7845 | 72 | ·0092 | 3578 | 27 | ·0075 | 40-44 |
| 45-49 | 9123 | 80 | ·0088 | 2192·5 | 22 | ·0100 | 6930·5 | 58 | ·0084 | 4903 | 51 | ·0104 | 45-49 |
| 50-54 | 7760 | 82 | ·0106 | 1751·5 | 16 | ·0091 | 6008·5 | 66 | ·0110 | 9216 | 114 | ·0124 | 50-54 |
| 55-59 | 6454 | 95 | ·0147 | 1394 | 22 | ·0158 | 5060 | 73 | ·0144 | 15896 | 224 | ·0141 | 55-59 |
| 60-64 | 5180 | 102 | ·0203 | 1070·5 | 29 | ·0271 | 4109·5 | 73 | ·0178 | 25540 | 521 | ·0209 | 60-64 |
| 65-69 | 4010 | 122 | ·0304 | 794 | 29 | ·0365 | 3216 | 93 | ·0289 | 32988 | 1089 | ·0332 | 65-69 |
| 70-74 | 2768 | 131 | ·0473 | 535·5 | 27 | ·0504 | 2232·5 | 104 | ·0466 | 32922 | 1628 | ·0493 | 70-74 |
| 75-79 | 1715 | 132 | ·0770 | 337 | 20 | ·0594 | 1378 | 112 | ·0813 | 25145 | 2039 | ·0811 | 75-79 |
| 80-84 | 865 | 103 | ·1191 | 178 | 26 | ·1461 | 687 | 77 | ·1121 | 14270 | 1888 | ·1323 | 80-84 |
| 85-89 | 292 | 51 | ·1747 | 57 | 10 | ·1754 | 235 | 41 | ·1745 | 5502 | 1063 | ·1932 | 85-89 |
| 90-94 | 74 | 15 | ·2027 | 11 | 3 | ·2727 | 63 | 12 | ·1905 | 1377 | 380 | ·2760 | 90-94 |
| 95-99 | 17 | 3 | ·1765 | 2 | ... | ... | 15 | 3 | ·2000 | 169 | 54 | ·3195 | 95-99 |
| 100-104 | ... | ... | ... | ... | ... | ... | ... | ... | ... | 15 | 6 | ·4000 | 100-104 |
| Total | 123,565 | 1,485 | ... | 55,908 | 448 | ... | 67,657 | 1,037 | ... | 174,406 | 9,107 | ... | Total |

Comparing the rates of mortality observable among the peerage females with those of the office annuitants, as shown in columns (4) and (13), it will be noticed that, while the latter are throughout (with the exception of ages 20-24, 30-34, 40-44, and 55-59) greater than the former, at no point is there any great divergence, and that, generally speaking, the rates in the two classes run very closely to each other, bearing in mind that both are unadjusted. The figures given for the married in columns (8) and (9), are the differences between columns (2) and (5), and (3) and (6), respectively. It will be observed that from ages 15 to 44 inclusive the rate of mortality for females is higher than that for spinsters, and of course the difference is still greater among the married; while from age 45 to the end of the table the rate of mortality for females is, with two exceptions (50-54 and 75-79) less than that for spinsters. I can think of no good reason why the mortality among females at ages 50-54 should be higher than among spinsters at these ages. I have therefore assumed this irregularity to be accidental, and have treated it accordingly.

An examination of the table will further show that, while the rate of mortality in the case of females proceeds with considerable regularity up to age 94, there is a distinct tendency, in the case of the spinsters, for the rate to decrease from ages 15 to 29. The higher rate of mortality observable among females as compared with spinsters at these early ages, is clearly due to the dangers of childbearing.

The following Table 8 shows the number of spinsters exposed to risk of death at each age, the unadjusted and adjusted annual death-rate, and the comparison of the expected and actual deaths :—

TABLE 8.—*Spinsters—Probability of Dying in a Year, and Comparison of Expected and Actual Deaths.*

| Age. $x$ | At Risk. | $q_x$ | | Deaths. | | Difference. | | Age. $x$ |
|---|---|---|---|---|---|---|---|---|
| | | Unadjusted. | Adjusted. | Actual. | Expected. | (6)−(5) | Accumulated. | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 15 | 3665·5 | ·0041 | ·0041 | 15 | 15·03 | + ·03 | + ·03 | 15 |
| 16 | 3610 | ·0039 | ·0046 | 14 | 16·61 | +2·61 | +2·64 | 16 |
| 17 | 3588·5 | ·0057 | ·0049 | 20 | 17·34 | −2·66 | − ·02 | 17 |
| 18 | 3401 | ·0044 | ·0052 | 15 | 17·69 | +2·69 | +2·67 | 18 |
| 19 | 3199 | ·0075 | ·0054 | 24 | 17·29 | −6·71 | −4·04 | 19 |
| 20 | 2951·5 | ·0044 | ·0054 | 13 | 15·94 | +2·94 | −1·10 | 20 |
| 21 | 2695 | ·0048 | ·0052 | 13 | 14·01 | +1·01 | − ·09 | 21 |
| 22 | 2447·5 | ·0037 | ·0049 | 9 | 11·99 | +2·99 | +2·90 | 22 |
| 23 | 2193 | ·0055 | ·0047 | 12 | 10·31 | −1·69 | +1·21 | 23 |
| 24 | 1960 | ·0041 | ·0046 | 8 | 9·02 | +1·02 | +2·23 | 24 |
| 25 | 1762 | ·0045 | ·0045 | 8 | 7·93 | − ·07 | +2·16 | 25 |
| 26 | 1586·5 | ·0038 | ·0044 | 6 | 6·98 | + ·98 | +3·14 | 26 |
| 27 | 1420·5 | ·0049 | ·0044 | 7 | 6·25 | − ·75 | +2·39 | 27 |
| 28 | 1278·5 | ·0047 | ·0045 | 6 | 5·73 | − ·27 | +2·12 | 28 |
| 29 | 1150·5 | ·0035 | ·0046 | 4 | 5·29 | +1·29 | +3·41 | 29 |
| 30 | 1068 | ·0000 | ·0047 | 0 | 5·02 | +5·02 | +8·43 | 30 |
| 31 | 988 | ·0081 | ·0049 | 8 | 4·84 | −3·16 | +5·27 | 31 |
| 32 | 907·5 | ·0066 | ·0051 | 6 | 4·63 | −1·37 | +3·90 | 32 |
| 33 | 852 | ·0082 | ·0053 | 7 | 4·52 | −2·48 | +1·42 | 33 |
| 34 | 804·5 | ·0062 | ·0055 | 5 | 4·42 | − ·58 | + ·84 | 34 |
| 35 | 753 | ·0027 | ·0058 | 2 | 4·37 | +2·37 | +3·21 | 35 |
| 36 | 718·5 | ·0070 | ·0061 | 5 | 4·35 | − ·65 | +2·56 | 36 |
| 37 | 674 | ·0045 | ·0064 | 3 | 4·31 | +1·31 | +3·87 | 37 |
| 38 | 641·5 | ·0109 | ·0067 | 7 | 4·30 | −2·70 | +1·17 | 38 |
| 39 | 615 | ·0081 | ·0070 | 5 | 4·31 | − ·69 | + ·48 | 39 |
| 40 | 588·5 | ·0068 | ·0073 | 4 | 4·30 | + ·30 | + ·78 | 40 |
| 41 | 569·5 | ·0035 | ·0076 | 2 | 4·33 | +2·33 | +3·11 | 41 |
| 42 | 543 | ·0055 | ·0078 | 3 | 4·24 | +1·24 | +4·35 | 42 |
| 43 | 518 | ·0116 | ·0080 | 6 | 4·14 | −1·86 | +2·49 | 43 |
| 44 | 495 | ·0141 | ·0082 | 7 | 4·06 | −2·94 | − ·45 | 44 |
| 45 | 477·5 | ·0063 | ·0084 | 3 | 4·01 | +1·01 | + ·56 | 45 |
| 46 | 455·5 | ·0066 | ·0087 | 3 | 3·96 | + ·96 | +1·52 | 46 |
| 47 | 442 | ·0068 | ·0091 | 3 | 4·02 | +1·02 | +2·54 | 47 |
| 48 | 420·5 | ·0214 | ·0096 | 9 | 4·04 | −4·96 | −2·42 | 48 |
| 49 | 397 | ·0101 | ·0103 | 4 | 4·09 | + ·09 | −2·33 | 49 |
| 50 | 381 | ·0131 | ·0110 | 5 | 4·19 | − ·81 | −3·14 | 50 |
| 51 | 365 | ·0082 | ·0117 | 3 | 4·27 | +1·27 | −1·87 | 51 |
| 52 | 350·5 | ·0114 | ·0124 | 4 | 4·35 | + ·35 | −1·52 | 52 |
| 53 | 337 | ·0089 | ·0131 | 3 | 4·42 | +1·42 | − ·10 | 53 |
| 54 | 318 | ·0031 | ·0139 | 1 | 4·41 | +3·41 | +3·31 | 54 |
| 55 | 305 | ·0066 | ·0147 | 2 | 4·48 | +2·48 | +5·79 | 55 |
| 56 | 293·5 | ·0170 | ·0156 | 5 | 4·58 | − ·42 | +5·37 | 56 |
| C. f. | 52,127·5 | ... | ... | 289 | 294·37 | +5·37 | ... | C. f. |

TABLE 8.—*Continued.*

| Age. $x$ | At Risk. | $q_x$. Un-adjusted. | $q_x$. Adjusted. | Deaths. Actual. | Deaths. Expected. | Difference. (6)−(5). | Difference. Accumulated. | Age. $x$ |
|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| B. f. | 52,127·5 | ... | ... | 289 | 294·87 | +5·87 | ... | B. f. |
| 57 | 281·5 | ·0213 | ·0166 | 6 | 4·67 | −1·33 | +4·04 | 57 |
| 58 | 264 | ·0227 | ·0179 | 6 | 4·72 | −1·28 | +2·76 | 58 |
| 59 | 250 | ·0120 | ·0195 | 3 | 4·88 | +1·88 | +4·64 | 59 |
| 60 | 238 | ·0294 | ·0212 | 7 | 5·05 | −1·95 | +2·69 | 60 |
| 61 | 223·5 | ·0358 | ·0230 | 8 | 5·14 | −2·86 | − ·17 | 61 |
| 62 | 211 | ·0142 | ·0250 | 3 | 5·28 | +2·28 | +2·11 | 62 |
| 63 | 203 | ·0148 | ·0270 | 3 | 5·48 | +2·48 | +4·59 | 63 |
| 64 | 195 | ·0410 | ·0291 | 8 | 5·68 | −2·32 | +2·27 | 64 |
| 65 | 179 | ·0223 | ·0313 | 4 | 5·60 | +1·60 | +3·87 | 65 |
| 66 | 169 | ·0414 | ·0336 | 7 | 5·68 | −1·32 | +2·55 | 66 |
| 67 | 155 | ·0129 | ·0360 | 2 | 5·58 | +3·58 | +6·13 | 67 |
| 68 | 151 | ·0464 | ·0385 | 7 | 5·82 | −1·18 | +4·95 | 68 |
| 69 | 140 | ·0643 | ·0411 | 9 | 5·75 | −3·25 | +1·70 | 69 |
| 70 | 128 | ·0547 | ·0439 | 7 | 5·61 | −1·39 | + ·31 | 70 |
| 71 | 116·5 | ·0687 | ·0469 | 8 | 5·46 | −2·54 | −2·23 | 71 |
| 72 | 107 | ·0280 | ·0501 | 3 | 5·36 | +2·36 | + ·13 | 72 |
| 73 | 98 | ·0612 | ·0535 | 6 | 5·24 | − ·76 | − ·63 | 73 |
| 74 | 86 | ·0349 | ·0572 | 3 | 4·92 | +1·92 | +1·29 | 74 |
| 75 | 81 | ·0494 | ·0614 | 4 | 4·97 | + ·97 | +2·26 | 75 |
| 76 | 76 | ·0789 | ·0664 | 6 | 5·05 | − ·95 | +1·31 | 76 |
| 77 | 68 | ·0882 | ·0725 | 6 | 4·93 | −1·07 | + ·24 | 77 |
| 78 | 58 | ·0517 | ·0797 | 3 | 4·62 | +1·62 | +1·86 | 78 |
| 79 | 54 | ·0185 | ·0881 | 1 | 4·76 | +3·76 | +5·62 | 79 |
| 80 | 52 | ·1154 | ·0975 | 6 | 5·07 | − ·93 | +4·69 | 80 |
| 81 | 46 | ·1957 | ·1073 | 9 | 4·93 | −4·07 | + ·62 | 81 |
| 82 | 34 | ·1177 | ·1173 | 4 | 3·99 | − ·01 | + ·61 | 82 |
| 83 | 26 | ·1539 | ·1277 | 4 | 3·32 | − ·68 | − ·07 | 83 |
| 84 | 20 | ·1500 | ·1388 | 3 | 2·77 | − ·23 | − ·30 | 84 |
| 85 | 16 | ·0625 | ·1492 | 1 | 2·39 | +1·39 | +1·09 | 85 |
| 86 | 15 | ·0667 | ·1611 | 1 | 2·40 | +1·40 | +2·49 | 86 |
| 87 | 13 | ·3846 | ·1750 | 5 | 2·28 | −2·72 | − ·23 | 87 |
| 88 | 8 | ·2500 | ·1905 | 2 | 1·52 | − ·48 | − ·71 | 88 |
| 89 | 5 | ·2000 | ·2060 | 1 | 1·03 | + ·03 | − ·68 | 89 |
| 90 | 4 | ·5000 | ·2220 | 2 | ·89 | −1·11 | −1·79 | 90 |
| 91 | 2 | ·0000 | ·2360 | 0 | ·47 | + ·47 | −1·32 | 91 |
| 92 | 2 | ·0000 | ·2490 | 0 | ·50 | + ·50 | − ·82 | 92 |
| 93 | 2 | ·5000 | ·2610 | 1 | ·52 | − ·48 | −1·30 | 93 |
| 94 | 1 | ·0000 | ·2730 | 0 | ·27 | + ·27 | −1·03 | 94 |
| 95 | 1 | ·0000 | ·2860 | 0 | ·29 | + ·29 | − ·74 | 95 |
| 96 | 1 | ·0000 | ·3060 | 0 | ·31 | + ·31 | − ·43 | 96 |
| Total | 55,908 | ... | ... | 448 | 447·57 | − ·43 | ... | Total |

TABLE 9.—*Adjusted Probability of dying in a year, Females, Spinsters, and Married Women.*

| Age. $x$ | Females. | Spinsters. | Married. | Age. $x$ | Females. | Spinsters. | Married. |
|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| 15 | ·0041 | ·0041 | ·0041 | 58 | ·0159 | ·0179 | ·0153 |
| 16 | ·0046 | ·0046 | ·0046 | 59 | ·0167 | ·0195 | ·0158 |
| 17 | ·0050 | ·0049 | ·0062 | 60 | ·0175 | ·0212 | ·0163 |
| 18 | ·0053 | ·0052 | ·0075 | 61 | ·0184 | ·0230 | ·0170 |
| 19 | ·0056 | ·0054 | ·0080 | 62 | ·0194 | ·0250 | ·0177 |
| 20 | ·0058 | ·0054 | ·0085 | 63 | ·0206 | ·0270 | ·0186 |
| 21 | ·0059 | ·0052 | ·0089 | 64 | ·0221 | ·0291 | ·0200 |
| 22 | ·0059 | ·0049 | ·0090 | 65 | ·0241 | ·0318 | ·0219 |
| 23 | ·0060 | ·0047 | ·0088 | 66 | ·0266 | ·0336 | ·0245 |
| 24 | ·0060 | ·0046 | ·0084 | 67 | ·0296 | ·0360 | ·0277 |
| 25 | ·0061 | ·0045 | ·0082 | 68 | ·0331 | ·0385 | ·0315 |
| 26 | ·0061 | ·0044 | ·0080 | 69 | ·0369 | ·0411 | ·0357 |
| 27 | ·0062 | ·0044 | ·0078 | 70 | 0409 | ·0439 | ·0400 |
| 28 | ·0063 | ·0045 | ·0077 | 71 | 0451 | ·0469 | ·0446 |
| 29 | ·0065 | ·0046 | ·0078 | 72 | ·0495 | ·0501 | ·0493 |
| 30 | ·0068 | ·0047 | ·0081 | 73 | ·0541 | ·0535 | ·0543 |
| 31 | ·0072 | ·0049 | ·0085 | 74 | ·0589 | ·0572 | ·0594 |
| 32 | ·0075 | ·0051 | ·0088 | 75 | 0639 | ·0614 | ·0646 |
| 33 | ·0077 | ·0053 | ·0089 | 76 | ·0692 | ·0664 | ·0700 |
| 34 | ·0078 | ·0055 | ·0089 | 77 | ·0748 | ·0725 | ·0755 |
| 35 | ·0079 | ·0058 | ·0088 | 78 | ·0808 | ·0797 | ·0811 |
| 36 | ·0080 | ·0061 | ·0088 | 79 | ·0880 | ·0881 | ·0880 |
| 37 | ·0081 | ·0064 | ·0088 | 80 | ·0970 | ·0975 | ·0969 |
| 38 | ·0082 | ·0067 | ·0088 | 81 | ·1070 | ·1078 | ·1069 |
| 39 | ·0088 | ·0070 | ·0088 | 82 | ·1170 | ·1178 | ·1169 |
| 40 | ·0084 | ·0073 | ·0088 | 83 | ·1275 | ·1277 | ·1274 |
| 41 | ·0085 | ·0076 | ·0088 | 84 | ·1388 | † | † |
| 42 | ·0086 | ·0078 | ·0089 | 85 | ·1492 | ... | ... |
| 43 | ·0087 | ·0080 | ·0090 | 86 | ·1611 | ... | ... |
| 44 | ·0088 | ·0082 | ·0090 | 87 | ·1750 | ... | ... |
| 45 | ·0088 | ·0084 | ·0090 | 88 | ·1905 | ... | ... |
| 46 | ·0089 | ·0087 | ·0090 | 89 | ·2060 | ... | .. |
| 47 | ·0090 | ·0091 | ·0090 | 90 | ·2220 | ... | ... |
| 48 | ·0092 | ·0096 | ·0091 | 91 | ·2360 | ... | ... |
| 49 | ·0095 | ·0103 | ·0092 | 92 | ·2490 | ... | ... |
| 50 | ·0099 | ·0110 | ·0095 | 93 | ·2610 | ... | .. |
| 51 | ·0105 | ·0117 | ·0101 | 94 | ·2730 | ... | ... |
| 52 | ·0112 | ·0124 | ·0108 | 95 | ·2860 | ... | .. |
| 53 | ·0119 | ·0131 | ·0115 | 96 | ·3060 | ... | ... |
| 54 | ·0127 | ·0139 | ·0123 | 97 | ·3700 | ... | ... |
| 55 | ·0135 | ·0147 | ·0131 | 98 | ·5400 | .. | . |
| 56 | ·0143 | ·0156 | ·0139 | 99 | 1·0000 | ... | .. |
| 57 | ·0151 | ·0166 | ·0146 | | | | |

† From this point to the end of the Table, the probability of death for both Spinsters and Married is the same as for Females.

I now bring the adjusted probabilities of death for females and spinsters, and also the probabilities of death among the married, together in Table 9. As the numbers unmarried from age 84 onwards are comparatively small, I have assumed that the mortality among them will be the same as that for females.

We now proceed to consider the probabilities of marriage. It is more convenient in this case to deduce the central marriage rate, than the probability of marrying in a year; and for this purpose we require to know the number of spinsters exposed to the risk of marriage in the middle of the year. This is obtained by deducting from the number who entered upon a year of life, half the number who married, and half the number who died, during that year. For example, we find from Table 6 that at age 20, 3061 spinsters entered upon a year of life at that age, of whom 219 married, and 13 died, between ages 20 and 21. Therefore the number at risk of marriage in the middle of the year is $3061 - 109{\cdot}5 - 6{\cdot}5 = 2945$; and $219 \div 2945 = {\cdot}0744$, is the central marriage rate.

In order to compare the rates of marriage got in this way with other statistics, I desired to obtain the probabilities of marriage among the general population. It appeared at first as if the necessary facts might be obtained from the Census Returns and the Annual Reports of the Registrar-General for England and Wales; but I was disappointed to find that this was not possible, owing to the classification according to age being different in the two sets of publications. There is no inconsistency of the kind in the corresponding Scottish statistics, and I have accordingly deduced the rate of marriage among spinsters from the numbers marrying, as grouped quinquennially in the detailed Annual Reports of the Registrar-General for Scotland, taking the average of the numbers marrying for the three years 1890, 1891, and 1892, and the enumerated spinsters as shown in the Census Returns for 1891. The results are given in the table on p. 277, along with the corresponding figures for peerage spinsters:—

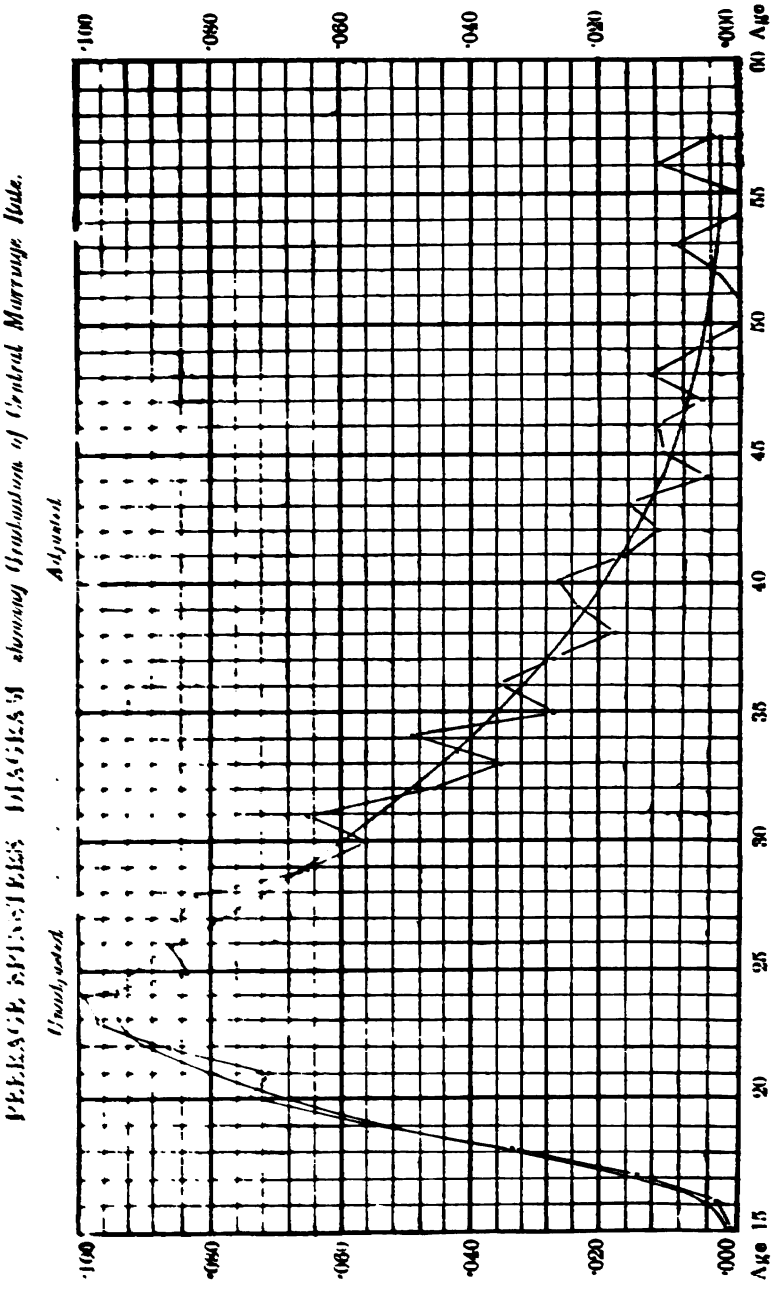
Unadjusted
Adjusted
·100
·080
·060
·040
·020
·000
Age 15
20
25
30
35
40
45
50
55
60 Age

TABLE 10.—*Comparison of Marriage Rate of Peerage Spinsters with that of General Population (Scotland).*

| Ages. | General Population. | | | Peerage Spinsters. | | | Middle Age. |
|---|---|---|---|---|---|---|---|
| | Spinsters enumerated in 1891. | Average number of Spinsters Married 1890-1892. | Central Marriage Rate. | At Risk. | Numbers Married. | Central Marriage Rate. | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 15– | 204,855 | 3,212 | ·01568 | 17,370 | 350 | ·02015 | 17½ |
| 20– | 144,825 | 12,510 | ·08638 | 12,219·5 | 1,028 | ·08413 | 22½ |
| 25– | 79,988 | 7,037 | ·08998 | 7,177·5 | 584 | ·08136 | 27½ |
| 30– | 44,143 | 2,254 | ·05106 | 4,607 | 234 | ·05079 | 32½ |
| 35– | 29,674 | 848 | ·02858 | 3,386 | 88 | ·02599 | 37½ |
| 40– | 22,194 | 366·3 | ·01650 | 2,703 | 40 | ·01480 | 42½ |
| 45– | 18,452 | 142·3 | ·00771 | 2,181·5 | 19 | ·00871 | 47½ |
| 50– | 16,380 | 65 | ·00397 | 1,743·5 | 3 | ·00172 | 52½ |
| 55– | 12,394 | 26·3 | ·00212 | 1,383 | 4 | ·00289 | 57½ |
| 60– | 12,254 | 8·7 | ·00071 | 1,056 | 1 | ·00095 | 62½ |
| 65– | 8,629 | 2 | ·00023 | 779·5 | 0 | ·00000 | 67½ |
| 70– | 7,186 | 2 | ·00028 | 522 | 1 | ·00192 | 72½ |

As was to be expected, the rate of marriage among the spinsters of the general population is at the principal marrying ages greater than the rate for peerage spinsters. On comparing the rates of marriage shown in columns (4) and (7), it will be seen that the maximum marriage rate in the case of the general population is attained at 25-30, whereas in the case of the peerage spinsters it is reached at 20-25; and that throughout, the progression of the marriage rate for peerage spinsters bears a close relation to that for the spinsters of the general population.

I have graduated the probabilities of marriage among the peerage spinsters by the graphic method, as shown by the appended diagram.

Table 11 shows the numbers at risk of marriage in the middle of the year of age, the unadjusted and adjusted central marriage rates, and the comparison of the expected and actual marriages.

TABLE 11.—*Spinsters—Central Marriage Rate, and comparison of Expected and Actual Marriages.*

| Age. $x$ | At risk in middle of year. | Central Marriage Rate. Unadjusted. | Central Marriage Rate. Adjusted. | Marriages. Actual. | Marriages. Expected. | Difference. (6)-(5) | Difference. Accumulated. | Age. $x$ |
|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 15 | 3658 | ·0008 | ·0011 | 3 | 4·02 | + 1·02 | + 1·02 | 15 |
| 16 | 3608 | ·0022 | ·0033 | 8 | 11·89 | + 3·89 | + 4·91 | 16 |
| 17 | 3528·5 | ·0139 | ·0147 | 49 | 51·87 | + 2·87 | + 7·78 | 17 |
| 18 | 3398·5 | ·0336 | ·0360 | 114 | 122·17 | + 8·17 | + 15·95 | 18 |
| 19 | 3187 | ·0552 | ·0531 | 176 | 169·23 | − 6·77 | + 9·18 | 19 |
| 20 | 2945 | ·0744 | ·0677 | 219 | 199·38 | − 19·62 | − 10·44 | 20 |
| 21 | 2688·5 | ·0714 | ·0797 | 192 | 214·27 | + 22·27 | + 11·83 | 21 |
| 22 | 2443 | ·0864 | ·0888 | 211 | 216·93 | + 5·93 | + 17·76 | 22 |
| 23 | 2187 | ·0969 | ·0940 | 212 | 205·58 | − 6·42 | + 11·34 | 23 |
| 24 | 1956 | ·0992 | ·0944 | 194 | 184·64 | − 9·36 | + 1·98 | 24 |
| 25 | 1758 | ·0831 | ·0904 | 146 | 158·92 | + 12·92 | + 14·90 | 25 |
| 26 | 1583·5 | ·0865 | ·0856 | 137 | 135·55 | − 1·45 | + 13·45 | 26 |
| 27 | 1417 | ·0854 | ·0800 | 121 | 113·36 | − 7·64 | + 5·81 | 27 |
| 28 | 1270·5 | ·0826 | ·0737 | 105 | 93·64 | − 11·36 | − 5·55 | 28 |
| 29 | 1148·5 | ·0653 | ·0668 | 75 | 76·72 | + 1·72 | − 3·83 | 29 |
| 30 | 1068 | ·0562 | ·0605 | 60 | 64·61 | + 4·61 | + ·78 | 30 |
| 31 | 984 | ·0650 | ·0548 | 64 | 53·93 | − 10·07 | − 9·29 | 31 |
| 32 | 904·5 | ·0453 | ·0492 | 41 | 44·50 | + 3·50 | − 5·79 | 32 |
| 33 | 848·5 | ·0354 | ·0440 | 30 | 37·33 | + 7·33 | + 1·54 | 33 |
| 34 | 802 | ·0486 | ·0396 | 39 | 31·76 | − 7·24 | − 5·70 | 34 |
| 35 | 752 | ·0266 | ·0356 | 20 | 26·77 | + 6·77 | + 1·07 | 35 |
| 36 | 711 | ·0352 | ·0318 | 25 | 22·61 | − 2·39 | − 1·32 | 36 |
| 37 | 672·5 | ·0268 | ·0280 | 18 | 18·83 | + ·83 | − ·49 | 37 |
| 38 | 638 | ·0172 | ·0244 | 11 | 15·57 | + 4·57 | + 4·08 | 38 |
| 39 | 612·5 | ·0229 | ·0212 | 14 | 12·99 | − 1·01 | + 3·07 | 39 |
| 40 | 586·5 | ·0256 | ·0184 | 15 | 10·79 | − 4·21 | − 1·14 | 40 |
| 41 | 568·5 | ·0158 | ·0160 | 9 | 9·10 | + ·10 | − 1·04 | 41 |
| 42 | 541·5 | ·0111 | ·0140 | 6 | 7·58 | + 1·58 | + ·54 | 42 |
| 43 | 515 | ·0155 | ·0120 | 8 | 6·18 | − 1·82 | − 1·28 | 43 |
| 44 | 491·5 | ·0041 | ·0104 | 2 | 5·11 | + 3·11 | + 1·83 | 44 |
| 45 | 476 | ·0105 | ·0092 | 5 | 4·38 | − ·62 | + 1·21 | 45 |
| 46 | 454 | ·0110 | ·0080 | 5 | 3·63 | − 1·37 | − ·16 | 46 |
| 47 | 440·5 | ·0045 | ·0069 | 2 | 3·04 | + 1·04 | + ·88 | 47 |
| 48 | 416 | ·0120 | ·0059 | 5 | 2·45 | − 2·55 | − 1·67 | 48 |
| 49 | 395 | ·0051 | ·0050 | 2 | 1·98 | − ·02 | − 1·69 | 49 |
| 50 | 378·5 | ... | ·0042 | 0 | 1·59 | + 1·59 | − ·10 | 50 |
| 51 | 363·5 | ... | ·0035 | 0 | 1·27 | + 1·27 | + 1·17 | 51 |
| 52 | 348·5 | ·0029 | ·0029 | 1 | 1·01 | + ·01 | + 1·18 | 52 |
| 53 | 335·5 | ·0089 | ·0025 | 2 | ·84 | − 1·16 | + ·02 | 53 |
| 54 | 317·5 | ... | ·0023 | 0 | ·73 | + ·73 | + ·75 | 54 |
| C. f. | 51,888 | ... | ... | 2,346 | 2,346·75 | + ·75 | ... | C. f. |

TABLE 11.—*Continued.*

| Age. $x$ | At risk in middle of year. | Central Marriage Rate. Un-adjusted. | Central Marriage Rate. Ad-justed. | Marriages. Actual. | Marriages. Expected. | Difference. (6 – 5) | Difference. Accumu-lated. | Age. $x$ |
|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| B. f. | 51,388 | ... | ... | 2,846 | 2,846·75 | + ·75 | ... | ... |
| 55 | 304 | ... | ·0021 | 0 | ·64 | + ·64 | +1·39 | 55 |
| 56 | 291 | ·0103 | ·0020 | 3 | ·58 | −2·42 | −1·03 | 56 |
| 57 | 278·5 | ·0086 | ·0019 | 1 | ·53 | − ·47 | −1·50 | 57 |
| 58 | 261 | ... | ·0018 | 0 | ·47 | + ·47 | −1·03 | 58 |
| 59 | 248·5 | ... | ·0017 | 0 | ·42 | + ·42 | − ·61 | 59 |
| 60 | 234·5 | ... | ·0016 | 0 | ·38 | + ·38 | − ·23 | 60 |
| 61 | 219·5 | ·0046 | ·0015 | 1 | ·33 | − ·67 | − ·90 | 61 |
| 62 | 209·5 | ... | ·0014 | 0 | ·29 | + ·29 | − ·61 | 62 |
| 63 | 201·5 | ... | ·0013 | 0 | ·26 | + ·26 | − ·35 | 63 |
| 64 | 191 | ... | ·0012 | 0 | ·23 | + ·23 | − ·12 | 64 |
| 65 | 177 | ... | ·0011 | 0 | ·19 | + ·19 | + ·07 | 65 |
| 66 | 165·5 | ... | ·0010 | 0 | ·17 | + ·17 | + ·24 | 66 |
| 67 | 154 | ... | ·0009 | 0 | ·14 | + ·14 | + ·38 | 67 |
| 68 | 147·5 | ... | ·0008 | 0 | ·12 | + ·12 | + ·50 | 68 |
| 69 | 135·5 | ... | ·0007 | 0 | ·10 | + ·10 | + ·60 | 69 |
| 70 | 124·5 | ... | ·0006 | 0 | ·07 | + ·07 | + ·67 | 70 |
| 71 | 112·5 | ·0089 | ·0005 | 1 | ·06 | − ·94 | − ·27 | 71 |
| 72 | 105·5 | ... | ·0004 | 0 | ·04 | + ·04 | − ·23 | 72 |
| 73 | 95 | ... | ·0003 | 0 | ·03 | + ·03 | − ·20 | 73 |
| 74 | 84·5 | ... | ·0002 | 0 | ·02 | + ·02 | − ·18 | 74 |
| 75 | 79 | ... | ·0001 | 0 | ·01 | + ·01 | − ·17 | 75 |
| Total | 55,207·5 | ... | ... | 2,352 | 2,351·83 | − ·17 | ... | Total |

Having obtained the adjusted central marriage rate for spinsters, the corresponding central death rate was calculated by the formula $\mu_2 = \frac{2q_2}{2-q_2}$. The numbers tabulated in columns (2), (4), and (5) of Table 12 were then calculated by means of Dr. Sprague's well-known formulas

$$(sm)_x = (sl)_x \cdot \frac{\mu_1}{1+\frac{1}{2}(\mu_1+\mu_2)}; \text{ and}$$

$$(sd)_x = (sl)_x \cdot \frac{\mu_2}{1+\frac{1}{2}(\mu_1+\mu_2)};$$

where $(sl)_x$ denotes the number of spinsters living at age $x$, $(sm)_x$ the number marrying between ages $x$ and $x+1$, and $(sd)_x$ the number dying unmarried between these ages. These numbers were checked by the formula ${}^1l = l \cdot \frac{1-\frac{1}{2}(\mu_1+\mu_2)}{1+\frac{1}{2}(\mu_1+\mu_2)}$. The numbers tabulated in columns (3) and (6) were obtained by deducting the numbers in columns (2) and (4) from the numbers at corresponding ages in columns (2) and (4) of Table 4.

TABLE 12.—*Combined Marriage and Mortality Table.*

| Age. | Numbers Living. | | Decrements caused by | | | Age. |
|---|---|---|---|---|---|---|
| | Spinsters. | Married. | Spinsters Dying. | Spinsters Marrying. | Married Dying. | |
| $x$ | $(sl)_x$ | $(ml)_x$ | $(sd)_x$ | $(sm)_x$ | $(md)_x$ | $x$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 0 | 1,135,588 | ..... | 58,599 | ..... | ..... | 0 |
| 1 | 1,076,989 | ..... | 16,257 | ..... | ..... | 1 |
| 2 | 1,060,732 | ..... | 8,694 | ..... | ..... | 2 |
| 3 | 1,052,038 | ..... | 6,523 | ..... | ..... | 3 |
| 4 | 1,045,515 | ..... | 5,537 | ..... | ..... | 4 |
| 5 | 1,039,978 | ..... | 4,783 | ..... | ..... | 5 |
| 6 | 1,035,195 | ..... | 4,246 | ..... | ..... | 6 |
| 7 | 1,030,949 | ..... | 4,126 | ..... | ..... | 7 |
| 8 | 1,026,823 | ..... | 4,007 | ..... | ..... | 8 |
| 9 | 1,022,816 | ..... | 3,889 | ..... | ..... | 9 |
| 10 | 1,018,927 | ..... | 3,773 | ..... | ..... | 10 |
| 11 | 1,015,154 | ..... | 3,658 | ..... | ..... | 11 |
| 12 | 1,011,496 | ..... | 3,746 | ..... | ..... | 12 |
| 13 | 1,007,750 | ..... | 3,832 | ..... | ..... | 13 |
| 14 | 1,003,918 | ..... | 3,918 | ..... | ..... | 14 |
| 15 | 1,000,000 | 0 | 4,100 | 1,097 | 2 | 15 |
| 16 | 994,803 | 1,095 | 4,567 | 3,270 | 13 | 16 |
| 17 | 986,966 | 4,352 | 4,849 | 14,367 | 72 | 17 |
| 18 | 967,750 | 18,647 | 4,990 | 34,134 | 267 | 18 |
| 19 | 928,626 | 52,514 | 4,886 | 47,907 | 612 | 19 |
| 20 | 875,833 | 99,809 | 4,571 | 57,203 | 1,091 | 20 |
| 21 | 814,059 | 155,921 | 4,069 | 62,239 | 1,657 | 21 |
| 22 | 747,751 | 216,503 | 3,507 | 63,428 | 2,232 | 22 |
| 23 | 680,816 | 277,699 | 3,056 | 60,987 | 2,697 | 23 |
| 24 | 616,773 | 335,989 | 2,706 | 55,475 | 3,056 | 24 |
| 25 | 558,589 | 388,408 | 2,405 | 48,209 | 3,373 | 25 |
| 26 | 507,975 | 433,244 | 2,144 | 41,609 | 3,646 | 26 |
| 27 | 464,222 | 471,207 | 1,964 | 35,634 | 3,836 | 27 |
| 28 | 426,624 | 503,005 | 1,852 | 30,259 | 4,005 | 28 |
| 29 | 394,513 | 529,259 | 1,756 | 25,445 | 4,247 | 29 |
| 30 | 367,312 | 550,457 | 1,675 | 21,520 | 4,563 | 30 |
| 31 | 344,117 | 567,414 | 1,641 | 18,311 | 4,926 | 31 |
| 32 | 324,165 | 580,799 | 1,613 | 15,527 | 5,176 | 32 |
| 33 | 307,025 | 591,150 | 1,591 | 13,184 | 5,325 | 33 |
| 34 | 292,250 | 599,009 | 1,577 | 11,318 | 5,374 | 34 |
| 35 | 279,355 | 604,953 | 1,593 | 9,743 | 5,392 | 35 |
| 36 | 268,019 | 609,304 | 1,610 | 8,364 | 5,407 | 36 |
| 37 | 258,045 | 612,261 | 1,629 | 7,108 | 5,418 | 37 |
| 38 | 249,313 | 613,946 | 1,650 | 5,990 | 5,425 | 38 |
| 39 | 241,673 | 614,511 | 1,678 | 5,052 | 5,430 | 39 |
| 40 | 234,948 | 614,133 | 1,700 | 4,268 | 5,436 | 40 |
| 41 | 228,980 | 612,965 | 1,727 | 3,621 | 5,433 | 41 |
| 42 | 223,632 | 611,153 | 1,732 | 3,097 | 5,449 | 42 |
| 43 | 218,803 | 608,801 | 1,739 | 2,600 | 5,463 | 43 |
| 44 | 214,464 | 605,938 | 1,749 | 2,210 | 5,471 | 44 |
| 45 | 210,505 | 602,677 | 1,761 | 1,920 | 5,436 | 45 |
| 46 | 206,824 | 599,161 | 1,792 | 1,641 | 5,382 | 46 |
| 47 | 203,391 | 595,420 | 1,844 | 1,392 | 5,344 | 47 |
| 48 | 200,155 | 591,468 | 1,916 | 1,172 | 5,365 | 48 |
| 49 | 197,067 | 587,275 | 2,024 | 978 | 5,423 | 49 |

TABLE 12.—*Continued.*

| Age. | Numbers Living. | | Decrements caused by | | | Age. |
|---|---|---|---|---|---|---|
| | Spinsters. | Married. | Spinsters Dying. | Spinsters Marrying. | Married Dying. | |
| $x$ | $(sl)_x$ | $(ml)_x$ | $(sd)_x$ | $(sm)_x$ | $(md)_x$ | $x$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 50 | 194,065 | 582,830 | 2,130 | 809 | 5,562 | 50 |
| 51 | 191,126 | 578,077 | 2,233 | 664 | 5,847 | 51 |
| 52 | 188,229 | 572,894 | 2,331 | 542 | 6,191 | 52 |
| 53 | 185,356 | 567,245 | 2,425 | 460 | 6,530 | 53 |
| 54 | 182,471 | 561,175 | 2,534 | 416 | 6,910 | 54 |
| 55 | 179,521 | 554,681 | 2,637 | 374 | 7,273 | 55 |
| 56 | 176,510 | 547,782 | 2,750 | 350 | 7,605 | 56 |
| 57 | 173,410 | 540,527 | 2,876 | 326 | 7,901 | 57 |
| 58 | 170,208 | 532,952 | 3,044 | 304 | 8,138 | 58 |
| 59 | 166,860 | 525,118 | 3,251 | 280 | 8,305 | 59 |
| 60 | 163,329 | 517,093 | 3,461 | 258 | 8,443 | 60 |
| 61 | 159,610 | 508,908 | 3,669 | 236 | 8,631 | 61 |
| 62 | 155,705 | 500,513 | 3,891 | 215 | 8,839 | 62 |
| 63 | 151,599 | 491,889 | 4,091 | 194 | 9,162 | 63 |
| 64 | 147,314 | 482,921 | 4,284 | 174 | 9,646 | 64 |
| 65 | 142,856 | 473,449 | 4,469 | 155 | 10,381 | 65 |
| 66 | 138,232 | 463,223 | 4,641 | 136 | 11,362 | 66 |
| 67 | 133,455 | 451,997 | 4,802 | 118 | 12,525 | 67 |
| 68 | 128,525 | 439,590 | 4,947 | 101 | 13,860 | 68 |
| 69 | 123,487 | 425,831 | 5,073 | 85 | 15,195 | 69 |
| 70 | 118,329 | 410,721 | 5,194 | 69 | 16,443 | 70 |
| 71 | 113,066 | 394,347 | 5,302 | 55 | 17,582 | 71 |
| 72 | 107,709 | 376,820 | 5,396 | 42 | 18,590 | 72 |
| 73 | 102,271 | 358,272 | 5,470 | 30 | 19,448 | 73 |
| 74 | 96,771 | 338,854 | 5,535 | 19 | 20,125 | 74 |
| 75 | 91,217 | 318,748 | 5,590 | 9 | 20,598 | 75 |
| 76 | 85,608 | 298,159 | 5,684 | ... | 20,873 | 76 |
| 77 | 79,924 | 277,286 | 5,795 | ... | 20,926 | 77 |
| 78 | 74,129 | 253,360 | 5,908 | ... | 20,795 | 78 |
| 79 | 68,221 | 235,565 | 6,011 | ... | 20,722 | 79 |
| 80 | 62,210 | 214,843 | 6,065 | ... | 20,811 | 80 |
| 81 | 56,145 | 194,032 | 6,024 | ... | 20,745 | 81 |
| 82 | 50,121 | 173,287 | 5,880 | ... | 20,259 | 82 |
| 83 | 44,241 | 153,028 | 5,649 | ... | 19,502 | 83 |
| 84 | 38,592 | 133,526 | 5,337 | ... | 18,466 | 84 |
| 85 | 33,255 | 115,060 | 4,962 | ... | 17,167 | 85 |
| 86 | 28,293 | 97,893 | 4,558 | ... | 15,771 | 86 |
| 87 | 23,735 | 82,122 | 4,154 | ... | 14,372 | 87 |
| 88 | 19,581 | 67,750 | 3,731 | ... | 12,907 | 88 |
| 89 | 15,850 | 54,843 | 3,265 | ... | 11,299 | 89 |
| 90 | 12,585 | 43,544 | 2,793 | ... | 9,666 | 90 |
| 91 | 9,792 | 33,878 | 2,311 | ... | 7,996 | 91 |
| 92 | 7,481 | 25,882 | 1,863 | ... | 6,444 | 92 |
| 93 | 5,618 | 19,438 | 1,466 | ... | 5,074 | 93 |
| 94 | 4,152 | 14,364 | 1,134 | ... | 3,921 | 94 |
| 95 | 3,018 | 10,443 | 863 | ... | 2,986 | 95 |
| 96 | 2,155 | 7,457 | 659 | ... | 2,282 | 96 |
| 97 | 1,496 | 5,175 | 554 | ... | 1,914 | 97 |
| 98 | 942 | 3,261 | 508 | ... | 1,761 | 98 |
| 99 | 434 | 1,500 | 434 | ... | 1,500 | 99 |

Before proceeding to give the values of benefits calculated by means of Table 12, I think it will be of interest to compare the total probability that a spinster will marry, with the probability that a bachelor will marry. I have therefore calculated for each age the probability that a spinster will marry, $\frac{\Sigma(sm)_x}{(sl)_x}$; and in the following table I have placed these alongside the probability that a bachelor will marry, as deduced by Dr. Sprague.

TABLE 13.—*Comparison of Total Probability of Marriage of a Spinster with that of a Bachelor.*

| Age. $x$ | Total Probability of Marriage. | | Age. $x$ | Total Probability of Marriage. | | Age. $x$ | Total Probability of Marriage. | |
|---|---|---|---|---|---|---|---|---|
| | Bachelor. | Spinster. | | Bachelor. | Spinster. | | Bachelor. | Spinster. |
| 15 | ·70463 | ·72670 | 38 | ·38292 | ·16189 | 60 | ·03696 | ·01161 |
| 16 | ·70814 | ·72989 | 39 | ·35382 | ·14223 | 61 | ·03220 | ·01026 |
| 17 | ·71208 | ·73187 | 40 | ·32551 | ·12479 | 62 | ·02795 | ·00900 |
| 18 | ·71649 | ·73155 | 41 | ·29825 | ·10941 | 63 | ·02412 | ·00783 |
| 19 | ·72148 | ·72562 | 42 | ·27223 | ·09588 | 64 | ·02084 | ·00674 |
| 20 | ·72656 | ·71466 | 43 | ·24772 | ·08379 | 65 | ·01800 | ·00573 |
| 21 | ·73115 | ·69862 | 44 | ·22488 | ·07336 | 66 | ·01563 | ·00480 |
| 22 | ·73085 | ·67734 | 45 | ·20387 | ·06425 | 67 | ·01362 | ·00396 |
| 23 | ·72539 | ·65076 | 46 | ·18484 | ·05611 | 68 | ·01187 | ·00319 |
| 24 | ·71696 | ·61946 | 47 | ·16766 | ·04898 | 69 | ·01030 | ·00250 |
| 25 | ·70502 | ·58467 | 48 | ·15205 | ·04282 | 70 | ·00888 | ·00189 |
| 26 | ·69010 | ·54802 | 49 | ·13775 | ·03755 | 71 | ·00753 | ·00137 |
| 27 | ·67304 | ·51004 | 50 | ·12459 | ·03309 | 72 | ·00633 | ·00093 |
| 28 | ·65424 | ·47146 | 51 | ·11225 | ·02936 | 73 | ·00520 | ·00057 |
| 29 | ·63345 | ·43314 | 52 | ·10067 | ·02629 | 74 | ·00422 | ·00029 |
| 30 | ·61074 | ·39594 | 53 | ·08991 | ·02377 | 75 | ·00329 | ·00010 |
| 31 | ·58606 | ·36009 | 54 | ·08002 | ·02168 | 76 | ·00253 | ... |
| 32 | ·55963 | ·32576 | 55 | ·07094 | ·01966 | 77 | ·00181 | ... |
| 33 | ·53174 | ·29338 | 56 | ·06270 | ·01788 | 78 | ·00125 | ... |
| 34 | ·50260 | ·26310 | 57 | ·05524 | ·01618 | 79 | ·00074 | ... |
| 35 | ·47274 | ·23473 | 58 | ·04851 | ·01457 | 80 | ·00039 | ... |
| 36 | ·44265 | ·20830 | 59 | ·04240 | ·01304 | 81 | ·00010 | ... |
| 37 | ·41261 | ·18394 | | | | | | |

It will be observed from this table that the total probability that a bachelor will marry some time after age $x$, has its maximum at age 21, and decreases slowly; while the corresponding probability that a spinster will marry, has its maximum at age 17, and decreases slowly for a few ages but rapidly thereafter.

Table 14 is a commutation table for benefits depending on marriage and mortality, calculated at 3 per cent interest. The notation used is similar to that employed by Dr. Sprague, the letter $s$ standing for spinster.

In Table 15 I give the values of certain benefits depending on marriage and mortality at 3 per cent interest. These are:—

$(ma)_x$ = the value of an annuity of 1 to a married woman.
$(mA)_x$ = the value of an assurance of 1 payable on the death of a married woman.
$(sa)_x$ = the value of an annuity of 1 to a spinster, so long as she remains unmarried.
$(sA)_x$ = the value of an assurance of 1 payable on the death of a spinster, provided she dies unmarried.
$(smE)_x$ = the value of an endowment of 1 payable at the end of the year in which a spinster shall marry.
$(sma)_x$ = the value of an annuity of 1 to commence on the marriage of a spinster.
$(smA)_x$ = the value of an assurance of 1 payable on the death of a spinster, provided she shall have married.

It will be observed, if we compare columns (3) and (5), that the single premiums in column (5), $(sA)_x$, are lower than those in column (3), $(mA)_x$, up to age 48 inclusive. If the rates of mortality among the married and unmarried had been the same, $(sA)_x$ would have continued to be less than $(mA)_x$, so long as the marriage contingency involved in the spinster assurance continues. This is, however, more than counterbalanced by the heavier mortality among the unmarried, with the result that from ages 49 to 68 inclusive, the premiums in column (5) are greater than those in column (3).

Dr. Sprague has shown, in the paper already referred to, various relations subsisting between functions corresponding to those tabulated in Tables 14 and 15. On page 444 of his paper he gives the following equation:—

$$(bA)_x + \sqrt{(1+i)}(bmA)_x = \frac{1-i\{(ba)_x + \sqrt{(1+i)}(bma)_x\}}{1+i},$$

and says "this equation shows us that the assurance $(bA)_x +$ " $\sqrt{(1+i)}(bmA)_x$ corresponds to the annuity $(ba)_x + \sqrt{(1+i)}(bma)_x$. " This suggests that these quantities may be the values of the

TABLE 14.—*Commutation Table for Benefits depending on Marriage and Mortality, 3 per cent interest.*

| $x$ | $(sD)_x$ | $(sN)_x$ | $(sM)_x$ | $(smEN)_x$ | $(smN)_x$ | $(smM)_x$ |
|---|---|---|---|---|---|---|
| | $(sl)_x v^x$ | $\Sigma\{(sl)_{x+1} v^{x+1}\}$ | $\Sigma\{(sd)_x v^{x+1}\}$ | $\Sigma\{(sm)_x v^{x+1}\}$ | $\Sigma\{(sm)_x v^{x+\frac{1}{2}} (ma)_{x+\frac{1}{2}}\}$ | $\Sigma\{(sm)_x v^{x+\frac{1}{2}} (mA)_{x+\frac{1}{2}}\}$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 0 | 1,135,588 | 20,041,272 | 180,112·4 | † | † | † |
| 1 | 1,045,620 | 18,995,652 | 123,220·2 | ..... | ..... | ..... |
| 2 | 999,842 | 17,995,810 | 107,896·4 | ..... | ..... | ..... |
| 3 | 962,764 | 17,033,046 | 99,940·12 | ..... | ..... | ..... |
| 4 | 928,927 | 16,104,119 | 94,144·52 | ..... | ..... | ..... |
| 5 | 897,094 | 15,207,025 | 89,368·25 | ..... | ..... | ..... |
| 6 | 866,959 | 14,340,066 | 85,362·56 | ..... | ..... | ..... |
| 7 | 838,256 | 13,501,810 | 81,910·18 | ..... | ..... | ..... |
| 8 | 810,584 | 12,691,226 | 78,653·08 | ..... | ..... | ..... |
| 9 | 783,903 | 11,907,323 | 75,582·05 | ..... | ..... | ..... |
| 10 | 758,177 | 11,149,146 | 72,688·27 | ..... | ..... | ..... |
| 11 | 733,369 | 10,415,777 | 69,962·57 | ..... | ..... | ..... |
| 12 | 709,443 | 9,706,334 | 67,396·92 | ..... | ..... | ..... |
| 13 | 686,229 | 9,020,105 | 64,846·07 | ..... | ..... | ..... |
| 14 | 663,708 | 8,356,397 | 62,312·66 | † | † | † |
| 15 | 641,863 | 7,714,534 | 59,797·84 | 338,673·9 | 7,565,071 | 113,352·4 |
| 16 | 619,927 | 7,094,607 | 57,242·86 | 337,990·3 | 7,548,827 | 113,151·9 |
| 17 | 597,130 | 6,497,477 | 54,479·75 | 336,011·9 | 7,502,196 | 112,560·2 |
| 18 | 568,451 | 5,929,026 | 51,631·47 | 327,572·8 | 7,304,776 | 109,993·6 |
| 19 | 529,582 | 5,399,444 | 48,785·74 | 308,106·7 | 6,852,294 | 103,991·1 |
| 20 | 484,929 | 4,914,515 | 46,080·48 | 281,581·7 | 6,239,204 | 95,711·0 |
| 21 | 437,597 | 4,476,918 | 43,623·35 | 250,832·3 | 5,532,282 | 86,001·8 |
| 22 | 390,245 | 4,086,673 | 41,499·77 | 218,350·2 | 4,789,379 | 75,634·6 |
| 23 | 344,964 | 3,741,709 | 39,722·80 | 186,211·7 | 4,058,121 | 65,269·0 |
| 24 | 303,411 | 3,438,298 | 38,219·45 | 156,210·1 | 3,379,171 | 55,485·9 |
| 25 | 266,785 | 3,171,513 | 36,925·62 | 129,715·0 | 2,783,123 | 46,742·3 |
| 26 | 235,546 | 2,935,967 | 35,810·44 | 107,360·7 | 2,283,497 | 39,270·0 |
| 27 | 208,988 | 2,726,979 | 34,845·23 | 88,628·8 | 1,867,762 | 32,923·1 |
| 28 | 186,467 | 2,540,512 | 33,986·81 | 73,054·0 | 1,524,681 | 27,570·6 |
| 29 | 167,410 | 2,373,102 | 33,200·92 | 60,213·7 | 1,244,088 | 23,091·9 |
| 30 | 151,328 | 2,221,774 | 32,477·47 | 49,730·8 | 1,016,920 | 19,379·4 |
| 31 | 137,642 | 2,084,132 | 31,807·50 | 41,123·1 | 831,980 | 16,284·5 |
| 32 | 125,885 | 1,958,247 | 31,170·23 | 34,012·2 | 680,515 | 13,689·7 |
| 33 | 115,757 | 1,842,490 | 30,562·09 | 28,158·1 | 556,899 | 11,522·3 |
| 34 | 106,977 | 1,735,513 | 29,979·71 | 23,332·2 | 455,899 | 9,709·2 |
| 35 | 99,278 | 1,636,235 | 29,419·27 | 19,310·0 | 372,498 | 8,175·5 |
| 36 | 92,475 | 1,543,760 | 28,869·64 | 15,948·3 | 303,473 | 6,873·8 |
| 37 | 86,441 | 1,457,319 | 28,330·31 | 13,146·5 | 246,533 | 5,771·8 |
| 38 | 81,083 | 1,376,236 | 27,800·52 | 10,836·4 | 200,092 | 4,848·5 |
| 39 | 76,309 | 1,299,927 | 27,279·53 | 8,945·1 | 162,500 | 4,079·9 |
| 40 | 72,025 | 1,227,902 | 26,766·66 | 7,396·3 | 132,086 | 3,439·9 |
| 41 | 68,151 | 1,159,751 | 26,260·69 | 6,126·1 | 107,452 | 2,905·8 |
| 42 | 64,621 | 1,095,130 | 25,761·65 | 5,079·7 | 87,429 | 2,458·2 |
| 43 | 61,383 | 1,033,747 | 25,275·75 | 4,210·9 | 71,032 | 2,080·0 |
| 44 | 58,414 | 975,333 | 24,802·10 | 3,502·7 | 57,861 | 1,765·7 |
| 45 | 55,666 | 919,667 | 24,339·60 | 2,918·3 | 47,157 | 1,501·7 |
| 46 | 53,099 | 866,568 | 23,887·48 | 2,425·4 | 38,274 | 1,274·8 |
| 47 | 50,697 | 815,871 | 23,440·81 | 2,016·4 | 31,028 | 1,082·8 |
| 48 | 48,437 | 767,434 | 22,994·57 | 1,679·5 | 25,167 | 921.6 |
| 49 | 46,301 | 721,133 | 22,544·40 | 1,404·1 | 20,468 | 787·2 |

† For ages 0-14, the values are the same as for age 15.

TABLE 14.—*Continued.*

| $x$ | $(sD)_x$ | $(sN)_x$ | $(sM)_x$ | $(smEN)_x$ | $(smN)_x$ | $(smM)_x$ |
|---|---|---|---|---|---|---|
| | $(sl)_x v^x$ | $\Sigma\{(sl)_{x+1} v^{x+1}\}$ | $\Sigma\{(sd)_x v^{x+1}\}$ | $\Sigma\{(sm)_x v^{x+1}\}$ | $\Sigma\{(sm)_x v^{x+\frac{1}{2}} (ma)_{x+\frac{1}{2}}\}$ | $\Sigma\{(sm)_x v^{x+\frac{1}{2}} (mA)_{x+\frac{1}{2}}\}$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 50 | 44,268 | 676,865 | 22,082·71 | 1,181·0 | 16,736 | 676·1 |
| 51 | 42,327 | 634,538 | 21,610·99 | 1,001·9 | 13,802 | 585·1 |
| 52 | 40,472 | 594,066 | 21,130·87 | 859·1 | 11,514 | 511·0 |
| 53 | 38,693 | 555,373 | 20,644·27 | 746·0 | 9,742 | 451·2 |
| 54 | 36,981 | 518,392 | 20,152·80 | 652·7 | 8,316 | 400·9 |
| 55 | 35,324 | 483,068 | 19,654·19 | 570·9 | 7,094 | 355·8 |
| 56 | 33,720 | 449,348 | 19,150·43 | 499·4 | 6·054 | 315·7 |
| 57 | 32,163 | 417,185 | 18,640·38 | 434·5 | 5,132 | 278·6 |
| 58 | 30,649 | 386,536 | 18,122·50 | 375·9 | 3,322 | 244·4 |
| 59 | 29,171 | 357,365 | 17,590·33 | 322·7 | 3,608 | 212·8 |
| 60 | 27,722 | 329,643 | 17,038·53 | 275·1 | 2,989 | 184·0 |
| 61 | 26,302 | 303,341 | 16,468·19 | 232·6 | 2,452 | 157·8 |
| 62 | 24,911 | 278,430 | 15,881·19 | 194·9 | 1,991 | 134·0 |
| 63 | 23,548 | 254,882 | 15,276·80 | 161·5 | 1,597 | 112·6 |
| 64 | 22,216 | 232,666 | 14,659·85 | 132·2 | 1,265 | 93·4 |
| 65 | 20,916 | 211,750 | 14,032·62 | 106·7 | 987·6 | 76·4 |
| 66 | 19,649 | 192,101 | 13,397·36 | 84·7 | 757·2 | 61·4 |
| 67 | 18,418 | 173,683 | 12,756·86 | 66·0 | 569·3 | 48·4 |
| 68 | 17,222 | 156,461 | 12,113·45 | 50·1 | 418·0 | 37·2 |
| 69 | 16,064 | 140,397 | 11,469·91 | 37·0 | 297·8 | 27·8 |
| 70 | 14,945 | 125,452 | 10,829·20 | 26·3 | 204·1 | 20·0 |
| 71 | 13,864 | 111,588 | 10,192·32 | 17·8 | 133·5 | 13·6 |
| 72 | 12,822 | 98,766 | 9,561·13 | 11·2 | 81·49 | 8·7 |
| 73 | 11,820 | 86,946 | 8,937·46 | 6·4 | 44·72 | 5·0 |
| 74 | 10,859 | 76,087 | 8,323·65 | 3·0 | 20·41 | 2·4 |
| 75 | 9,938 | 66,149 | 7,720·64 | 1·0 | 6·21 | ·8 |
| 76 | 9,055 | 57,094 | 7,128·32 | ..... | ..... | ..... |
| 77 | 8,207 | 48,887 | 6,544·62 | ..... | ..... | ..... |
| 78 | 7,391 | 41,496 | 5,966·86 | ..... | ..... | ..... |
| 79 | 6,604 | 34,892 | 5,394·98 | ..... | ..... | ..... |
| 80 | 5,846 | 29,046 | 4,830·09 | ..... | ..... | ..... |
| 81 | 5,123 | 23,923 | 4,276·72 | ..... | ..... | ..... |
| 82 | 4,440 | 19,483 | 3,743·10 | ..... | ..... | ..... |
| 83 | 3,805 | 15,678 | 3,237·40 | ..... | ..... | ..... |
| 84 | 3,222 | 12,456 | 2,765·73 | ..... | ..... | ..... |
| 85 | 2,696 | 9,760 | 2,333·08 | ..... | ..... | ..... |
| 86 | 2,227 | 7,533 | 1,942·55 | ..... | ..... | ..... |
| 87 | 1,814 | 5,719 | 1,594·26 | ..... | ..... | ..... |
| 88 | 1,453 | 4,266 | 1,286·09 | ..... | ..... | ..... |
| 89 | 1,142 | 3,124 | 1,017·37 | ..... | ..... | ..... |
| 90 | 880·0 | 2,243·9 | 789·05 | ..... | ..... | ..... |
| 91 | 664·8 | 1,579·1 | 599·43 | ..... | ..... | ..... |
| 92 | 493·1 | 1,086·0 | 447·11 | ..... | ..... | ..... |
| 93 | 359·5 | 726·5 | 327·88 | ..... | ..... | ..... |
| 94 | 258·0 | 468·5 | 236·80 | ..... | ..... | ..... |
| 95 | 182·0 | 286·5 | 168·40 | ..... | ..... | ..... |
| 96 | 126·2 | 160·32 | 117·86 | ..... | ..... | ..... |
| 97 | 85·06 | 75·26 | 80·39 | ..... | ..... | ..... |
| 98 | 52·00 | 23·26 | 49·81 | ..... | ..... | ..... |
| 99 | 23·26 | ..... | 22·58 | ..... | ..... | ..... |

TABLE 15.—*Values of Benefits depending on Marriage and Mortality, at 3 per cent interest.*

| $x$ | Married. | | Spinsters. | | | | |
|---|---|---|---|---|---|---|---|
| | $(ma)_x$ | $(mA)_x$ | $(sa)_x$ | $(sA)_x$ | $(smE_x)$ | $(sma)_x$ | $(smA)_x$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 0 | ..... | ..... | 17·648 | ·15861 | ·29824 | 6·662 | ·09982 |
| 1 | ..... | ..... | 18·167 | ·11784 | ·32390 | 7·235 | ·10841 |
| 2 | ..... | ..... | 17·999 | ·10791 | ·33873 | 7·566 | ·11337 |
| 3 | ..... | ..... | 17·692 | ·10381 | ·35177 | 7·858 | ·11774 |
| 4 | ..... | ..... | 17·336 | ·10135 | ·36459 | 8·144 | ·12203 |
| 5 | ..... | ..... | 16·952 | ·09962 | ·37752 | 8·433 | ·12636 |
| 6 | ..... | ..... | 16·541 | ·09846 | ·39064 | 8·726 | ·13075 |
| 7 | ..... | ..... | 16·107 | ·09772 | ·40402 | 9·025 | ·13522 |
| 8 | ..... | ..... | 15·657 | ·09703 | ·41782 | 9·333 | ·13984 |
| 9 | ..... | ..... | 15·190 | ·09642 | ·43203 | 9·651 | ·14460 |
| 10 | ..... | ..... | 14·705 | ·09587 | ·44670 | 9·978 | ·14951 |
| 11 | ..... | ..... | 14·203 | ·09540 | ·46180 | 10·316 | ·15456 |
| 12 | ..... | ..... | 13·682 | ·09500 | ·47738 | 10·663 | ·15978 |
| 13 | ..... | ..... | 13·144 | ·09450 | ·49353 | 11·024 | ·16518 |
| 14 | ..... | ..... | 12·591 | ·09389 | ·51028 | 11·398 | ·17079 |
| 15 | 23·510 | ·28610 | 12·019 | ·09316 | ·52764 | 11·786 | ·17660 |
| 16 | 23·315 | ·29179 | 11·444 | ·09234 | ·54521 | 12·177 | ·18253 |
| 17 | 23·126 | ·29731 | 10·881 | ·09124 | ·56271 | 12·564 | ·18850 |
| 18 | 22·969 | ·30187 | 10·430 | ·09083 | ·57626 | 12·850 | ·19350 |
| 19 | 22·836 | ·30574 | 10·196 | ·09212 | ·58179 | 12·939 | ·19637 |
| 20 | 22·711 | ·30938 | 10·135 | ·09503 | ·58067 | 12·866 | ·19737 |
| 21 | 22·593 | ·31283 | 10·231 | ·09969 | ·57320 | 12·642 | ·19653 |
| 22 | 22·479 | ·31615 | 10·472 | ·10634 | ·55952 | 12·273 | ·19381 |
| 23 | 22·363 | ·31952 | 10·847 | ·11515 | ·53980 | 11·764 | ·18921 |
| 24 | 22·237 | ·32318 | 11·332 | ·12597 | ·51485 | 11·137 | ·18287 |
| 25 | 22·099 | ·32723 | 11·888 | ·13841 | ·48622 | 10·432 | ·17521 |
| 26 | 21·949 | ·33158 | 12·465 | ·15203 | ·45580 | 9·695 | ·16672 |
| 27 | 21·791 | ·33620 | 13·049 | ·16673 | ·42409 | 8·937 | ·15754 |
| 28 | 21·622 | ·34111 | 13·624 | ·18227 | ·39178 | 8·177 | ·14786 |
| 29 | 21·444 | ·34629 | 14·175 | ·19832 | ·35968 | 7·431 | ·13794 |
| 30 | 21·262 | ·35160 | 14·682 | ·21462 | ·32863 | 6·720 | ·12806 |
| 31 | 21·079 | ·35692 | 15·142 | ·23109 | ·29877 | 6·045 | ·11831 |
| 32 | 20·898 | ·36218 | 15·556 | ·24761 | ·27018 | 5·406 | ·10875 |
| 33 | 20·716 | ·36749 | 15·917 | ·26402 | ·24325 | 4·811 | ·09954 |
| 34 | 20·530 | ·37292 | 16·223 | ·28025 | ·21811 | 4·262 | ·09076 |
| 35 | 20·335 | ·37859 | 16·481 | ·29633 | ·19450 | 3·752 | ·08235 |
| 36 | 20·132 | ·38451 | 16·694 | ·31219 | ·17246 | 3·282 | ·07433 |
| 37 | 19·920 | ·39067 | 16·859 | ·32774 | ·15209 | 2·852 | ·06677 |
| 38 | 19·700 | ·39709 | 16·973 | ·34286 | ·13365 | 2·468 | ·05980 |
| 39 | 19·471 | ·40376 | 17·035 | ·35749 | ·11722 | 2·130 | ·05347 |
| 40 | 19·233 | ·41068 | 17·048 | ·37163 | ·10269 | 1·834 | ·04776 |
| 41 | 18·986 | ·41787 | 17·017 | ·38533 | ·08989 | 1·577 | ·04264 |
| 42 | 18·730 | ·42533 | 16·947 | ·39866 | ·07861 | 1·353 | ·03804 |
| 43 | 18·465 | ·43305 | 16·841 | ·41177 | ·06860 | 1·157 | ·03388 |
| 44 | 18·191 | ·44104 | 16·697 | ·42459 | ·05996 | ·991 | ·03023 |
| 45 | 17·907 | ·44930 | 16·521 | ·43725 | ·05243 | ·847 | ·02698 |
| 46 | 17·612 | ·45790 | 16·320 | ·44987 | ·04563 | ·721 | ·02401 |
| 47 | 17·305 | ·46686 | 16·093 | ·46237 | ·03977 | ·612 | ·02436 |
| 48 | 16·985 | ·47617 | 15·844 | ·47473 | ·03467 | ·520 | ·01903 |
| 49 | 16·654 | ·48579 | 15·575 | ·48691 | ·03033 | ·442 | ·01700 |

TABLE 15.—*Continued.*

| $x$ | Married. | | Spinsters. | | | | |
|---|---|---|---|---|---|---|---|
| | $(ma)_x$ | $(mA)_x$ | $(sa)_x$ | $(sA)_x$ | $(smE)_x$ | $(sma)_x$ | $(smA)_x$ |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 50 | 16·314 | ·49571 | 15·290 | ·49885 | ·02668 | ·378 | ·01527 |
| 51 | 15·965 | ·50587 | 14·991 | ·51057 | ·02367 | ·326 | ·01382 |
| 52 | 15·612 | ·51616 | 14·679 | ·52212 | ·02123 | ·285 | ·01263 |
| 53 | 15·256 | ·52653 | 14·353 | ·53354 | ·01928 | ·252 | ·01166 |
| 54 | 14·896 | ·53700 | 14·018 | ·54495 | ·01765 | ·225 | ·01084 |
| 55 | 14·535 | ·54754 | 13·675 | ·55640 | ·01616 | ·201 | ·01007 |
| 56 | 14·169 | ·55817 | 13·326 | ·56794 | ·01481 | ·180 | ·00936 |
| 57 | 13·800 | ·56894 | 12·971 | ·57957 | ·01351 | ·160 | ·00866 |
| 58 | 13·425 | ·57987 | 12·612 | ·59129 | ·01226 | ·141 | ·00797 |
| 59 | 13·042 | ·59102 | 12·251 | ·60301 | ·01106 | ·124 | ·00730 |
| 60 | 12·649 | ·60247 | 11·891 | ·61462 | ·00993 | ·108 | ·00664 |
| 61 | 12·244 | ·61424 | 11·533 | ·62612 | ·00884 | ·093 | ·00600 |
| 62 | 11·829 | ·62634 | 11·177 | ·63751 | ·00782 | ·080 | ·00538 |
| 63 | 11·403 | ·63875 | 10·824 | ·64874 | ·00686 | ·068 | ·00478 |
| 64 | 10·968 | ·65142 | 10·473 | ·65988 | ·00595 | ·057 | ·00421 |
| 65 | 10·527 | ·66426 | 10·124 | ·67091 | ·00510 | ·047 | ·00365 |
| 66 | 10·086 | ·67711 | 9·776 | ·68181 | ·00431 | ·039 | ·00313 |
| 67 | 9·650 | ·68981 | 9·430 | ·69263 | ·00358 | ·031 | ·00263 |
| 68 | 9·223 | ·70226 | 9·085 | ·70336 | ·00291 | ·024 | ·00216 |
| 69 | 8·808 | ·71432 | 8·740 | ·71402 | ·00230 | ·019 | ·00173 |
| 70 | 8·408 | ·72597 | 8·394 | ·72462 | ·00176 | ·014 | ·00133 |
| 71 | 8·022 | ·73723 | 8·049 | ·73516 | ·00128 | ·010 | ·00098 |
| 72 | 7·648 | ·74812 | 7·702 | ·74565 | ·00088 | ·006 | ·00068 |
| 73 | 7·286 | ·75866 | 7·355 | ·75610 | ·00054 | ·004 | ·00042 |
| 74 | 6·936 | ·76887 | 7·007 | ·76652 | ·00028 | ·002 | ·00022 |
| 75 | 6·595 | ·77880 | 6·656 | ·77691 | ·00010 | ·001 | ·00008 |
| 76 | 6·262 | ·78850 | 6·305 | ·78723 | ..... | ..... | ..... |
| 77 | 5·935 | ·79801 | 5·956 | ·79739 | ..... | ..... | ..... |
| 78 | 5·612 | ·80742 | 5·614 | ·80735 | ..... | ..... | ..... |
| 79 | 5·291 | ·81678 | 5·284 | ·81698 | ..... | ..... | ..... |
| 80 | 4·975 | ·82597 | 4·968 | 82618 | ..... | ..... | ..... |
| 81 | 4·674 | ·83474 | 4·670 | 83486 | ..... | ..... | ..... |
| 82 | 4·390 | ·84300 | 4·388 | 84307 | ..... | ..... | ..... |
| 83 | 4·121 | ·85086 | 4·120 | ·85087 | ..... | ..... | ..... |
| 84 | 3·865 | ·85830 | 3·865 | ·85830 | ..... | ..... | ..... |
| 85 | 3·620 | ·86544 | 3·620 | ·86544 | ..... | ..... | ..... |
| 86 | 3·382 | ·87236 | 3·382 | ·87236 | ..... | ..... | ..... |
| 87 | 3·153 | ·87904 | 3·153 | 87904 | ..... | ..... | ..... |
| 88 | 2·936 | ·88535 | 2·936 | ·88535 | ..... | ..... | ..... |
| 89 | 2·736 | ·89117 | 2·736 | ·89117 | ..... | ..... | ..... |
| 90 | 2·550 | ·89661 | 2·550 | ·89661 | ..... | ..... | ..... |
| 91 | 2·375 | ·90169 | 2·375 | ·90169 | ..... | ..... | ..... |
| 92 | 2·203 | ·90672 | 2·203 | ·90672 | ..... | ..... | ..... |
| 93 | 2·021 | ·91201 | 2·021 | ·91201 | ..... | ..... | ..... |
| 94 | 1·816 | ·91797 | 1·816 | ·91797 | ..... | ..... | ..... |
| 95 | 1·574 | ·92503 | 1·574 | ·92503 | ..... | ..... | ..... |
| 96 | 1·270 | ·93388 | 1·270 | ·93388 | ..... | ..... | ..... |
| 97 | ·885 | ·94510 | ·885 | ·94510 | ..... | ..... | ..... |
| 98 | ·447 | ·95786 | ·447 | ·95786 | ..... | ..... | ..... |
| 99 | ·000 | ·97087 | ·000 | ·97087 | ..... | ..... | ..... |

"assurance and the annuity on the life of $x$ independent of "marriage; but I have not been able to satisfy myself whether "this is the case or not."

The assurance referred to by Dr. Sprague consists of 1 payable on the death of a bachelor, provided he dies unmarried, and $\sqrt{(1+i)}$ payable on his death provided he has married. We can, however, deduce from the equations he gives the value of a *uniform* assurance of 1 payable on the death of a bachelor whether he dies married or unmarried, without introducing the $\sqrt{(1+i)}$. In other words, from the benefits dependent on marriage we can find the values of an assurance and an annuity on the life of $x$ independent of marriage, provided the married and unmarried are subject to the same rate of mortality. We have from Dr. Sprague's paper,

$$(bl)_x \times (ba)_x = (bl)_{x+1}v + (bl)_{x+2}v^2 + \ldots$$
$$(bl)_x \times (b\mathrm{A})_x = (bd)_x v + (bd)_{x+1}v^2 + \ldots$$
$$(bl)_x \times (bm\mathrm{E})_x = (bm)_x v + (bm)_{x+1}v^2 + \ldots$$

Now, for any age $z$ we have

$$(bl)_z = (bl)_{z+1} + (bd)_z + (bm)_z.$$

Hence, by addition we get

$$(bl)_x\{(ba)_x + (b\mathrm{A})_x + (bm\mathrm{E})_x\} = (bl)_x v + (bl)_{x+1}v^2 + \ldots$$
$$= (bl)_x v\{1 + (ba)_x\};$$

so that $(ba)_x + (b\mathrm{A})_x + (bm\mathrm{E})_x = v + v(ba)_x$,

and $$(b\mathrm{A})_x + (bm\mathrm{E})_x = v - (1-v)(ba)_x = \frac{1 - i(ba)_x}{1+i} \quad \ldots\ldots\ldots \quad (\alpha).$$

Again $(bl)_x(bm\mathrm{A})_x = (bm)_x v^{\frac{1}{2}}(m\mathrm{A})_{x+\frac{1}{2}} + (bm)_{x+1}v^{\frac{3}{2}}(m\mathrm{A})_{x+\frac{3}{2}} + \ldots$

also $$(bl)_x(bma)_x = (bm)_x v^{\frac{1}{2}}(ma)_{x+\frac{1}{2}} + (bm)_{x+1}v^{\frac{3}{2}}(ma)_{x+\frac{3}{2}} + \ldots$$

Now, observing that, whatever the age, $(1+i)(m\mathrm{A}) + i(ma) = 1$, we get $(bl)_x\{(1+i)(bm\mathrm{A})_x + i(bma)_x\} = (bm)_x v^{\frac{1}{2}} + (bm)_{x+1}v^{\frac{3}{2}} + \ldots$
$$= (bl)_x \sqrt{(1+i)}\,(bm\mathrm{E})_x,$$

whence $$(1+i)(bm\mathrm{A})_x + i(bma)_x = \sqrt{(1+i)}(bm\mathrm{E})_x \quad \ldots\ldots\ldots \quad (\beta).$$

At this point Dr. Sprague eliminates $(bm\mathrm{E})_x$ from the two equations, by multiplying $(\alpha)$ by $\sqrt{(1+i)}$ and adding it to $(\beta)$, obtains $$(b\mathrm{A})_x + \sqrt{(1+i)}\,(bm\mathrm{A})_x = \frac{1 - i\{(ba)_x + \sqrt{(1+i)}(bma)_x\}}{1+i}.$$

If, instead of eliminating $(bm\mathrm{E})_x$, we retain it, we get, on adding the two equations $\alpha$ and $\beta$ together and transposing,

$$(1+i)\{(b\mathrm{A})_x+(bm\mathrm{A})_x\}=1-i\{(ba)_x+(bma)_x\}+\sqrt{(1+i)}(bm\mathrm{E})_x$$
$$-(1+i)(bm\mathrm{E})_x,$$

and as $\sqrt{(1+i)}=1+\frac{1}{2}i$ very nearly,

$$\text{we have } (b\mathrm{A})_x+(bm\mathrm{A})_x=\frac{1-i\{(ba)_x+(bma)_x\}}{1+i}-\frac{\frac{1}{2}i\,(bm\mathrm{E})_x}{1+i},$$
$$=\frac{1-i\{(ba)_x+\frac{1}{2}(bm\mathrm{E})_x+(bma)_x\}}{1+i}.$$

Here we have the relation of the assurance to the annuity in the familiar form, and it may also be established by the following reasoning. We have an annuity of which the value is $(ba)_x$, payable so long as a bachelor aged $x$ remains alive and unmarried; an endowment of $\frac{1}{2}$ payable certainly at the end of the year of his marriage, which is assumed to take place in the middle of the year; and an annuity to commence on his marriage, and to continue for the remainder of life, the first payment being made on the first anniversary of his marriage. On the other hand, $(b\mathrm{A})_x+(bm\mathrm{A})_x$ is the value of an assurance payable on his death, whether he dies a bachelor or has meantime married. The following relation also subsists:—

$$(1+i)\{(b\mathrm{M})_x+(bm\mathrm{M})_x\}+i\{(b\mathrm{N})_x+\tfrac{1}{2}(bm\mathrm{EN})_x+(bm\mathrm{N})_x\}=(b\mathrm{D})_x$$

approximately.

As showing one of the purposes to which my results may be applied, I may mention that by means of the values in Table 15, column (6), we are able to say what the single premium should be for a policy effected on the life of a girl, for a sum payable by way of a dowry on her marriage. For example, at age 1 the net single premium would be 32·390 per cent. If it were desired that the premium should be returnable, in the event of her death unmarried, we find with the help of column 5, that the increased premium, to provide this, should be 36·716.

TABLE 16.—*Peerage Females (Mortality only)—Commutation Table, 3 per cent interest.*

| $x$ | $D_x$ | $N_x$ | $M_x$ | $x$ | $D_x$ | $N_x$ | $M_x$ |
|---|---|---|---|---|---|---|---|
| 0 | 1,135,588 | 27,776,998 | 293,474·2 | 50 | 177,216 | 2,863,071 | 88,663·22 |
| 1 | 1,045,620 | 26,731,878 | 236,582·0 | 51 | 170,350 | 2,692,721 | 86,959·72 |
| 2 | 999,842 | 25,731,536 | 221,258·18 | 52 | 163,651 | 2,529,070 | 85,222·42 |
| 3 | 962,764 | 24,768,772 | 213,801·94 | 53 | 157,106 | 2,371,964 | 83,443·45 |
| 4 | 928,927 | 23,839,845 | 207,506·34 | 54 | 150,715 | 2,221,249 | 81,628·54 |
| 5 | 897,094 | 22,942,751 | 202,730·07 | 55 | 144,467 | 2,076,782 | 79,770·27 |
| 6 | 866,959 | 22,075,792 | 198.724·38 | 56 | 138,365 | 1,938,417 | 77,877·10 |
| 7 | 838,256 | 21,237,536 | 195,272·00 | 57 | 132,415 | 1,806,002 | 75,956·54 |
| 8 | 810,584 | 20,426,952 | 192,014·90 | 58 | 126,618 | 1,679,384 | 74,015·93 |
| 9 | 783,903 | 19,643,049 | 188,943·87 | 59 | 120,975 | 1,558,409 | 72,061·04 |
| 10 | 758,177 | 18,884,872 | 186,050·09 | 60 | 115,490 | 1,442,919 | 70,099·60 |
| 11 | 733,369 | 18,151,503 | 183,324·39 | 61 | 110,165 | 1,332,754 | 68,137·95 |
| 12 | 709,443 | 17,442,060 | 180,758·74 | 62 | 104,988 | 1,227,766 | 66,170·08 |
| 13 | 686,229 | 16,755,831 | 178,207·89 | 63 | 99,958 | 1,127,818 | 64,192·73 |
| 14 | 663,708 | 16,092,123 | 175,674·48 | 64 | 95,043 | 1,032,770 | 62,194·10 |
| 15 | 641,863 | 15,450,260 | 173,159·66 | 65 | 90,235 | 942,535 | 60,154·57 |
| 16 | 620,610 | 14,829,650 | 170,603·43 | 66 | 85,496 | 857,039 | 58,043·67 |
| 17 | 599,764 | 14,229,886 | 167,832·46 | 67 | 80,797 | 776,242 | 55,835·12 |
| 18 | 579,405 | 13,650,481 | 164,941·89 | 68 | 76,123 | 700,119 | 53,513·49 |
| 19 | 559,581 | 13,090,950 | 161,943·90 | 69 | 71,459 | 628,660 | 51,066·95 |
| 20 | 540,190 | 12,550,760 | 158,899·79 | 70 | 66,818 | 561,842 | 48,507·15 |
| 21 | 521,412 | 12,029,348 | 155,856·19 | 71 | 62,218 | 499,624 | 45,854·05 |
| 22 | 503,237 | 11,526,111 | 152,867·84 | 72 | 57,682 | 441,942 | 43,129·77 |
| 23 | 485,672 | 11,040,439 | 149,959·93 | 73 | 53,229 | 388,713 | 40,357·46 |
| 24 | 468,697 | 10,571,742 | 147,129·88 | 74 | 48,888 | 339,830 | 37,561·33 |
| 25 | 452,291 | 10,119,451 | 144,376·48 | 75 | 44,664 | 295,166 | 34,765·79 |
| 26 | 436,439 | 9,683,012 | 141,697·20 | 76 | 40,592 | 254,574 | 31,994·78 |
| 27 | 421,120 | 9,261,892 | 139,090·60 | 77 | 36,682 | 217,891 | 29,267·60 |
| 28 | 406,320 | 8,855,572 | 136,555·55 | 78 | 32,950 | 184,941 | 26,603·50 |
| 29 | 392,000 | 8,463,572 | 134,070·15 | 79 | 29,405 | 155,536 | 24,018·75 |
| 30 | 378,109 | 8,085,463 | 131,597·00 | 80 | 26,037 | 129,499 | 21,506·45 |
| 31 | 364,601 | 7,720,862 | 129,101·88 | 81 | 22,826 | 106,673 | 19,054·28 |
| 32 | 351,431 | 7,369,431 | 126,551·67 | 82 | 19,790 | 86,888 | 16,683·02 |
| 33 | 338,636 | 7,030,795 | 123,992·08 | 83 | 16,966 | 69,917 | 14,435·01 |
| 34 | 326,241 | 6,704,554 | 121,460·47 | 84 | 14,371 | 55,546 | 12,334·97 |
| 35 | 314,269 | 6,390,285 | 118,990·20 | 85 | 12,028 | 43,523·1 | 10,405·37 |
| 36 | 302,705 | 6,087,580 | 116,580·15 | 86 | 9,931·4 | 33,591·7 | 8,663·72 |
| 37 | 291,538 | 5,796,042 | 114,229·58 | 87 | 8,088·7 | 25,503·0 | 7,110·34 |
| 38 | 280,755 | 5,515,287 | 111,937·71 | 88 | 6,478·8 | 19,024·2 | 5,735·96 |
| 39 | 270,343 | 5,244,944 | 109,703·76 | 89 | 5,091·7 | 13,932·5 | 4,537·60 |
| 40 | 260,292 | 4,984,652 | 107,526·29 | 90 | 3,925·0 | 10,007·5 | 3,519·17 |
| 41 | 250,587 | 4,734,065 | 105,402·41 | 91 | 2,964·8 | 7,042·7 | 2,673·32 |
| 42 | 241,219 | 4,492,846 | 103,333·46 | 92 | 2,199·1 | 4,343·6 | 1,998·95 |
| 43 | 232,178 | 4,260,668 | 101,318·88 | 93 | 1,603·4 | 3,240·2 | 1,462·35 |
| 44 | 223,455 | 4,037,213 | 99,357·26 | 94 | 1,150·4 | 2,089·84 | 1,056·02 |
| 45 | 215,037 | 3,822,176 | 97,448·01 | 95 | 811·97 | 1,277·87 | 751·10 |
| 46 | 206,926 | 3,615,250 | 95,600·27 | 96 | 562·91 | 714·96 | 525·69 |
| 47 | 199,110 | 3,416,140 | 93,812·09 | 97 | 379·30 | 335·66 | 358·47 |
| 48 | 191,572 | 3,224,568 | 92,072·60 | 98 | 232·01 | 103·65 | 222·24 |
| 49 | 184,281 | 3,040,287 | 90,361·93 | 99 | 103·65 | ..... | 100·63 |

TABLE 17.—*Peerage Females (Mortality only)—Commutation Table, 4 per cent interest.*

| $x$ | $D_x$ | $N_x$ | $M_x$ | $x$ | $D_x$ | $N_x$ | $M_x$ |
|---|---|---|---|---|---|---|---|
| 0 | 1,135,588 | 22,580,951 | 223,413·7 | 50 | 109,319 | 1,571,286 | 44,682·21 |
| 1 | 1,035,567 | 21,545,384 | 167,068·4 | 51 | 104,074 | 1,467,162 | 43,641·48 |
| 2 | 980,706 | 20,564,678 | 152,087·87 | 52 | 99,020 | 1,368,142 | 42,590·30 |
| 3 | 935,260 | 19,629,418 | 144,308·93 | 53 | 94,145 | 1,278,997 | 41,524·25 |
| 4 | 893,708 | 18,785,710 | 138,783·05 | 54 | 89,447 | 1,184,550 | 40,447·13 |
| 5 | 854,786 | 17,880,924 | 134,182·04 | 55 | 84,915 | 1,099,635 | 39,354·88 |
| 6 | 818,130 | 17,062,794 | 130,401·97 | 56 | 80,546 | 1,019,089 | 38,252·82 |
| 7 | 783,437 | 16,279,357 | 127,175·36 | 57 | 76,341 | 942,748 | 37,145·56 |
| 8 | 750,290 | 15,529,067 | 124,160·54 | 58 | 72,297 | 870,451 | 36,037·50 |
| 9 | 718,617 | 14,810,450 | 121,345·28 | 59 | 68,411 | 802,040 | 34,932·02 |
| 10 | 688,350 | 14,122,100 | 118,718·02 | 60 | 64,681 | 737,359 | 33,833·50 |
| 11 | 659,424 | 13,462,676 | 116,267·15 | 61 | 61,105 | 676,254 | 32,745·43 |
| 12 | 631,777 | 12,830,899 | 113,982·37 | 62 | 57,674 | 618,580 | 31,664·40 |
| 13 | 605,230 | 12,225,669 | 111,732·62 | 63 | 54,380 | 564,200 | 30,588·61 |
| 14 | 579,737 | 11,645,932 | 109,519·74 | 64 | 51,212 | 512,988 | 29,511·70 |
| 15 | 555,265 | 11,090,667 | 107,344·21 | 65 | 48,154 | 464,834 | 28,423·31 |
| 16 | 531,718 | 10,558,949 | 105,154·12 | 66 | 45,186 | 419,648 | 27,307·67 |
| 17 | 508,916 | 10,050,033 | 102,802·88 | 67 | 42,292 | 377,356 | 26,151·65 |
| 18 | 486,918 | 9,563,120 | 100,378·74 | 68 | 39,462 | 337,894 | 24,948·12 |
| 19 | 465,691 | 9,097,429 | 97,878·54 | 69 | 36,688 | 301,206 | 23,692·08 |
| 20 | 445,271 | 8,652,158 | 95,369·32 | 70 | 33,975 | 267,231 | 22,390·43 |
| 21 | 425,660 | 8,226,498 | 92,884·64 | 71 | 31,332 | 235,899 | 21,054·36 |
| 22 | 406,872 | 7,819,626 | 90,468·53 | 72 | 28,769 | 207,130 | 19,695·63 |
| 23 | 388,896 | 7,430,730 | 88,140·07 | 73 | 26,293 | 180,837 | 18,326·25 |
| 24 | 371,693 | 7,059,037 | 85,895·70 | 74 | 23,914 | 156,923 | 16,958·38 |
| 25 | 355,235 | 6,703,802 | 83,733·15 | 75 | 21,639 | 135,284 | 15,603·95 |
| 26 | 339,488 | 6,364,314 | 81,649·09 | 76 | 19,478 | 115,806 | 14,274·31 |
| 27 | 324,422 | 6,039,892 | 79,641·02 | 77 | 17,432 | 98,374 | 12,978·29 |
| 28 | 310,010 | 5,729,882 | 77,706·85 | 78 | 15,508 | 82,866 | 11,724·42 |
| 29 | 296,209 | 5,433,673 | 75,828·80 | 79 | 13,707 | 69,159 | 10,519·59 |
| 30 | 282,966 | 5,150,707 | 73,977·96 | 80 | 12,020 | 57,139 | 9,359·80 |
| 31 | 270,232 | 4,880,475 | 72,128·64 | 81 | 10,436·3 | 46,703·0 | 8,238·65 |
| 32 | 257,967 | 4,622,508 | 70,256·67 | 82 | 8,961·2 | 37,741·8 | 7,164·91 |
| 33 | 246,135 | 4,376,323 | 68,395·84 | 83 | 7,608·4 | 30,133·4 | 6,156·77 |
| 34 | 234,893 | 4,141,430 | 66,573·12 | 84 | 6,383·0 | 23,750·4 | 5,224·04 |
| 35 | 224,098 | 3,917,332 | 64,811·63 | 85 | 5,288·7 | 18,461·7 | 4,375·25 |
| 36 | 213,776 | 3,703,556 | 63,109·61 | 86 | 4,326·6 | 14,135·1 | 3,616·51 |
| 37 | 203,910 | 3,499,646 | 61,465·55 | 87 | 3,490·0 | 10,645·1 | 2,946·29 |
| 38 | 194,480 | 3,305,166 | 59,877·96 | 88 | 2,768·4 | 7,876·7 | 2,359·01 |
| 39 | 185,467 | 3,119,699 | 58,345·87 | 89 | 2,154·8 | 5,721·9 | 1,851·86 |
| 40 | 176,854 | 2,942,845 | 56,865·89 | 90 | 1,645·1 | 4,076·8 | 1,425·01 |
| 41 | 168,628 | 2,774,222 | 55,436·71 | 91 | 1,230·7 | 2,846·1 | 1,078·89 |
| 42 | 160,759 | 2,613,463 | 54,057·87 | 92 | 904·06 | 1,942·07 | 794·60 |
| 43 | 153,246 | 2,460,217 | 52,728·18 | 93 | 652·85 | 1,289·22 | 578·15 |
| 44 | 146,070 | 2,314,147 | 51,445·89 | 94 | 463·89 | 825·33 | 414·30 |
| 45 | 139,216 | 2,174,931 | 50,209·84 | 95 | 324·27 | 501·06 | 292·53 |
| 46 | 132,676 | 2,042,255 | 49,025·11 | 96 | 222·65 | 278·41 | 203·37 |
| 47 | 126,438 | 1,915,817 | 47,889·59 | 97 | 148·58 | 129·83 | 137·87 |
| 48 | 120,481 | 1,795,336 | 46,795·61 | 98 | 90·01 | 39·82 | 85·02 |
| 49 | 114,781 | 1,680,555 | 45,730·10 | 99 | 39·82 | ..... | 38·29 |

TABLE 18.—*Peerage Females (Mortality only)—Commutation Table, 5 per cent interest.*

| $x$ | $D_x$ | $N_x$ | $M_x$ | $x$ | $D_x$ | $N_x$ | $M_x$ |
|---|---|---|---|---|---|---|---|
| 0 | 1,135,588 | 18,887,438 | 182,111·0 | 50 | 67,748 | 873,360 | 22,933·51 |
| 1 | 1,025,705 | 17,861,733 | 126,302·4 | 51 | 63,883 | 809,477 | 22,294·68 |
| 2 | 962,115 | 16,899,618 | 111,556·75 | 52 | 60,202 | 749,275 | 21,655·59 |
| 3 | 908,790 | 15,990,828 | 104,046·55 | 53 | 56,694 | 692,581 | 21,013·63 |
| 4 | 860,148 | 15,130,680 | 98,680·06 | 54 | 53,351 | 639,230 | 20,371·17 |
| 5 | 814,850 | 14,315,830 | 94,341·67 | 55 | 50,165 | 589,065 | 19,725·89 |
| 6 | 772,478 | 13,543,352 | 90,772·53 | 56 | 47,132 | 541,933 | 19,081·02 |
| 7 | 732,677 | 12,810,675 | 87,754·98 | 57 | 44,245 | 497,688 | 18,439·28 |
| 8 | 694,995 | 12,115,680 | 84,962·34 | 58 | 41,503 | 456,185 | 17,803·19 |
| 9 | 659,315 | 11,456,365 | 82,379·40 | 59 | 38,898 | 417,287 | 17,174·63 |
| 10 | 625,533 | 10,830,832 | 79,991·89 | 60 | 36,427 | 380,860 | 16,555·97 |
| 11 | 593,540 | 10,237,292 | 77,785·90 | 61 | 34,085 | 346,775 | 15,949·03 |
| 12 | 563,239 | 9,674,053 | 75,748·99 | 62 | 31,865 | 314,910 | 15,351·76 |
| 13 | 534,431 | 9,139,622 | 73,762·40 | 63 | 29,759 | 285,151 | 14,763·05 |
| 14 | 507,047 | 8,632,575 | 71,826·98 | 64 | 27,758 | 257,393 | 14,179·34 |
| 15 | 481,018 | 8,151,557 | 69,942·35 | 65 | 25,852 | 231,541 | 13,595·02 |
| 16 | 456,232 | 7,695,325 | 68,063·18 | 66 | 24,028 | 207,513 | 13,001·78 |
| 17 | 432,509 | 7,262,816 | 66,064·94 | 67 | 22,274 | 185,239 | 12,392·92 |
| 18 | 409,869 | 6,852,947 | 64,020·16 | 68 | 20,586 | 164,653 | 11,765·08 |
| 19 | 388,270 | 6,464,677 | 61,939·79 | 69 | 18,957 | 145,696 | 11,116·06 |
| 20 | 367,710 | 6,096,967 | 59,867·65 | 70 | 17,388 | 128,308 | 10,449·93 |
| 21 | 348,168 | 5,748,799 | 57,835·32 | 71 | 15,883 | 112,425 | 9,772·66 |
| 22 | 329,630 | 5,419,169 | 55,877·89 | 72 | 14,444 | 97,981 | 9,090·48 |
| 23 | 312,065 | 5,107,104 | 54,009·44 | 73 | 13,075 | 84,906 | 8,409·49 |
| 24 | 295,421 | 4,811,683 | 52,225·62 | 74 | 11,779 | 73,127 | 7,735·73 |
| 25 | 279,652 | 4,532,031 | 50,523·20 | 75 | 10,557 | 62,570·4 | 7,074·95 |
| 26 | 264,710 | 4,267,321 | 48,898·19 | 76 | 9,412·0 | 53,158·4 | 6,432·44 |
| 27 | 250,553 | 4,016,768 | 47,347·35 | 77 | 8,343·5 | 44,814·9 | 5,812·14 |
| 28 | 237,143 | 3,779,625 | 45,867·81 | 78 | 7,351·3 | 37,463·1 | 5,217·72 |
| 29 | 224,427 | 3,555,198 | 44,444·87 | 79 | 6,435·9 | 31,027·2 | 4,652·00 |
| 30 | 212,351 | 3,342,847 | 43,055·91 | 80 | 5,590·1 | 25,437·1 | 4,112·61 |
| 31 | 200,865 | 3,141,982 | 41,681·31 | 81 | 4,807·5 | 20,629·6 | 3,596·15 |
| 32 | 189,921 | 2,952,061 | 40,303·12 | 82 | 4,088·6 | 16,541·0 | 3,106·25 |
| 33 | 179,520 | 2,772,541 | 38,946·19 | 83 | 3,438·3 | 13,102·7 | 2,650·66 |
| 34 | 169,656 | 2,602,885 | 37,629·70 | 84 | 2,857·1 | 10,245·6 | 2,233·16 |
| 35 | 160,317 | 2,442,568 | 36,369·55 | 85 | 2,344·7 | 7,900·9 | 1,856·85 |
| 36 | 151,476 | 2,291,092 | 35,163·54 | 86 | 1,899·9 | 6,001·0 | 1,523·67 |
| 37 | 143,109 | 2,147,983 | 34,009·70 | 87 | 1,517·9 | 4,483·1 | 1,232·17 |
| 38 | 135,191 | 2,012,792 | 32,906·10 | 88 | 1,192·7 | 3,290·43 | 979·16 |
| 39 | 127,698 | 1,885,094 | 31,850·88 | 89 | 919·45 | 2,370·98 | 762·77 |
| 40 | 120,609 | 1,764,485 | 30,841·93 | 90 | 695·26 | 1,675·72 | 582·36 |
| 41 | 113,900 | 1,650,585 | 29,876·56 | 91 | 515·18 | 1,160·54 | 435·38 |
| 42 | 107,553 | 1,543,032 | 28,954·07 | 92 | 374·84 | 785·70 | 319·58 |
| 43 | 101,550 | 1,441,482 | 28,072·93 | 93 | 268·10 | 517·60 | 230·69 |
| 44 | 95,878 | 1,345,609 | 27,231·29 | 94 | 188·69 | 328·91 | 164·04 |
| 45 | 90,504 | 1,255,105 | 26,427·73 | 95 | 130·65 | 198·26 | 114·98 |
| 46 | 85,432 | 1,169,673 | 25,664·87 | 96 | 88·85 | 109·41 | 79·41 |
| 47 | 80,639 | 1,089,034 | 24,940·67 | 97 | 58·73 | 50·68 | 53·52 |
| 48 | 76,109 | 1,012,925 | 24,249·60 | 98 | 35·24 | 15·44 | 32·82 |
| 49 | 71,817 | 941,108 | 23,582·92 | 99 | 15·44 | ..... | 14·71 |

TABLE 19.—*Peerage Females—Annuity and Assurance Values.*

| $x$ | 3 per cent. | | 4 per cent. | | 5 per cent. | | $x$ |
|---|---|---|---|---|---|---|---|
| | $a_x$ | $A_x$ | $a_x$ | $A_x$ | $a_x$ | $A_x$ | |
| 0 | 24·460 | ·25843 | 19·885 | ·19674 | 16·632 | ·16037 | 0 |
| 1 | 25·565 | ·22626 | 20·805 | ·16133 | 17·414 | ·12314 | 1 |
| 2 | 25·736 | ·22129 | 20·969 | ·15503 | 17·565 | ·11595 | 2 |
| 3 | 25·727 | ·22155 | 20·988 | ·15430 | 17·596 | ·11449 | 3 |
| 4 | 25·664 | ·22338 | 20·964 | ·15523 | 17·591 | ·11472 | 4 |
| 5 | 25·575 | ·22599 | 20·919 | ·15698 | 17·569 | ·11578 | 5 |
| 6 | 25·463 | ·22922 | 20·856 | ·15939 | 17·532 | ·11751 | 6 |
| 7 | 25·335 | ·23295 | 20·779 | ·16233 | 17·485 | ·11977 | 7 |
| 8 | 25·200 | ·23689 | 20·697 | ·16548 | 17·433 | ·12225 | 8 |
| 9 | 25·058 | ·24103 | 20·610 | ·16886 | 17·376 | ·12495 | 9 |
| 10 | 24·908 | ·24539 | 20·516 | ·17247 | 17·315 | ·12788 | 10 |
| 11 | 24·751 | ·24998 | 20·416 | ·17632 | 17·248 | ·13105 | 11 |
| 12 | 24·586 | ·25479 | 20·309 | ·18042 | 17·176 | ·13449 | 12 |
| 13 | 24·417 | ·25969 | 20·200 | ·18461 | 17·102 | ·13802 | 13 |
| 14 | 24·246 | ·26469 | 20·088 | ·18891 | 17·025 | ·14166 | 14 |
| 15 | 24·071 | ·26978 | 19·974 | ·19332 | 16·947 | ·14541 | 15 |
| 16 | 23·895 | ·27490 | 19·858 | ·19776 | 16·867 | ·14919 | 16 |
| 17 | 23·726 | ·27983 | 19·748 | ·20200 | 16·792 | ·15275 | 17 |
| 18 | 23·560 | ·28468 | 19·640 | ·20614 | 16·720 | ·15620 | 18 |
| 19 | 23·396 | ·28943 | 19·535 | ·21018 | 16·650 | ·15953 | 19 |
| 20 | 23·234 | ·29416 | 19·431 | ·21418 | 16·581 | ·16281 | 20 |
| 21 | 23·071 | ·29891 | 19·327 | ·21821 | 16·512 | ·16611 | 21 |
| 22 | 22·904 | ·30377 | 19·219 | ·22235 | 16·440 | ·16952 | 22 |
| 23 | 22·732 | ·30877 | 19·107 | ·22664 | 16·366 | ·17307 | 23 |
| 24 | 22·556 | ·31391 | 18·992 | ·23109 | 16·288 | ·17678 | 24 |
| 25 | 22·374 | ·31921 | 18·872 | ·23571 | 16·206 | ·18067 | 25 |
| 26 | 22·186 | ·32467 | 18·747 | ·24051 | 16·121 | ·18472 | 26 |
| 27 | 21·994 | ·33029 | 18·617 | ·24549 | 16·032 | ·18897 | 27 |
| 28 | 21·795 | ·33608 | 18·483 | ·25066 | 15·938 | ·19342 | 28 |
| 29 | 21·591 | ·34202 | 18·344 | ·25600 | 15·841 | ·19804 | 29 |
| 30 | 21·384 | ·34804 | 18·203 | ·26144 | 15·742 | ·20276 | 30 |
| 31 | 21·176 | ·35409 | 18·060 | ·26691 | 15·642 | ·20751 | 31 |
| 32 | 20·970 | ·36010 | 17·919 | ·27235 | 15·544 | ·21221 | 32 |
| 33 | 20·762 | ·36615 | 17·777 | ·27782 | 15·444 | ·21695 | 33 |
| 34 | 20·551 | ·37230 | 17·631 | ·28342 | 15·342 | ·22180 | 34 |
| 35 | 20·334 | ·37863 | 17·481 | ·28921 | 15·236 | ·22686 | 35 |
| 36 | 20·111 | ·38513 | 17·324 | ·29521 | 15·125 | ·23214 | 36 |
| 37 | 19·881 | ·39182 | 17·163 | ·30144 | 15·009 | ·23765 | 37 |
| 38 | 19·645 | ·39870 | 16·995 | ·30789 | 14·889 | ·24340 | 38 |
| 39 | 19·401 | ·40580 | 16·821 | ·31459 | 14·762 | ·24942 | 39 |
| 40 | 19·150 | ·41310 | 16·640 | ·32154 | 14·630 | ·25572 | 40 |
| 41 | 18·892 | ·42062 | 16·452 | ·32876 | 14·492 | ·26231 | 41 |
| 42 | 18·626 | ·42838 | 16·257 | ·33627 | 14·347 | ·26921 | 42 |
| 43 | 18·351 | ·43638 | 16·054 | ·34408 | 14·195 | ·27644 | 43 |
| 44 | 18·067 | ·44464 | 15·843 | ·35220 | 14·035 | ·28403 | 44 |
| 45 | 17·775 | ·45317 | 15·623 | ·36066 | 13·868 | ·29201 | 45 |
| 46 | 17·471 | ·46200 | 15·393 | ·36951 | 13·691 | ·30041 | 46 |
| 47 | 17·157 | ·47116 | 15·152 | ·37876 | 13·505 | ·30929 | 47 |
| 48 | 16·832 | ·48062 | 14·901 | ·38841 | 13·309 | ·31862 | 48 |
| 49 | 16·498 | ·49035 | 14·641 | ·39841 | 13·104 | ·32837 | 49 |

TABLE 19.—*Continued.*

| $x$ | 3 per cent. | | 4 per cent. | | 5 per cent. | | $x$ |
|---|---|---|---|---|---|---|---|
| | $a_x$ | $A_x$ | $a_x$ | $A_x$ | $a_x$ | $A_x$ | |
| 50 | 16·156 | ·50031 | 14·373 | ·40873 | 12·891 | ·33851 | 50 |
| 51 | 15·807 | ·51048 | 14·097 | ·41933 | 12·671 | ·34899 | 51 |
| 52 | 15·454 | ·52076 | 13·817 | ·43012 | 12·446 | ·35972 | 52 |
| 53 | 15·098 | ·53113 | 13·532 | ·44107 | 12·216 | ·37065 | 53 |
| 54 | 14·738 | ·54161 | 13·243 | ·45219 | 11·982 | ·38183 | 54 |
| 55 | 14·376 | ·55217 | 12·950 | ·46347 | 11·743 | ·39322 | 55 |
| 56 | 14·009 | ·56284 | 12·652 | ·47492 | 11·498 | ·40485 | 56 |
| 57 | 13·639 | ·57362 | 12·349 | ·48657 | 11·248 | ·41675 | 57 |
| 58 | 13·263 | ·58456 | 12·040 | ·49847 | 10·992 | ·42897 | 58 |
| 59 | 12·882 | ·59567 | 11·724 | ·51062 | 10·728 | ·44153 | 59 |
| 60 | 12·494 | ·60698 | 11·400 | ·52308 | 10·456 | ·45450 | 60 |
| 61 | 12·098 | ·61851 | 11·067 | ·53589 | 10·174 | ·46792 | 61 |
| 62 | 11·694 | ·63026 | 10·725 | ·54902 | 9·883 | ·48178 | 62 |
| 63 | 11·283 | ·64223 | 10·375 | ·56250 | 9·582 | ·49609 | 63 |
| 64 | 10·866 | ·65438 | 10·017 | ·57627 | 9·273 | ·51082 | 64 |
| 65 | 10·448 | ·66664 | 9·653 | ·59026 | 8·957 | ·52588 | 65 |
| 66 | 10·024 | ·67890 | 9·287 | ·60434 | 8·637 | ·54112 | 66 |
| 67 | 9·607 | ·69105 | 8·923 | ·61836 | 8·316 | ·55637 | 67 |
| 68 | 9·197 | ·70299 | 8·563 | ·63221 | 7·998 | ·57151 | 68 |
| 69 | 8·798 | ·71464 | 8·210 | ·64577 | 7·686 | ·58639 | 69 |
| 70 | 8·409 | ·72596 | 7·865 | ·65902 | 7·379 | ·60099 | 70 |
| 71 | 8·030 | ·73699 | 7·529 | ·67197 | 7·079 | ·61531 | 71 |
| 72 | 7·662 | ·74772 | 7·200 | ·68462 | 6·784 | ·62936 | 72 |
| 73 | 7·303 | ·75818 | 6·878 | ·69701 | 6·494 | ·64316 | 73 |
| 74 | 6·952 | ·76839 | 6·562 | ·70915 | 6·208 | ·65675 | 74 |
| 75 | 6·609 | ·77839 | 6·252 | ·72109 | 5·927 | ·67015 | 75 |
| 76 | 6·272 | ·78821 | 5·946 | ·73286 | 5·648 | ·68343 | 76 |
| 77 | 5·940 | ·79786 | 5·643 | ·74449 | 5·371 | ·69661 | 77 |
| 78 | 5·613 | ·80739 | 5·343 | ·75602 | 5·096 | ·70973 | 78 |
| 79 | 5·289 | ·81682 | 5·046 | ·76748 | 4·821 | ·72282 | 79 |
| 80 | 4·973 | ·82601 | 4·754 | ·77870 | 4·550 | ·73570 | 80 |
| 81 | 4·673 | ·83476 | 4·475 | ·78942 | 4·291 | ·74804 | 81 |
| 82 | 4·390 | ·84300 | 4·212 | ·79955 | 4·046 | ·75973 | 82 |
| 83 | 4·121 | ·85084 | 3·961 | ·80921 | 3·811 | ·77092 | 83 |
| 84 | 3·865 | ·85830 | 3·721 | ·81843 | 3·586 | ·78162 | 84 |
| 85 | 3·620 | ·86514 | 3·491 | ·82728 | 3·370 | ·79192 | 85 |
| 86 | 3·382 | ·87236 | 3·267 | ·83588 | 3·159 | ·80197 | 86 |
| 87 | 3·153 | ·87904 | 3·050 | ·84422 | 2·953 | ·81174 | 87 |
| 88 | 2·936 | ·88535 | 2·845 | ·85211 | 2·759 | ·82100 | 88 |
| 89 | 2·736 | ·89117 | 2·655 | ·85941 | 2·579 | ·82959 | 89 |
| 90 | 2·550 | ·89661 | 2·478 | ·86623 | 2·410 | ·83761 | 90 |
| 91 | 2·375 | ·90169 | 2·313 | ·87259 | 2·253 | ·84511 | 91 |
| 92 | 2·203 | ·90672 | 2·148 | ·87892 | 2·096 | ·85257 | 92 |
| 93 | 2·021 | ·91201 | 1·975 | ·88559 | 1·931 | ·86045 | 93 |
| 94 | 1·818 | ·91797 | 1·779 | ·89311 | 1·748 | ·86938 | 94 |
| 95 | 1·574 | ·92503 | 1·545 | ·90211 | 1·518 | ·88012 | 95 |
| 96 | 1·270 | ·93388 | 1·251 | ·91344 | 1·231 | ·89374 | 96 |
| 97 | ·885 | ·94510 | ·874 | ·92793 | ·863 | ·91129 | 97 |
| 98 | ·447 | ·95786 | ·442 | ·94452 | ·438 | ·93151 | 98 |
| 99 | ... | ·97087 | ... | ·96154 | ... | ·95238 | 99 |

## DISCUSSION.

THE PRESIDENT (Mr. G. M. LOW), in inviting discussion, said—I think we may congratulate ourselves to-day on having heard a paper of the kind which is most acceptable to a Society like ours—one containing the results of a careful independent examination of a particular problem that has not been investigated before. Mr. Lees has further presented us, in the latter part of his paper, with some very interesting formulas and demonstrations, which from the merely theoretical point of view cannot fail to be of use to the student. He has worked out in a masterly manner the statistics that form the subject of his investigation, so that we have the results in a very clear and simple form; but looking underneath the surface we can see what a vast amount of labour he has undertaken, in order that he might be able to present a paper of this kind for the benefit of his professional brethren.

Mr. J. CHATHAM.—I heartily endorse the remarks our President has made regarding Mr. Lees's paper. It is, I think, the first time that the results of an important original research have been submitted to the Faculty, and I venture to think that Mr. Lees has done it well, and that his paper will take a high place among investigations of this kind. Any one who has conducted a similar investigation must know the labour involved in writing, sorting, and tabulating the cards; and certainly the results here brought out are of the highest importance. Mr. Lees tells us that the last volume of the Peerage used by him was 1898; but he does not give the date of the close of his observations, and I think it is desirable to know this. I understand that he followed the method adopted by Dr. Sprague in his investigation; but there is another plan available, namely, to form first of all a table of mortality, and then a table of marriage, and to combine the two in the same way as one would form a joint-life table of male and female mortality. It is simpler in working, and might have saved some time. In Table 3, where the rate of mortality obtained by Mr. Lees for females is compared with other tables, undoubtedly the most important comparison is that with Dr. Thiele's rates. It should not be forgotten, however, that that gentleman merely graduated the observations of Messrs. Bailey and Day, and that these related to the period 1800-1855. Now, if we compare the rate which Mr. Lees has brought out with that of Dr. Thiele, it will be seen that for ages 10 to 30 it is, roughly speaking, about 70 per cent, from 30 to 50 about 77 per cent, and from 50 to 70 about 70 per cent, of Dr. Thiele's. The author has not assigned any cause for this remarkable reduction in the rate of mortality; but it seems to me that it may be due to some extent to the manner in which the statistics have been brought together. No doubt when a peerage has subsisted for some time the particulars given will be correct, but where it is of recent creation they are not so likely to be correct; and I should like to know whether these new creations were taken into account, and if so, whether they form a substantial portion of the statistics. There are also the extinct peerages, before 1863, which would not be taken into account at all by the method of extraction adopted, which tends to a

process of selection. Undoubtedly part of the improvement in mortality is due to the same causes as the improvement which is observable in other classes of lives. I looked about for some means of ascertaining this, and I found a table in the *Clergy Mutual* experience, which is divided into three periods—1750-1860 (entrants in the eighteenth century), 1800-1860 (entrants in the nineteenth century), and 1829-1887 (the Society's experience). The rates of mortality among the clergy during these three periods show a substantial improvement, and there is no doubt that a similar improvement has taken place in the rates of mortality of the peerage females. Mr. Lees started in 1863; but, from the method he adopted, his observations must extend back to a much earlier period—in fact, back as far as the eighteenth century. It is satisfactory to find that at the older ages his rates agree so closely with those deduced by Mr. Campbell in his recent paper, and this shows once more how unsuitable for the valuation of reversions the Carlisle Table is. I looked for some other experience with which to test Mr. Lees's results. The nearest I could get to them were those given by Mr. Hewat in the Scottish Ministers' experience, 1845-1900. Dealing first of all with children, the rate of mortality among the children of Scottish Ministers (their experience being given only from 1860 to 1900, and both sexes being included) is in the first year of life 619 per 10,000, as compared with 516 per 10,000 in Mr. Lees's experience. The higher rate is no doubt due to the inclusion of the males, who are subject to a heavier mortality in the first year than females. The comparative rates for the first 11 years are as follows:—

| Age. | Scottish Ministers' Children. | Peerage Females. | Age. | Scottish Ministers' Children. | Peerage Females. |
|---|---|---|---|---|---|
| 0 | ·0619 | ·0516 | 6 | ·0058 | ·0041 |
| 1 | ·0157 | ·0151 | 7 | ·0048 | ·0040 |
| 2 | ·0094 | ·0082 | 8 | ·0044 | ·0039 |
| 3 | ·0084 | ·0062 | 9 | ·0041 | ·0038 |
| 4 | ·0077 | ·0053 | 10 | ·0040 | ·0037 |
| 5 | ·0068 | ·0046 | | | |

The rate of mortality among the ministers' children is throughout higher than among the peerage females, although towards the end they approach each other, and it is perhaps at that particular part of the table that the error in the statistics to which I have alluded might tell with greatest effect. Coming next to adults, in the ministers' wives and widows we have a class corresponding exactly with Mr. Lees's married females. The following are the comparative rates of mortality:—

| Age. | Scottish Ministers' Wives and Widows. | British Peerage Wives and Widows. | Difference. |
|---|---|---|---|
| 20 | ·0075 | ·0085 | + ·0010 |
| 30 | ·0102 | ·0081 | − ·0021 |
| 40 | ·0081 | ·0088 | + ·0007 |
| 50 | ·0104 | ·0095 | − ·0009 |
| 60 | ·0167 | ·0163 | − ·0004 |
| 70 | ·0493 | ·0400 | − ·0093 |
| 80 | ·1043 | ·0969 | − ·0074 |
| 90 | ·3539 | ·2220 | − ·1319 |

Of course no great importance is to be attached to the rates at the high ages. Mr. Hewat kills off his females at 95: Mr. Lees is a little more gallant, and gives them 4 years longer. All through there is a very close agreement in the statistics of the married. Incidentally I may remark that there is also a very close agreement between the rates of mortality of

Scottish Ministers and those deduced from the *Clergy Mutual* office experience. Mr. Lees's results may also be compared with those for assured lives. In a recent paper I gave the net premiums for females in the British Offices whole-life experience with profits, calculated at 3½ per cent interest and deduced from the unadjusted observations. Taking Mr. Lees's unadjusted data also, the comparison of net premiums for the assurance of 100 is as follows :—

| Age. | British Offices Females. | British Peerage Females. | Difference. |
|---|---|---|---|
| 22 | 1·547 | 1·180 | ·367 |
| 32 | 1·853 | 1·534 | ·319 |
| 42 | 2·469 | 2·042 | ·427 |
| 52 | 3·566 | 3·034 | ·532 |
| 62 | 5·749 | 4·838 | ·911 |

In every case the premium according to the British Peerage Females experience is very much less than that of the British Offices Females, notwithstanding that there is no medical examination. Taking even the best class of whole-life policyholder in the latter experience, namely, the limited-payment class, the results are not so good as those of the British Peerage Females. So that the ladies of the peerage must be splendid lives indeed; and ought to be received with open arms by insurance offices. Then to show how the comparison is affected when temporary assurances are taken, I have calculated the net annual premium for temporary assurances ceasing at 50, as follows :—

| Age. | British Offices Females. | Peerage Females. | | |
|---|---|---|---|---|
| | | Spinsters. | Married. | Combined. |
| 22 | 1·096 | ·585 | ·831 | ·710 |
| 27 | 1·061 | ·646 | ·833 | ·762 |
| 32 | 1·075 | ·751 | ·835 | ·807 |
| 37 | 1·015 | ·835 | ·798 | ·805 |
| 42 | 1·103 | ·980 | ·778 | ·825 |

The higher rate of premium is of course required for the married, but for ages 37 and 42 the premium for spinsters is the greater here. These results are very different from the results of the British Offices experience, and also from those for male lives. Mr. Lees has given us a combined marriage and mortality table, and deduced a number of commutation columns from it. He enables us to determine the single premium for a spinster marrying, and shows that at age 1, for a dowry of £100 payable on marriage, £32 would be required; but I would ask, will any one give £32 at age 1 for £100 on a daughter's marriage? Even if the premium were returnable, it is doubtful if a policy of the kind would be of much practical use, because it might not be possible to return the premium for a very long time indeed. Mr. Lees has given also a very interesting formula showing that the relation of the assurance to the annuity can be expressed in the ordinary familiar form, a problem which Dr. Sprague attempted some time ago and came to no definite conclusion upon. I think Mr. Lees has arrived at a satisfactory result, and that it is very neat indeed. Mr. Lees says that he does not on this occasion deal with various other functions that might be deduced from his facts, but I certainly hope that he will favour us with them on another occasion.

Mr. R. Murrie.—I desire to add my tribute of commendation and admiration for Mr. Lees's masterly work. The paper is indeed a monument, no less of his skill, than of his industry. We all know the constant

care and watchfulness required in working with the Peerage volumes, to see that no facts are admitted, and that none are left out, which would vitiate our data for the object in view. Mr. Lees is to be congratulated on the fact that he has worked out his tables on a solid and homogeneous basis, his rates of mortality and marriage being derived from the same body of lives. No doubt it is permissible, and may give very good practical results, to combine the rate of mortality in one body with the rate of marriage in another; but at best it must be looked upon as a makeshift. I may illustrate this. Suppose that mortality tables are constructed from two bodies of lives $A$ and $B$, and that in both tables $l_{x+n}=10{,}000$, but that $d_{x+n}$ is 105 for the $A$ lives and only 100 for the $B$ lives. Now, the observed rate of marriage among the 10,000 $A$ lives is not strictly applicable to the 10,000 $B$ lives, because the former include more lives who will die in the year, and who therefore cannot have the same chance of marriage as the others. Mr. Lees begins very modestly with about 4000 lives, and then by a wave of his wand he turns them into a million. No doubt he has got many precedents for this operation. If we go back to the middle ages, we have the case of St. Ursula and her eleven followers, who, by an error of the mediæval copyist, were turned into 11,000, to the admiration of succeeding ages. Mr. Lees has more recent and very eminent actuarial sanction for what he has done, but the device seems to be somewhat theatrical. It gives to the results an appearance of extreme mathematical accuracy, which they certainly do not possess, and which in point of fact no one wants—at all events in connection with probabilities of marriage. Another advantage in keeping the numbers small, would have been that Bremiker's or Sang's tables might have been discarded in favour of the Institute of Actuaries four-figure logarithm card or Erskine Scott's book, which would have lightened the work to some extent. Mr. Lees has compared his results with a great many tables, but there is another to which he might have referred, namely, the table of the rates of mortality among married females of the Peerage, given by Mr. J. R. Hart (*T.A.S.E.*, iv, 41). I may give a few extracts showing the comparison :—

| Age. | Lees. | Hart. | Age. | Lees. | Hart. |
|---|---|---|---|---|---|
| 16 | ·0046 | ·0041 | 35 | ·0088 | ·0091 |
| 18 | ·0075 | ·0039 | 45 | ·0090 | ·0099 |
| 20 | ·0085 | ·0037 | 55 | ·0131 | ·0127 |
| 22 | ·0090 | ·0052 | 65 | ·0219 | ·0276 |
| 25 | ·0082 | ·0076 | 75 | ·0646 | ·0680 |

It will be seen that Mr. Hart's results are widely different from Mr. Lees's at young ages, this being due no doubt to the smallness of the numbers under observation. From age 25 the two sets of results, while not closely agreeing, cross each other several times, the general impression left on one's mind being that for whole-life functions the two tables would give fairly close results. It is perhaps ungracious, after Mr. Lees has given us so much, that we should ask for more. We learn from his paper that he has a great deal of information on his cards which would form ample material for another paper. Especially one would like to see his facts as to the issue of marriages worked up into tables. In most cases when we are asked to make calculations in connection with trust funds or property, the question we want answered is—will a spinster marry and have children?—the fact of the marriage *per se* not affecting the value.

Mr. A. E. Sprague.—It would appear from the demonstration given by Mr. Lees in the second last paragraph, that the assurance on a male life independent of marriage is equal to $(b\mathrm{A})_x+(bm\mathrm{A})_x$; but I am inclined to think there may be some fallacy in the argument, for the following reason.

The assurance independent of marriage, and the assurance $(bA)_x$, are both payable at the end of the year of death—that is, an integral number of years after entry; but the assurance $(bmA)_x$ is payable on the anniversary of the marriage, which is assumed to take place in the middle of the year. The last-mentioned assurance is therefore payable an integral number of years plus six months, after entry; and from this it appears to me as if it should be multiplied by $\sqrt{(1+i)}$, to allow for the delay of six months in payment. It would, I think, add to the value of the paper if tables of the forces of mortality and the force of marriage were given—for integral ages. The forces of mortality might be denoted by $(s\mu)$ and $(m\mu)$, to distinguish between spinsters and married women; and as the Greek letter $\mu$ has been adopted to denote the force of mortality, from the initial letter of "mortality", I would suggest that the Greek letter $\nu$ would be an appropriate symbol to adopt for the force of marriage, from the initial letter of "nuptials".

Mr. W. Young.—I think Mr. Lees's figures lose some of their value by his adoption throughout of one year as the unit in his table, in getting the probabilities of death and marriage. In measuring averages, the important part of the fraction is the numerator, and in the case of Mr. Lees's numerators they never go into three figures. When the figures are small, there is a danger that any trustworthy information they contain may be lost or altered by too great refinement. I think it would have been more satisfactory to take five or more years throughout as the unit. This will be evident if we compare the rates of mortality in Table 7 with those in Table 2. The rates in Table 7 are much smoother, and, although unadjusted, agree more closely with the graduated rates in Table 2 than with the ungraduated rates. It is often forgotten, in dealing with a small body of facts, that if you have a fraction with a numerator of two figures, you cannot get a probability with more than two significant figures. Any other figures obtained are meaningless, and it would be better to leave them out or substitute zeros for them. In the present case there are one or two figures in the numerator, and consequently only one or two significant figures in the resulting probabilities have any meaning. Mr. Lees says it is more convenient to deduce the central marriage rate, than the probability of marrying in a year; but the only reason for doing this is that, by using the former, it is easier to allow for those who die without being exposed to the contingency of marriage for the whole year, and also for those who marry and then die in the year. It is quite as easy, with a small body of facts, to calculate the probability of marriage; because from Table 6 we can see that the deaths could not have affected the second significant figure of the probability of marriage, and that is the last significant figure in the results. In the monetary tables it would have been an advantage not to give more figures than there are significant figures in the original data—giving, say, the annuity values to one decimal place and the assurance values to two or three places. There would be some advantage in this if the tables were applied to any practical purpose, for then the number of significant figures in the result would bear some relation to the confidence that could be placed in the figures.

Mr. V. Marr.—Like Mr. Chatham, I have been struck with the close agreement between Mr. Lees's rates of the probabilities of dying in a year for married women, and those relating to ministers' wives published in the supplement to Mr. Hewat's book on Widows' and Pension Funds, and also with the agreement between the rates of mortality in childhood as given by Mr. Lees and Mr. Hewat. The only case where there is any considerable difference is at age 0, where Mr. Lees has ·0516 against Mr. Hewat's ·0619. In the latter statistics I think there was difficulty experienced in eliminating

the cases of infants still-born, and deciding whether cases where the dates of birth and of death were identical should be included or excluded. In many cases it was impossible to tell from the facts whether the child had been still-born or had died on the day of its birth. As to the radix of Mr. Lees's tables, I think it is giving too much importance to the facts to raise a small number like 4500 in such a ratio as he does. It means that 1934 are supposed to die in a year at the last age of the table. I have noticed the same feature in other tables where a large radix has been taken; and it must be caused by either the radix being too high, or the graduated values of $q_x$ above age 80 being too low. I presume Mr. Lees satisfied himself that his graduated values were the best that could be got, before he availed himself of them. Leaving out of account the calculation of the "expectation of life", the column $l_x$ does not serve any practical use if one uses logarithms in calculations, when $q_x$ is tabulated, except in the construction of a "combined" table and the relative monetary values such as Mr. Lees gives in his Tables 12 and 15. I am glad Mr. Lees has given a column of log $l_x$ in Table 4.

Mr. N. Campbell.—I do not quite see why it is "to be expected" that the rate of marriage among the spinsters of the general population should at the principal marrying ages be greater than the rate for peerage spinsters. In any case little reliance can be placed upon statistics of this nature in regard to the general population, on account of the fact that spinsters are not always, in stating their ages, sufficiently imbued with that love of truth and accuracy which is so desirable in the case of statistics.

Mr. J. J. M'Lauchlan.—In reading this valuable paper, it occurred to me to examine some of the tables, to see whether any light could be thrown on a question that is always interesting in connection with life insurance—the risk of childbirth. We have here an opportunity of comparing the mortality among spinsters and among married women respectively, all belonging to the same class, and therefore placed in very much the same circumstances. I put down the probabilities of dying in a year for ages between 20 and 50, with the differences between the two sets:—

| Age. | Married. | Spinsters. | Difference. |
|---|---|---|---|
| 20 | ·0085 | ·0054 | ·0031 |
| 25 | ·0082 | ·0045 | ·0037 |
| 30 | ·0081 | ·0047 | ·0034 |
| 35 | ·0088 | ·0058 | ·0030 |
| 40 | ·0088 | ·0073 | ·0015 |
| 45 | ·0090 | ·0084 | ·0006 |

That progression of those differences is just what might be expected, and gives us an interesting idea of the additional risk under an insurance on the life of a married woman as compared with one on a spinster. A comparison of the probability of dying for females and spinsters, shows what, taking all females together, married and unmarried, is the extra mortality at each age due to the risks that accompany marriage:—

| Age. | Females. | Spinsters. | Difference. |
|---|---|---|---|
| 20 | ·0058 | ·0054 | ·0004 |
| 25 | ·0061 | ·0045 | ·0016 |
| 30 | ·0068 | ·0047 | ·0021 |
| 35 | ·0079 | ·0058 | ·0021 |
| 40 | ·0084 | ·0073 | ·0011 |
| 45 | ·0088 | ·0084 | ·0004 |

The two sets of differences are quite consistent with each other. It has

not been the practice, in charging female extras, to make any distinction between married and single women, but those figures would enable us to do so, if it were thought desirable.

Mr. B. B. Maxwell.—I agree with the opinion expressed by Mr. Marr, that the rates of mortality towards the end of Table 2 are rather too low. It seems rather a pity that the values of $q_x$ have not been tabulated to another place of decimals, as that would have enabled one to form a better opinion of the graduation, and would have tended to reduce the large fluctuations seen in Table 4 in the differences of the $d_x$ column. Thus, at age 13 the difference is 86, but at age 15 it is 478; at age 26 it is 10, but at age 30 it is 329; at age 45 it is $-23$, and at age 53 it is 489. This view appears to be confirmed by the curious oscillating movement of the mortality curve, as shown in the amount of the accumulated deaths in Tables 2 and 8.

Mr. J. Nicoll.—I feel some disappointment that Mr. Lees has been unable to include in his observations the five children under six months of age, and I would suggest in this connection whether it is not better to follow the existing in all cases to the anniversary of their birth beyond the date on which the observations closed, instead of merely treating them as under observation for the nearest integral number of years. In connection with Table 5, it is interesting to see the values brought out for reversions, though it appears very doubtful whether, in the present state of the market, any offices offering these prices would secure much of the business. Table 7 again shows very clearly the necessity for the imposition of the usual female extra, and it is somewhat remarkable that there are still so many offices that do not charge that extra, apparently on the ground that the improvement in female life after 50 more than counterbalances the heavy mortality before that age. Such reasoning seems, however, to be illogical, and it would appear to be fairer to all concerned to impose the extra in all cases under age 50; and, after that age is attained, to take off the extra, and even, if need be, to reduce somewhat in future the premiums of the survivors, if that were thought desirable. Mr. Campbell has very properly, in connection with Table 10, called attention to the untrustworthiness of the ages as given in the Registrar-General's returns, and more especially in regard to females. Dealing, however, with the table as it stands, and summing the first four terms of both sets of figures, it appears that for ages 15 to 30 the marriage rate over all of the peerage spinsters is higher than that deduced from the general population. In connection with Table 13, it is interesting to notice that, for the earlier ages at least, the total probability of marriage for a bachelor of any age agrees pretty well—as we should expect it to do—with the total probability for a spinster five years younger. Mr. Lees's monetary values will require to be used with caution, at ages beyond childhood at any rate, as the personal element and circumstances must, of course, count for a great deal in any single instance.

Mr. Lees, in reply, said—I have to thank you for the kind remarks which have been made. They go a long way to repay one for the trouble taken in the preparation of the paper I have to-day submitted. With reference to Mr. Chatham's remarks, I may say that I looked into the 1899 Peerage to get any information not contained in the 1898 volume at the date of publication; so that the observations may be taken as having closed on 31 December 1898. My illustration of the cost of a dowry was suggested to me by a letter that appeared in the public press from a father who desired to provide for the expenses of the marriages of his five daughters. If a man were to take out such a policy, the money could be used towards the expense of the marriage, or be given to the daughter, or he could put it into his own pocket as some recompense for the loss of his daughter. I took into account the new peerages where the dates were fully given;

but if the dates relating to one or two daughters only were given, the data relating to that particular family were excluded. Mr. Murrie has remarked about the large radix that has been used by me, as well as by others. That is required in connection with the combined marriage and mortality table; for the numbers dying married are very small at the younger ages. If a smaller radix had been used, there would have been no significant figures at these ages. As to the suggested comparison with Mr. Hart's table, he dealt only with the mortality among married women, and had not the advantage of an experience relating to both married and unmarried. The number of entrants into the married state at the earlier ages is very small, and there are no deaths at one or two ages, and Mr. Hart's rate of mortality appears to be too low at these ages, while at the end of the table it appears to be higher than is warranted by the original data. With reference to Mr. Young's remark, although the grouping is not shown in the paper, the graduation was not done for each age separately, but the data were suitably grouped for the purpose. As to the formula at the end of the paper, my view receives some confirmation from Dr. Sprague's own paper. We have the rate of mortality in his two classes (married and males) the same from age 45 to the end of the table. That being so, the value of an assurance for a married man should be the same as for a male; and I find that the addition of the values of the contingent assurance payable on death unmarried, and the assurance payable on death after marriage, as given by Dr. Sprague, produce to the last figure of decimals (with one or two exceptions) the value he gives for the married, for all ages from 45 to the end. It therefore appears to me that $(bA)_x + (bmA)_x$ is the value of the assurance independent of marriage.

The President.—It only remains for us to express our thanks to Mr. Lees in a formal way for his paper. One thing has struck me in regard to this paper, and to the paper we have had from Mr. Campbell since we commenced these meetings—a paper also based upon original research—and that is, how very remarkably their results corroborate each other at the point where they touch. That being so, I think we may take it that the labours of those two gentlemen have supplied us with an approximation as near as we are likely to obtain, to the mortality of daughters of the peerage and tenants for life—people who live in comfortable circumstances, and have the means of taking care of themselves, and getting every advantage in the way of medical attendance, change of air, and so forth. Mr. Nicoll has referred to the subject of reversions, and has expressed a doubt whether there is any use in working out problems of this kind, seeing that the quotations sometimes made for reversions have little or nothing to do with their real values. But I should say that probably the best means of bringing people who make such quotations to a wiser view, is to publish, as we propose, the results of such an investigation as this. In the meantime, those who wish to do business on a right footing will abide closely by the results indicated by tables such as Mr. Lees has given us. I am sure you will all heartily join in according to Mr. Lees our best thanks for his paper.

# PRESENTATION

TO

# MR. JAMES MEIKLE.

Mr. James Meikle, F.F.A., F.I.A., having recently retired from the position of Actuary to the Scottish Provident Institution after a period of sixty-three years' service with that office, was entertained to dinner, and presented with an address, in the North British Station Hotel, Edinburgh, on Friday, 12 December 1902, by his friends in the Faculty of Actuaries and among the officials of Life Assurance Offices in Scotland.

The chair was occupied by Mr. Andrew H. Turnbull, Manager of the Scottish Widows' Fund Life Assurance Society, and the vice-chairmen were Mr. David Deuchar and Mr. Gordon Douglas.

Dinner being over, and the company having pledged the health of the King,

The Chairman said—I feel highly honoured in being intrusted with the business for which we are assembled to-night, although I fear I shall hardly be able to express, as you would wish, to Mr. Meikle the pleasure that it affords us and his friends in Australasia to present him with a tangible proof of our regard, to thank him for all that he has done for our profession, and to wish him continued good health and many years in which to enjoy his well-earned leisure. When active men like Mr. Meikle look back upon their past, I think there is nothing that can give them greater satisfaction than the consciousness of having done something for others; and I congratulate Mr. Meikle on having a right to a very large share of such satisfaction. He has not merely left footprints on the field of knowledge through which he toiled as a student: he has very materially assisted in laying out a path through that field by which the students of the present day can pass with greater ease and celerity than was dreamt of fifty years ago. If any of those students are blest with the ability, the zeal, and the vitality that characterise Mr. Meikle, they may not only widen that path, but it is possible they may carry it further on. If they succeed in doing this, I am sure that nothing would be more gratifying to their pioneer.

Mr. Gordon Douglas then read the address, which was as follows:—

"Edinburgh, 12 *December* 1902.

"Dear Mr. Meikle,—In the presentation made to you in 1889, on the completion of half a century of official service, you already possess evidence in a permanent form of the respect and esteem entertained for you by your professional brethren.

"But now that you have retired from official life, it has appeared to your friends in the Faculty of Actuaries and among the officials of Life Assurance Offices in Scotland, that it would be fitting to renew the expression of their regard and appreciation. Your professional friends therefore now cordially assure you of the continuance of their high esteem, and offer their earnest wishes for your comfort and happiness in your retirement. Gratefully recalling the services rendered by you to the actuarial profession and to life assurance business during your long and honourable career, they desire further to express their admiration of the high talent and unwearying industry of which these were the evidence and the result.

"In the forefront among these services they venture to place the valuable work done by you in connection with the Actuarial Society of Edinburgh, which was founded by the Faculty of Actuaries for the benefit of actuarial students, and which after a useful existence of over forty years has now become merged in the Faculty. The papers on the elementary problems of life assurance which you contributed to that Society, at a time when there was no systematic text-book, were of inestimable value to the students of those days; and many actuaries, who have attained to positions of importance at home and abroad, have acknowledged their great indebtedness to those early papers. You also contributed a number of other papers on more advanced subjects during the later years of the Actuarial Society, and you held office as its Honorary President during four separate sessions.

"In connection with the Faculty of Actuaries you did valuable service as Examiner of Candidates, as a Member of Council, as Honorary Librarian, as Honorary Secretary, as Vice-President, and as President. You also conducted an elaborate investigation of the Mortality Experience of the Scottish Life Assurance Offices up to 1863; and you took part in the recent mortality investigation conducted by a joint committee of the Institute of Actuaries and Faculty of Actuaries.

"As evidence that you are not forgotten by those who have carried with them to distant places the knowledge of those manifold services, it will be of special interest to you to know that the casket intended to contain this document has been chiefly provided by the contributions of actuaries in Australasia who specially desired to be associated with this movement to do you honour.

"As a Committee chosen on behalf of your many friends in the profession to give expression to the sentiments contained in this Address, we have pleasure in subscribing ourselves,—Dear Mr. Meikle, yours very faithfully,

"Aw. H. Turnbull.
"Hugh Blair.
"D. Deuchar.
"Geo. M. Low.
"Gordon Douglas.
"James Watt."

The Chairman then continued: It only remains for me, Mr. Meikle, to hand you this handsome silver casket containing the address, with the best wishes of the donors. And of you, Gentlemen, I have to ask that you will join me in this toast: "The Guest of the evening, Mr. Meikle". The toast was pledged with enthusiasm.

Mr. Meikle, in the course of his reply, which included several poetical quotations, said—Mr. Chairman and Gentlemen, you almost strike me dumb by the great honour you have done me. I scarcely have a word to say, and I think the most eloquent thing I could do would be to bow myself to the ground and say nothing. With reference to my age, I do not admit that I am an old man. If you count age by the number of times that the earth has revolved round the sun during my life, I may perhaps be reckoned an old man. That, however, is not the right way of looking at the matter. You must measure a man's age by the love in his heart; and when measured in that way, I claim to be as young as any of you. But although not old, I suppose I must admit that I am so far up in years; and if you will allow me, I should like to refer a little to old times. There is certainly a melancholy sweetness in looking back over the long period of sixty-three years. I remember well the first day I entered my late office —it was the 21st of October 1839. I climbed to the top of a high stool, and my first duty was to fill up a referee's letter. The conditions of work were very different in these days from what they are now. The hours were from 10 to 4 and 7 to 9, but it was often 10 o'clock at night, and occasionally 11, before the day's work was over. There were no such

things as copying letter-books then. There were no messengers then as you have now. The apprentices had to carry all the parcels to the "Black Bull" inn, from which the mail-coach for London started.

After other interesting reminiscences Mr. Meikle went on to say—I always looked up with a reverent eye to the managers of the offices then, of whom perhaps many of you have never even heard the names. There were Gilbert Laurie Finlay, Henry David Dickie, James Borthwick, William Spens, Robert Christie, William Thomas Thomson, and others. I remember how I stood in great awe of Robert Christie—he was a very accurate man. And then there was David Chisholm, who died only the other day. All these men were big men. I like to think that in my time I have done some little things for love, and I believe I may justly claim this. These papers read to the Actuarial Society were all done for love to the Society and the Faculty. I was never happier than when working hard. The more I had to do, the more I got through. I am very pleased indeed if I have so far succeeded as to make the thin end of the wedge a little sharper for some of you. I have occasionally had the satisfaction of knowing that my work has been of use to individuals. An eminent actuary has told me that he would not have entered the business at all if it had not been for my "Rationale of Life Assurance Premiums" getting in the thin end of the wedge. By the way, it was to the late Mr. M'Candlish that I was indebted for that fine title. When my "Observations on the Rate of Mortality of Assured Lives, as experienced by ten Assurance Companies in Scotland" were in type, I was greatly gratified with what Mr. William Thomas Thomson said to me after perusing them, and with his going personally to ask Messrs. Blackwood to allow their names to appear as the publishers. As Edwin Waugh, the Lancashire poet, says in "The Moorland Flower":

> "Heaven loves the heart that lives and dies
> To bless its neighbouring waste."

Gentlemen, I feel that I have tried to do what lay in my power to bless the wilderness round about me. Again I thank you for the exceeding great honour you have conferred upon me.

Mr. James Chisholm proposed the toast of "The Committee", in the following terms—This is a unique occasion. I have great pleasure—and I take it as a great honour to be asked to do it—to put before you the toast of the Committee who have arranged this dinner. You have heard their names as they appear at the foot of the address that has been presented to Mr. Meikle. They include the names of gentlemen at the head of institutions known all over the world. Besides the arrangements for the dinner, a great deal of other work, such as correspondence and preparation of the address, has fallen upon the Committee, and especially on the honorary secretary, Mr. Gordon Douglas. I ask you therefore to join with me in drinking the health of the Committee of which our Chairman is at the head, and of which Mr. Deuchar, whose name I couple with the toast, has been an active member.

Mr. David Deuchar, replying on behalf of the Committee, said—I think there is some inappropriateness in connecting my name with this toast; because it was really Mr. Low the convener, and Mr. Gordon Douglas the honorary secretary, who took the chief share in all the arrangements. The other members of the Committee have however had very great satisfaction in being associated with Mr. Low and Mr. Douglas, in organising this dinner in honour of the guest of the evening, whom they have all known so long, but who in spite of Anno Domini remains ever young.

Mr. G. M. Low, President of the Faculty, proposed the toast of "The Chairman". He said—When the Committee were making their arrangements, it appeared to them that what was wanted to round off these arrangements and give them entire completeness was that our friend, Mr. Turnbull, should consent to take the chair. That feeling was entertained not merely because Mr. Turnbull is the senior life assurance manager in Scotland, but because in his own person he links us with the period in actuarial history in which our good friend Mr. Meikle began to show those conspicuous services to the profession for which, among many other things, we seek to honour him on this occasion. It was in the year 1860 that Mr. Meikle read his paper on "The Rationale of Life Assurance Premiums", with which his name will always be associated in grateful memory by actuarial students—at all events, by actuarial students of the period to which I myself and others here belonged. And it was in the same year (1860) that our friend Mr. Turnbull was admitted an associate of the Faculty of Actuaries, after an examination, and after he had given in a paper which you will not be surprised to hear was described by the examiners as "most creditable". Mr. Turnbull's qualities are very well known to you all, and I need not enlarge upon them; but I claim, on behalf of the Committee, that we exercised a wise discretion in asking him to preside to-night.

Mr. Turnbull, in reply, said—I thank you very much for the kind manner in which you have received Mr. Low's toast, and Mr. Low for the kind way in which he has proposed it. It is quite true, I am sure, that I am the senior present. I began my business career in 1850—rather more than fifty years ago. I remember many of the men who then held positions in the life offices—in fact, all those whom Mr. Meikle has mentioned; and I recollect a good deal about the customs of the offices in those days. It was exceedingly interesting to me to hear Mr. Meikle's reminiscences.

The proceedings terminated by the singing of "Auld Lang Syne".

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A fuller report of the proceedings will be found in the *Insurance Record* and in the *Post Magazine* of 20 December 1902.

No. [illegible] [illegible] Vol. I.

# TRANSACTIONS

OF THE

# FACULTY OF ACTUARIES.

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1856.

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## Some Practical Notes on the Valuation of Pension Funds. By David Cummin, F.F.A., M.I.A.

With Abstract of Discussion thereon.

LONDON:
CHARLES & EDWIN LAYTON,
PARTRIDGE STREET,
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GEORGE KING
ALEXANDER LAFF
ALEX. WILLIAM LAURIE
MUNGO MACKENZIE LEE
JOHN NORMAN LEWIS
GEORGE LINDE, C.A.
HUGH LINTON
JAMES H. MACARTHUR, C.A.
COLIN MCCAIG, C.A.
WILLIAM HAY MACDONALD
ARCHIBALD MCLEOD
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PHILIP CAMPBELL MACKAY
ALEX. GEORGE MACPHERSON
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JOHN MANSON, M.A.
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BENJAMIN HILL MASWELL
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JAMES MACMILLAN MILLIGAN
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DAVID FRANCIS PARK, C.A.
WM. [illegible]
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Members.

WILLIAM [illegible]
EDWARD [illegible]
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Honorary Members.

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WILLIAM [illegible]

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# TRANSACTIONS

OF THE

# FACULTY OF ACTUARIES.

No. 9.—Vol. I.

**Some Practical Notes on the Valuation of Pension Funds. By David Carment, F.F.A., F.I.A.**

**With Abstract of Discussion thereon.**

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TRANSACTIONS

OF THE

FACULTY OF ACTUARIES.

*Some Practical Notes on the Valuation of Pension Funds.* By DAVID CARMENT, F.F.A., F.I.A., *Assistant Actuary, Australian Mutual Provident Society, Sydney.*

[Read before the Faculty, 23 March 1903.]

ALTHOUGH the question of the valuation of Provident and Pension Funds has recently been so exhaustively treated by Mr. Manly in his two papers appearing in the *Journal of the Institute of Actuaries* (vol. xxxvi, p. 207, and vol. xxxvii, p. 193), I have been rash enough on the present occasion to undertake to contribute a few practical notes with reference to the subject, with the view of further elucidating, if possible, the effect of the assumption of varying rates of mortality before and after the pension age, of secessions, of different scales of salaries, and of different rates of interest.

The class of Funds principally referred to by Mr. Manly appears to be mainly those connected with the great Railway Companies of the United Kingdom, whereas those with which I am best acquainted in Australia have been in connection with Banks and Life Assurance institutions, as well as the Superannuation Funds and Widows' Funds founded for the benefit of aged and infirm Ministers and School Teachers and their widows and orphan children. One point in which several of these Funds differ from those described by Mr. Manly is that the compulsory age of retirement is either deferred until 70 (in place of 60 or 65), or else there is no age fixed at which retirement is compulsory; so that we may have men still in active work so late as 75 or 80 years

of age, and it is quite possible for the average age at retirement to be well over 65. Again, in the majority of the Funds with which I am acquainted here it is rather the exception than the rule for the employers to contribute directly any fixed proportion of the necessary contribution, although in several of the Bank Funds a similar result is no doubt attained by a comparatively high rate of interest being guaranteed on the accumulations, this rate being sometimes as high as 5 or 6 per cent. Further, it is by no means an invariable condition that there should always be a return of the whole or of some portion of the contributions in every possible event. In the Pension Fund of one large Life Assurance institution, in fact, there is no refund of contributions whatever in the event of an individual voluntarily leaving the service before becoming entitled to a pension; and on the whole I am inclined to agree with the wisdom of this provision, for otherwise there might, under certain circumstances, be some inducement thereby given to a member to transfer his services elsewhere by reason of his thus being enabled to secure a cash payment on resigning his position. In the Fund referred to, moreover, there is no refund of contributions under any circumstances whatever, the only benefits being a gratuity equal to one month's pay for each year of service to such as break down after 10 and before 20 years, with a pension after that length of time equal to one-sixtieth of average salary for each year of service, the maximum being forty-sixtieths. The rate of contribution, however, is on the very moderate scale of 2½ per cent on salary.

In valuing any of these Funds, great difficulty is necessarily experienced in determining, with any degree of certainty, the rates of retirement likely to be experienced in the future by the body of lives under consideration; as well as, in cases where the pension and the contributions are percentages of the salary, the scale of salary likely to prevail in the future. In the majority of instances it will be found that the past experience in these respects of the individual Fund in question, is hardly sufficient to enable us to predict its future with sufficient exactness; and therefore recourse must generally be had to some other experience or to more or less arbitrary hypotheses. Of course the rates of mortality and of retirement among different classes of persons will often vary widely, and the rates which might be reasonably assumed as suitable in the case of a Fund for a body of clergymen or of school teachers might be wholly unsuitable for another composed of life assurance officials, or again for one consisting of railway employees.

Further, it must be observed that the prosperity, or even the solvency of a Fund, depends very much on the manner in which it is managed in regard to the permission of retirements on the ground of ill-health or incapacity. In Funds connected with the

Civil Service of some of the Colonies, it is a notorious fact that many persons in the prime of life have been, for very insufficient reasons, placed as life-long pensioners on the Funds, thus completely and irrevocably destroying their solvency. There seems, in fact, to be an almost irresistible tendency in the case of Government Pension Funds, towards mismanagement of this description; whereas in the case of a Bank or an Assurance Office it is generally the case that an officer is not permitted to retire until he has either reached the age when retirement is optional, or until he is in reality permanently incapacitated, through mental or bodily infirmity or disease, for any further duty. It is thus very difficult, when a Fund is first started, to form any approximate estimate of the probable average age at which retirements will take place; for in one body of individuals this age might be somewhere between 60 and 65, while in another it might more closely approximate to 70. In a Fund connected with a large Australian Life Office, retirement is *optional* at age 60 and absolutely *compulsory* at 70. The number of actual retirements so far has been far too few to enable any one to judge as to the probable average retiring age; but from the fact that several members of the Fund are, or have been, in active work at ages from 65 to 70, it seems reasonable to suppose that comparatively few will voluntarily retire until the earlier of these ages has been passed, and no doubt a fair proportion will remain in harness even until 70 if they live so long.

One of the large Australian Banks has also a flourishing Provident Fund in operation, which has on each occasion shown a large surplus of assets over liabilities. In this case there is no fixed age at which retirement is compulsory, and some of the staff of the institution are still at work at ages over 70.

On the other hand, an Australian fund of considerable import ance, embracing provisions for superannuation and for widows as regards a large body of Clergymen, has had an experience of quite a different character, for the average age of retirement from active work has in the past been about 55; and one result of this adverse experience, combined no doubt with other factors, has been to necessitate a considerable reduction in the scale of benefits hitherto granted.

It appears to me, therefore, that in the case of all such Funds it is a vital necessity that the management should be abundantly satisfied, as regards each applicant for a pension, that he is in reality permanently incapacitated for further duty.

A signal instance of a hopelessly-mismanaged Fund is afforded by the Civil Service Superannuation Fund of New South Wales, which was started in the year 1884, and embraced practically the whole of the salaried employees of the Government, amounting eventually to about 10,000 in number. The contri-

bution was fixed at 4 per cent on salaries, and pensions were allowed at the rate of one-sixtieth for each year of service, based on the average salary of the last three years, for every year since appointment, *not* for each year of contribution to the Fund. There was, however, a provision that the back contributions, *without interest*, should be charged against the individual, who was permitted on retirement to commute the amount of arrears for an equivalent reduction of his annual pension. The rate of contribution, however, was lamentably insufficient to provide the promised benefits, even if it had really been paid by all the members from the commencement of their service, and even although the Fund was subsidised by the Government to the extent of £20,000 a year for the first five years of its existence—an amount evidently wholly inadequate for the purpose of securing its solvency. As a matter of fact, many old Civil Servants were almost at once retired on pensions, and within a few years many others, still comparatively young, were permitted to become pensioners, while in addition many more who were compelled to retire from the service owing to retrenchment were also added to the list. No wonder, then, that the Fund was soon perceived to be in a condition of hopeless insolvency. A few years ago it was decided to permit all such contributors as desired to do so, to withdraw from the Fund entirely, under a promise that the amount of their contributions thereto would be refunded to them on their finally leaving the service, or to their heirs at their death if still in active work; with interest at 3 per cent from the date of their ceasing to contribute. There are still, however, a considerable number of contributors as well as a large number of pensioners, and in a year or two at most the balance in hand will be completely exhausted; after which it is believed that the Government will simply ask Parliament for an annual vote of so much money as will be required to pay the annual deficiency.

Judging by the valuation report of Mr. M'Gowan upon the Civil Service Superannuation Fund of Cape Colony as at 31 December 1896, that Fund seems to have reached a similar condition, and apparently for somewhat similar reasons.

The encyclopædic papers of Mr. Manly on the subject of the valuation of Staff Pension Funds already referred to, have well-nigh exhausted the whole subject; but having had, prior to their appearance, some experience in connection with the founding and valuation of several Funds of this description, it occurred to me that possibly some account of the methods pursued might not be entirely without interest. In the necessary calculations I have generally employed Lubbock's formula of summation No. 17, as given in the *Institute of Actuaries Text-Book*, Part II, page 470, generally using 5 as the interval between the terms, and taking three or four orders of differences. Before, however, finally

deciding on the interval to be used, I generally made one or more trials to determine whether or not a longer interval, such as 7 or 9, might not be safely used in the particular case under consideration. Sometimes, however, if the progression of the differences in the terms to be summed is very irregular, the formula in question does not give good results, and occasionally the more orders of differences are included the further is the result from the truth. In such cases I have often found it much less laborious, as well as more accurate, to take the 3rd, 8th, 13th, etc., terms of the series to be summed, and to multiply the sum of these selected terms by 5, adding on any individual terms which there may be in excess of an exact multiple of 5. Thus, if the series consisted of 35 terms in all, I should sum the 3rd, 8th, 13th, 18th, 23rd, 28th, and 33rd terms, and multiply the result by 5. If there were 37 terms, I should add to the above result the 36th and 37th terms. Sometimes it might be found sufficiently accurate to take every 7th term, but in this case there would generally be a larger number of final terms to add on, unless indeed the final terms of the series should be comparatively so small as to be practically negligible. If on the other hand, as will generally be the case in dealing with the *benefits* under a Pension Fund, the final terms are unduly large as compared with the rest, it will usually be necessary to calculate them individually in every instance, instead of relying on their being safely averaged with the terms immediately preceding them. In all these calculations I find it generally sufficient to use logarithms to four places, the four-place table by Hannyngton being invaluable for this purpose.

As regards the rate of retirement which should be assumed in founding a Fund, or in valuing one already in existence, I might say that at one time, in some investigations into a large Clergymen's Fund, a table was made use of which had been formed by an Australian Wesleyan minister, the Rev. John Cope, from the experience of Wesleyan Ministers in England. An arbitrary modification of this table was used by myself in the preliminary calculations for the founding of a Fund among the School Teachers of one of the Australian States. These tables, however, gave rates of retirement which were no doubt considerably too great when applied to the clerical staff of a Bank or Insurance Office; and consequently an attempt was recently made to investigate the experience in this respect of a large Australian Bank and a large Australian Life Office, and the combined results are no doubt of some value in the way of indicating the probable rates of retirement in future among similar bodies of individuals. The rates are very much lighter than those among the Wesleyan clergymen; and in fact the actual retirements among the combined staffs of the two institutions referred to, were little more than *half*

the number expected by the table in question. In case, however, the actual rate found to prevail in the past should not be quite a safe guide for the future, a table was constructed following a course lying nearly midway between the Wesleyan clergymen's experience and that formed from the combined data afforded by the Bank and the Life Office, and these modified rates have been used in the following calculations.

I have combined these rates of retirement, which are shown in Table A on page 321, with the $H^M$ rate of mortality, and thus produced the Life and Labour Table No. I (Table B, page 322).

Next, in order to test the effect of varying rates of interest and of various suppositions as to mortality, I formed a hypothetical Fund, substantially based on the actual particulars of the staff of a large Australian Bank. In order to this end, the members of the staff as at a given date were arranged in "years of service", and the average salary in each *year of service* was determined. These average salaries were then roughly graduated, so as to form a fairly probable "salary scale" for the future according to years of service, as shown in Table E (page 325), in which there is also shown the rate of Pension, $B_n$, payable to a member retiring in each year of service after the 20th, calculated at $\frac{1}{60}$ of average salary over the whole period of service prior to the year of retirement, up to a maximum of forty-sixtieths. This table also shows, for each year of service, the amount of a contribution of 1 per cent on salary, $c_n$. The column $c_{n+\frac{1}{2}}$ is used in finding the value of the future contributions as hereafter described. The numbers and average ages of the members of the staff who were in each year of service, were approximately as in Table F (page 326), where the actual figures have been slightly altered so as to bring the total to an assumed number of 1000 in all, and to avoid fractions in the average attained age of each quinquennial group. It was assumed that retirement was permissible in the event of incapacity at any time after 20 years' service, and that it was compulsory at age 70. The Fund in question provided for several supplementary benefits, but it will be convenient to defer the consideration of these until a little later. The ages at entry were taken to be exact integral ages, and those now in their $n$th year of service were taken as having served for $n-\frac{1}{2}$ years. Thus, those who entered at age 19 and are now in their 3rd year of service, are taken as having on the average served for $2\frac{1}{2}$ years, their present average age being therefore $21\frac{1}{2}$. It is further assumed that retirements take place on the average in the middle of a year of service; except as regards those who are alive and in active work at 70, who are supposed to retire exactly on the attainment of that age. Further, the pensions are assumed to be payable by quarterly instalments, beginning three months after the date of retirement. Seeing that a member cannot retire on a

pension until his 21st year of service, the formula for the value of the pension benefit will be, for a member now in the $n$th year of service,

$$(1)\quad \frac{v^{21-n}\cdot r_{x+20}\cdot B_{20}\cdot a^{(4)}_{x+20\frac{1}{4}}+\ .\ .\ .\ +v^{70-x-n+\frac{1}{4}}\cdot l'_{70}\cdot B_{70-x}\cdot a^{(4)}_{70}}{l'_{x+n-\frac{1}{4}}},$$

where $x$ = age at entry;

$r_x$ = number retiring between ages $x$ and $x+1$, according to the Table No. I;

$B_n$ = annual pension to which a member will be entitled who retires after serving $n$ complete years, according to Table E; and

$l'_x$ = number alive in active work at age $x$, according to Life and Labour Table No. I.

For instance, in the case of an individual who entered at exact age 19 and who is now in his 3rd year of service, the formula for the value of his pension benefit will be

$$(2)\quad \frac{v^{18}\cdot r_{39}\cdot B_{20}\cdot a^{(4)}_{39\frac{1}{4}}+\ .\ .\ .\ +v^{48\frac{1}{4}}\cdot l'_{70}\cdot B_{51}\cdot a^{(4)}_{70}}{l'_{21\frac{1}{4}}},$$

where $r_{39}$ is the number who retire, according to the fundamental table, between age 39 and age 40; $B_{20}$ is the rate of pension applicable to a member who has had 20 completed years of service; and $a^{(4)}_{39\frac{1}{4}}$ is the value of an annuity payable quarterly at age $39\frac{1}{2}$ (taken as the mean between $a^{(4)}_{39}$ and $a^{(4)}_{40}$) by the $H^M$ table. On reaching age 70, the member is assumed to retire exactly on the attainment of that age. Of course, if the member has been already more than 20 years in the service he is entitled to retire *at once* on becoming incapacitated, and therefore the power of $v$ entering into the first term of the formula will be $v^0$. Thus, if he entered at age 19 and is now in his 21st year of service, his present age is taken at $39\frac{1}{2}$, and as he may retire at once the value of his pension will be

$$(3)\quad \frac{v^0\cdot r_{39}\cdot B_{20}\cdot a^{(4)}_{39\frac{1}{4}}+\ .\ .\ .\ +v^{30\frac{1}{4}}\cdot l'_{70}\cdot B_{51}\cdot a^{(4)}_{70}}{l'_{39\frac{1}{4}}}.$$

In order to determine the rate of contribution to be charged to an entrant of age $x$, we must, however, first find the value of the pension benefit as at the commencement of the first year of service, and thus for age 19 at entry we shall have

$$(4)\quad \frac{v^{20\frac{1}{4}}\cdot r_{39}\cdot B_{20}\cdot a^{(4)}_{39\frac{1}{4}}+\ .\ .\ .\ +v^{51}\cdot l'_{70}\cdot B_{51}\cdot a^{(4)}_{70}}{l'_{19}}.$$

It is also necessary to find the value, as at the same epoch, of a contribution of 1 per cent on salary; which will be, on the assumption that the contributions for each year of service will be paid in the middle of that year,

$$(5)\quad \frac{v^{\frac{1}{4}}\cdot l'_{19\frac{1}{4}}\cdot c_1+v^{1\frac{1}{4}}\ l'_{20\frac{1}{4}}\cdot c_2+\ .\ .\ .\ +v^{50\frac{1}{4}}\cdot l'_{69\frac{1}{4}}\cdot c_{51}}{l'_{19}},$$

where $c_n$ = the contribution payable in the $n$th year of service. Then the quotient resulting from the division of the former of these results by the latter, will give the necessary rate per cent of contribution required to provide the benefit.

The value of the contributions of those members now in their $n$th year of service will be found thus—assuming, as before, that the contributions for the next 12 months will be paid in the middle of that term, and observing that each member will therefore have to pay for the remaining half of the $n$th year upon his salary for that year, and for the first half of the $(n+1)$th year upon the next higher rate of salary. For a member aged 19 at entry and now in his 3rd year of service, we have

$$(6) \qquad \frac{v^{\frac{1}{2}} \cdot l'_{22} \cdot c_{3\frac{1}{2}} + \ . \ . \ . \ + v^{47\frac{1}{2}} \cdot l'_{69} \cdot c_{50\frac{1}{2}}}{l'_{21\frac{1}{2}}},$$

where $c_{3\frac{1}{2}}$ is taken from Table E, and indicates the amount of contributions which he will pay during the second half of his third year of service and the first half of his fourth year, that is during the succeeding twelve months from the date of valuation; and so on.

In valuing the future contributions, I have generally assumed that the necessary expenses of management, which in such Funds are usually on a very moderate scale, can safely be provided for out of the excess of interest actually realised over that assumed in the valuation. It will seldom happen that in any particular case the rates of contribution payable are exactly sufficient for the benefits secured under the rules. In most Funds a uniform rate per cent on the salary is charged to all entrants, and it may easily happen that this rate is too high for those entering at the lower ages and too low for those starting later in life. It will generally, however, be possible to fix a rate which shall *on the average* be sufficient; especially if, as in some services, the ages of entrants are confined by the regulations within narrow limits, such as between 16 and 18 years of age, or some such limitation. In any case where the contribution is at a uniform rate, negative values will be almost certain to arise at some points in a valuation, and should be excluded in arriving at the final result; for, although continued membership of the Fund may be compulsory on all those in the service, there is, of course, no certainty that these particular individuals will not leave the service while the value of their share in the Fund is still negative. If the contributions as a whole are insufficient for the promised benefits, it will, of course, be necessary to take some means to bring them more nearly into harmony; but it occasionally happens that the contribution is considerably *more* than what is necessary, especially where the Fund has in the past been to a considerable extent built up from extraneous sources other than the members' own contributions. In such a case it may be found that the contribution *alone* is

barely sufficient, or perhaps quite insufficient, to provide the benefits, although the Fund possesses a large surplus of assets over liabilities; and thus, as clearly pointed out by Mr. D. Deuchar in his "Notes on Widows' Funds" (*Transactions of the Actuarial Society of Edinburgh*, iii, 225), "the introduction of each new member may be regarded as representing a loss to the Fund". It may be perhaps considered that new members should therefore be admitted in future only on payment of a higher rate of contribution than the existing members are paying. This, however, might not in all cases be altogether advisable or practicable, and it may sometimes be possible to consider the annual interest on the existing surplus in the light of a perpetual subsidy which may suffice to make up the deficiency in the rate of contribution required from new entrants.

To return now to the Model Fund, particulars of which are set out in Table F (page 326): in order to determine the necessary rate of contribution to provide the pension benefit referred to above, I proceeded as follows. The average age at entry of the whole of the members having been found to be as nearly as possible 19, the value of the pension benefit for an entrant of that age was found by means of formula (4), the result being 131·0, taking interest at 4 per cent. The next step was to find, by means of formula (5), the value of a contribution of 1 per cent on salary, which was found to be 38·1. Then the former of these results being divided by the latter gives the necessary rate, namely, 3·438 per cent.

Next, in order to arrive at a valuation of the liabilities in respect of prospective pensions, the members were arranged in quinquennial groups of years of service, as shown in Table F, all those who were in the first to the fifth year inclusive being treated as being in the *third* year, and so on. The degree of accuracy of this method was first tested by analysing one or two of the groups, and calculating the liability for those in each year of service separately. The results, as regards the groups tested, showed that the error of the five-year average method was not more than from ·3 to ·4 per cent of the total value, and that the tendency on the whole is to *increase* slightly the value of the liabilities. The full calculation as regards one of the groups is shown on page 314, both in detail and by the method already described of taking every fifth term and multiplying the sum by 5. It will be seen that in the instance given, which is the longest group, the result to one decimal-place agrees exactly. Such a close agreement will, of course, not always occur; but an examination of several varying groups showed that in valuing the benefits the error would probably be less than 1 per cent in excess of the true value.

The future contributions have next to be valued according to formula (6), the calculation being made on the basis of a rate

## Table 1.—Valuation of Pension Benefit.

*Life and Labour Table No. I.—Salary Scale No. I.—Interest*, 4%.
*Year of Service*, 3.—*Age at Entry*, 18 = $x$.

| Exact Age at Retirement. $x+n+\frac{1}{2}$ (1) | log $v^{x+n-20}$ (2) | log $r_{x+n}$ (3) | log $a^{(4)}_{x+n}$ (4) | log $B_n$ (5) | (2)+(3)+(4)+(5) (6) | $\log^{-1}$(6) (7) |
|---|---|---|---|---|---|---|
| | | | **Detailed Calculation.** | | | |
| 38½ | $\bar{1}$·6934 | 2·2279 | 1·1998 | 1·7803 | 4·9014 | 79690 |
| 39½ | ·6764 | ·2625 | ·1937 | ·8116 | ·9442 | 87940 |
| 40½ | ·6593 | ·2945 | ·1875 | ·8407 | ·9820 | 95940 |
| 41½ | ·6423 | ·3263 | ·1807 | ·8692 | 5·0185 | 104400 |
| 42½ | ·6253 | ·3541 | ·1738 | ·8965 | ·0497 | 1121 |
| 43½ | ·6082 | ·3838 | ·1664 | ·9222 | ·0806 | 1204 |
| 44½ | ·5912 | ·4133 | ·1587 | ·9469 | ·1101 | 1289 |
| 45½ | ·5742 | ·4425 | ·1504 | ·9708 | ·1379 | 1374 |
| 46½ | ·5571 | ·4713 | ·1424 | ·9939 | ·1647 | 1461 |
| 47½ | ·5401 | ·5038 | ·1339 | 2·0162 | ·1940 | 1563 |
| 48½ | ·5231 | ·5403 | ·1248 | ·0374 | ·2256 | 1681 |
| 49½ | ·5060 | ·5809 | ·1156 | ·0580 | ·2605 | 1822 |
| 50½ | ·4890 | ·6232 | ·1062 | ·0785 | ·2969 | 1981 |
| 51½ | ·4720 | ·6665 | ·0962 | ·0980 | ·3327 | 2151 |
| 52½ | ·4549 | ·7110 | ·0856 | ·1166 | ·3681 | 2334 |
| 53½ | ·4379 | ·7789 | ·0748 | ·1351 | ·4267 | 2671 |
| 54½ | ·4209 | ·8716 | ·0633 | ·1532 | ·5090 | 3228 |
| 55½ | ·4038 | ·9903 | ·0515 | ·1708 | ·6164 | 4134 |
| 56½ | ·3868 | 3·1106 | ·0394 | ·1886 | ·7254 | 5314 |
| 57½ | ·3698 | ·1948 | ·0265 | ·2060 | ·7971 | 6268 |
| 58½ | ·3527 | ·2555 | ·0137 | ·2235 | ·8454 | 7005 |
| 59½ | ·3357 | ·2993 | 0·9998 | ·2299 | ·8647 | 7323 |
| 60½ | ·3187 | ·3296 | ·9856 | ·2360 | ·8699 | 7411 |
| 61½ | ·3016 | ·3487 | ·9710 | ·2418 | ·8631 | 7296 |
| 62½ | ·2846 | ·3583 | ·9559 | ·2472 | ·8460 | 7015 |
| 63½ | ·2676 | ·3600 | ·9404 | ·2524 | ·8204 | 6613 |
| 64½ | ·2505 | ·3596 | ·9244 | ·2572 | ·7917 | 6190 |
| 65½ | ·2335 | ·3558 | ·9079 | ·2617 | ·7589 | 5740 |
| 66½ | ·2165 | ·3520 | ·8907 | ·2660 | ·7252 | 5311 |
| 67½ | ·1994 | ·3448 | ·8728 | ·2702 | ·6872 | 4866 |
| 68½ | ·1824 | ·3351 | ·8539 | ·2739 | ·6453 | 441900 |
| 69½ | ·1654 | ·6715 | ·8342 | ·2778 | ·9489 | 889000 |
| 70 | ·1568 | 4·0022 | ·8240 | ·2813 | 6·2643 | 1838000 |
| | | | | | | 13,973,470 |
| | | | **Approximate Calculation.** | | | |
| 40½ | $\bar{1}$·6593 | 2·2945 | 1·1875 | 1·8407 | 4·9820 | 95940 |
| 45½ | ·5742 | ·4425 | ·1504 | ·9708 | 5·1379 | 137400 |
| 50½ | ·4890 | ·6232 | ·1062 | 2·0785 | ·2969 | 198100 |
| 55½ | ·4038 | ·9903 | ·0515 | ·1708 | ·6164 | 413400 |
| 60½ | ·3187 | 3·3296 | 0·9856 | ·2360 | ·8699 | 741100 |
| 65½ | ·2335 | ·3558 | ·9079 | ·2617 | ·7589 | 574000 |
| | | | | | | 2,159,940 |
| | | | | | | 5 |
| | | | | | | 10,799,700 |
| 68½ | $\bar{1}$·1824 | 3·3351 | 0·8539 | 2·2739 | 5·6453 | 441900 |
| 69½ | ·1654 | ·6715 | ·8342 | ·2778 | ·9489 | 889000 |
| 70 | ·1568 | 4·0022 | ·8240 | ·2813 | 6·2643 | 1838000 |
| | | | | | | 13,968,600 |

log 13,970,000 = 7·1452
log $l'_{20\frac{1}{2}}$ = 4·9897
$\bar{2}$·1555 = log 143·1

of contribution of 1 per cent on the yearly salary. I have not thought it necessary to give in this case the calculation worked out in detail year by year, but from one or two trials I satisfied myself that the error of the approximate method would not be more than about ·75 per cent of the total value, in excess of the truth.

Summarising the results of the whole of the groups we get the figures shown in Division I of Table G (page 327); where it is assumed that the actual rate charged to members of the Fund is 3·438 per cent on their yearly salaries, the negative values which thus arise in the earliest group being excluded in arriving at the final result. The total liability of the Fund is thus shown to be £116,856·0.

If the pension benefit be based on *final* salary instead of *average* salary, the required rate of contribution would be 5·126 per cent.

If 3 per cent interest be assumed in place of 4 per cent, the necessary rate of contribution will be 4·567 per cent on salary, in place of 3·438 per cent; and the result of the valuation will be a liability of £137,757·2, as shown in Division II of Table G.

Again, if we assume that the rate of mortality among those in active work will be more nearly represented by 75 per cent of the $H^M$ rate, which seems a probable supposition in view of the experience of similar bodies of lives, the rate of contribution required will be 3·817 per cent, on a 4 per cent interest basis; and the result of the valuation will be £125,861·7, as shown in Division IV of Table G (page 328).

Next, in order to test the effect of a different *scale of salary* upon the necessary contributions and the resulting valuation, the supposition was made that the initial and terminal salaries remained the same as before, but that between these points the increase was at a *uniform* rate per annum, instead of being, as in the former scale, greater in the earlier years and smaller afterwards. The resulting scale of salaries and rates of pension are shown in Table E (page 325); and the liability in Division III of Table G. The resulting rate of contribution is 3·776 per cent, and the total liability £85,772·2. If we assume a scale of salary which is *lower throughout* than our first assumption, say, for instance, that adopted by Mr. Manly (*J. I. A.*, xxxvi, 270), the rate of contribution would be 3·636 per cent.

Further, I decided to try the effect of the assumption of a *light* rate of mortality among those in active work, combined with a rate *heavier* than the normal among those who break down early. For this purpose I availed myself of the annuity-rates assumed by Mr. Manly as applying to early retirants, as given in his Table 8 (*J. I. A.*, xxxvi, 266). These were taken to be applicable as far as age 63½; from that point up to age 69½ the $H^M$ annuities were used, and for those retiring at age 70 the Government

Annuitants 1884 (four years after purchase); the idea being that those who are absolutely compelled by bad health to retire early are probably, as Mr. Manly points out, very bad lives; but that those who retire at about 65 are neither much better nor much worse than the average; while those who have been able to continue at work till 70 (when retirement is made compulsory irrespective of health) are probably somewhat similar in vitality to annuitants. The result of this investigation seems to show that these various assumptions very nearly counterbalance each other—at least when 75 per cent of the $H^M$ is taken as the rate of mortality prevailing among those in active work; for the resulting rate of premium is 3·494 per cent on salary, as against 3·438 per cent when the $H^M$ mortality was assumed to prevail throughout without any modification. It is, of course, possible that the rate of mortality experienced among those in active work might be even more favourable than that just assumed; for among the Australasian Wesleyan Ministers it was recently found by Mr. James Graham, F.I.A., that the death-rate actually experienced was less than 50 per cent of the $H^M$ rate, and that at most ages it ran pretty close to Dr. Sprague's Select Mortality, being on the whole perceptibly lighter than the Select Tables. I have not tried the effect of assuming so light a rate in connection with the present calculations, though the result would probably have thrown some additional light on the problem. Neither have I calculated the liability of the Fund on the assumption of varying rates of mortality during active work and during retirement, but have contented myself for the present with exhibiting the effect upon the rate of contribution required. Want of sufficient time must be my excuse.

In connection with this part of the subject, it may be instructive to quote some figures lately deduced by me, with the valuable assistance of Mr. Robert Thodey, F.I.A., from the experience of State School Teachers in Victoria, both as regards those in active work and those who had retired on pensions. Among those *in active work* the rate of mortality was much on a par with that found by Mr. Graham to have prevailed among the Ministers, the results being as shown in Table 2. The total actual deaths were 51 per cent of those expected by the $H^M$ Table; or 59 per cent of those by the $O^M$ Table; or 92 per cent of those by Dr. Sprague's Select Table, using $q_{[x]}$ throughout.

Among the *pensioners* taken as a whole, the actual deaths were 83 per cent of those expected by the $H^M$ Table, or 84 per cent of those expected by the Government Annuitants 1884 (four years after purchase). A very heavy rate, however, was experienced up to about age 50, thus corroborating Mr. Manly's assumption as to the high mortality among those who retire early. The results are shown in Table 3.

TABLE 2.—STATE SCHOOL TEACHERS OF VICTORIA—MALES.

*Rate of Mortality "in active work". Comparison of Actual Deaths with Expected Deaths by* $H^M$ *and* $O^M$ *Tables and by Dr. Sprague's Select* $q_{[x]}$.

| Ages. | Exposed to Risk. | Actual Deaths. | Expected Deaths, by | | |
|---|---|---|---|---|---|
| | | | $H^M$ Table. | $O^M$ Table. | Select $q_{[x]}$. |
| 15-19 | 791·1 | 1 | 4·3 | 3·0 | 3·9 |
| 20-24 | 7,223·1 | 22 | 48·3 | 31·5 | 34·2 |
| 25-29 | 9,848·3 | 33 | 68·5 | 51·5 | 43·8 |
| 30-34 | 9,661·7 | 36 | 78·2 | 62·7 | 44·9 |
| 35-39 | 8,805·9 | 56 | 83·0 | 70·7 | 45·1 |
| 40-44 | 7,368·9 | 40 | 79·8 | 73·6 | 42·9 |
| 45-49 | 5,659·9 | 46 | 77·0 | 72·1 | 39·0 |
| 50-54 | 4,147·2 | 30 | 72·8 | 70·0 | 36·1 |
| 55-59 | 2,594·5 | 27 | 61·6 | 60·2 | 30·3 |
| 60-64 | 881·8 | 13 | 29·5 | 28·3 | 14·1 |
| 65-69 | 219·8 | 8 | 10·7 | 10·6 | 5·1 |
| 70-74 | 57·8 | 3 | 4·1 | 4·1 | 1·9 |
| Total, . | 57,255·0 | 315 | 617·8 | 538·3 | 341·3 |

TABLE 3.—STATE SCHOOL TEACHERS OF VICTORIA—MALES.

*Rate of Mortality among Pensioners. Comparison of Actual and Expected Deaths.*

| Ages. | Exposed to Risk. | Actual Deaths. | Expected Deaths, by | |
|---|---|---|---|---|
| | | | $H^M$ Table. | Government Annuitants Table. |
| 30-34 | 36·0 | 3 | ·3 | ·5 |
| 35-39 | 75·2 | 3 | ·7 | 1·2 |
| 40-44 | 129·9 | 2 | 1·4 | 2·2 |
| 45-49 | 246·8 | 5 | 3·4 | 5·0 |
| 50-54 | 573·0 | 6 | 10·2 | 13·4 |
| 55-59 | 866·6 | 22 | 21·1 | 22·8 |
| 60-64 | 1,622·1 | 52 | 56·9 | 59·7 |
| 65-69 | 1,382·7 | 48 | 68·3 | 66·8 |
| 70-74 | 759·4 | 41 | 55·7 | 55·0 |
| 75-79 | 250·8 | 22 | 28·0 | 26·3 |
| 80-84 | 69·8 | 11 | 11·4 | 10·6 |
| 85-89 | 13·9 | 2 | 3·0 | 3·0 |
| Total, . | 6,026·2 | 217 | 260·4 | 257·5 |

### Effect of Withdrawals.

Although I believe that in all such calculations as the present it is preferable to ignore the effect of withdrawals in arriving at the proper rate of contribution to be exacted, yet I thought it would be interesting to see what effect the introduction of this element would have upon the contributions and the valuations. I therefore constructed a table embodying certain rates of *withdrawal*, which are shown on page 321, combined with the $H^M$ rate of mortality and the same rate of *retirement* as before. The rates of withdrawal employed are practically those actually experienced by the two institutions already referred to on page 309, and are considerably heavier than those assumed by Mr. Manly in his paper. Seeing, however, that the tendency of the establishment of a Pension Fund is to discourage secessions among the staff, it would be desirable, if such an element were to be reckoned upon at all, to assume a much more moderate rate than had been actually found to prevail in the past. Still, for our present purpose of indicating in what direction the condition of a Fund would be affected by such an assumption, the tables now employed may be taken as sufficient. On a 4 per cent interest basis, the rate of contribution required is only 2·182 per cent on salary, and the liability of the Fund, as shown in Division V of Table G, is £112,971·5.

### Method of Construction of Tables.

The manner in which the Life and Labour Tables were constructed, is exhibited in the specimen on page 319, in which there is shown the early portion of Table No. III, in which the decrement is made up of the dying, the retiring, and the withdrawing. The others were constructed on an exactly similar plan.

### Additional Benefits.

Having now found the necessary rates of contribution for a given scale of pension benefits on several different assumptions, I finally calculated what additional rate would be required in order to provide certain supplementary benefits, such as are usually provided in the case of some of the Bank Funds with which I have had to do. These are as follows:—*First*, the payment of the pension in case of retirement for a period of *five years certain*, even in case of the death of the pensioner during that period, any unpaid balance being payable at his death to his representatives; *Second*, a payment of a gratuity of one month's salary for each year of service, to such as are compelled to retire from ill-health after 10 years', and before the completion of 20

years', service; *Third*, payment to a member's representatives of an amount equal to five years' pension, should death occur after the optional age of retirement, say 60, and before the age of compulsory retirement, say 70; *Fourth*, a refund of contributions

TABLE 4.—LIFE AND LABOUR TABLE No. III, SHOWING METHOD OF CONSTRUCTION.

| AGE. $x$ | $l_x^{(3)}$ | $d_x^{(3)}$ | $r_x$ | $w_x$ | $d_x^{(3)}+r_x+w_x$ | $\log l_x^{(3)}$ | $\log d_x^{(3)}$ | $\log r_x$ | $\log w_x$ | $\log q_x$ ($H^M$) | $\log q(r)_x$ | $\log q(w)_x$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 100000 | 287 | | 3220 | 3507 | 5·0000 | 2·4581 | | $\bar{3}$·5079 | $\bar{3}$·4581 | | $\bar{2}$·5079 |
| 16 | 96493 | 318 | | 3580 | 3898 | 4·9845 | ·4960 | | ·5539 | ·5115 | | ·5694 |
| 17 | 92600 | 360 | | 3899 | 4259 | ·9666 | ·5557 | | ·5909 | ·5891 | | ·6243 |
| 18 | 88341 | 423 | | 4179 | 4602 | ·9462 | ·6267 | | ·6211 | ·6805 | | ·6749 |
| 19 | 83739 | 481 | | 4405 | 4886 | ·9229 | ·6822 | | ·6439 | ·7593 | | ·7210 |
| 20 | 78853 | 499 | | 4605 | 5104 | ·8968 | ·6981 | | ·6632 | ·8013 | | ·7664 |
| 21 | 73749 | 496 | | 4632 | 5128 | ·8678 | ·6955 | | ·6658 | ·8277 | | ·7980 |
| 22 | 68621 | 470 | | 4392 | 4862 | ·8365 | ·6718 | | ·6427 | ·8353 | | ·8062 |
| 23 | 63759 | 431 | | 4004 | 4435 | ·8045 | ·6347 | | ·6025 | ·8302 | | ·7980 |
| 24 | 59324 | 394 | | 3471 | 3865 | ·7732 | ·5953 | | ·5404 | ·8221 | | ·7672 |
| 25 | 55459 | 368 | | 3023 | 3391 | ·7440 | ·5655 | | ·4804 | ·8215 | | ·7364 |
| 26 | 52068 | 348 | | 2661 | 3009 | ·7166 | ·5417 | | ·4250 | ·8251 | | ·7084 |
| 27 | 49059 | 339 | 25 | 2369 | 2733 | ·6907 | ·5298 | 1·3897 | ·3746 | ·8391 | $\bar{4}$·6990 | ·6839 |
| 28 | 46326 | 332 | 27 | 2136 | 2495 | ·6659 | ·5215 | ·4293 | ·3296 | ·8556 | ·7684 | ·6637 |
| 29 | 43831 | 326 | 29 | 1929 | 2284 | ·6418 | ·5130 | ·4679 | ·2853 | ·8712 | ·8261 | ·6435 |
| 30 | 41547 | 321 | 32 | 1745 | 2098 | ·6186 | ·5064 | ·5107 | ·2418 | ·8878 | ·8921 | ·6232 |
| 31 | 39449 | | | | | | | | | | | |
| * | * | * | * | * | * | * | * | * | * | * | * | * |

paid by such as, having served for at least 20 years, die before attaining age 60. The values of these several benefits for an entrant of age 19 were arrived at on similar principles to those already described, the first portion being got by using $a^{(4)}_{\overline{5}|}+{}_{5|}a^{(4)}_{x}$ in formula (4), in place of $a^{(4)}_{x}$. To aid in finding the value of the second portion, a small subsidiary table is required, showing $\frac{1}{12}$th of the aggregate past salary for years of service 10 to 19 inclusive; for the third portion the amount of annual pension payable in any given year between ages 60 and 70, has to be multiplied by $a^{(4)}_{\overline{5}|}$ in place of by $a^{(4)}_{x}$; and for the fourth portion a subsidiary table is required, showing 1 per cent on the aggregate past salaries for durations over 20 years and up to age 60. As shown above, however, it is not necessary to calculate every individual term of any of these series, and therefore in the subsidiary tables only the required terms need to be inserted. The results at 4 per cent interest, and using $H^M$ mortality throughout, were as follows:—Value of Pension, assuming it to be payable in any case for 5 years certain, 137·8; Value of Gratuity, 1·2; Value of 5 years' Pension in case of death between age 60 and 70, 16·9; Value of refund of Contribution of 1 per cent on salary, in case of death before 60, 4·5. Now, as the rate of contribution to be charged depends partly on this last item, let us denote this rate by X; then we shall have for the total value of the proposed benefits $137{\cdot}8+1{\cdot}2+16{\cdot}9+4{\cdot}5X, = 155{\cdot}9+4{\cdot}5X$. And, as we found on page 313 that the value of a contribution of 1 per cent on salary for an entrant aged 19 was 38·1, we have the equation $38{\cdot}1X = 155{\cdot}9+4{\cdot}5X$; whence $33{\cdot}6X = 155{\cdot}9$, and $X = 4{\cdot}640$, the required rate of contribution.

In order to see what difference in the rates of contribution would be caused by an alteration of the age at entry, I calculated what the rate would be, on the first set of assumptions, for the pension benefit alone at age at entry 18 instead of 19. The result was 3·308 per cent, as against 3·438 per cent. I therefore estimated that age 20 at entry would require a rate of about 3·568 per cent. I further estimated that with all the additional benefits just enumerated, the rate of contribution for an entrant of age 20 would be as nearly as possible 4·816 per cent.

## Conclusion.

I have now endeavoured, to the best of my ability, to explain the methods I have been in the habit of using in dealing with Pension Funds, and believe that similar principles are easily applicable in nearly every case, even where the system of benefits may be unusually complicated; but, of course, each Fund would require a set of fundamental tables, calculated as far as possible

in accordance with its own circumstances and with the hypotheses which the actuary may see fit to use in either founding a new Fund or making a Valuation of an existing one.

I have moreover, I believe, indicated correctly the probable effect, upon the rate of contribution and upon the valuations, of variations in the rate of interest and in the rates of mortality employed; as well as the effect of withdrawals and of varying scales of salary. The field is no doubt an extensive one, and I feel that much remains to be done, both towards further elucidating some of the points I have dealt with and investigating others which I have not touched upon at all.

It only remains to acknowledge my great indebtedness to Mr. F. J. Jacobs, A.I.A., who has frequently assisted me in similar work, and with whose valuable assistance the numerous tables embodied in the present paper were constructed, and the somewhat laborious calculations completed which were necessary in order to arrive at the conclusions herein set forth.

TABLE A.—RATES OF MORTALITY, RETIREMENT, AND WITHDRAWAL.

| AGE. $x$ | $\cdot 75 q_x$ H$^M$ | $q(r)_x$ | $q(w)_x$ | AGE. $x$ | $\cdot 75 q_x$ H$^M$ | $q(r)_x$ | $q(w)_x$ |
|---|---|---|---|---|---|---|---|
| 15 | ·002153 | ..... | ·0322 | 43 | ·008345 | ·00306 | ·0217 |
| 16 | ·002435 | ..... | ·0371 | 44 | ·008667 | ·00332 | ·0206 |
| 17 | ·002911 | ..... | ·0421 | 45 | ·009144 | ·00360 | ·0195 |
| 18 | ·003594 | ..... | ·0478 | 46 | ·009705 | ·00391 | ·0184 |
| 19 | ·004309 | ..... | ·0526 | 47 | ·01028 | ·00429 | ·0173 |
| 20 | ·004747 | ..... | ·0584 | 48 | ·01083 | ·00475 | ·0162 |
| 21 | ·005044 | ..... | ·0628 | 49 | ·01142 | ·00531 | ·0151 |
| 22 | ·005133 | ..... | ·0640 | 50 | ·01196 | ·00598 | ·0140 |
| 23 | ·005073 | ..... | ·0628 | 51 | ·01250 | ·00676 | ·0130 |
| 24 | ·004980 | ..... | ·0585 | 52 | ·01316 | ·00766 | ·0120 |
| 25 | ·004972 | ..... | ·0545 | 53 | ·01395 | ·00920 | ·0110 |
| 26 | ·005014 | ..... | ·0511 | 54 | ·01480 | ·01170 | ·0100 |
| 27 | ·005178 | ·00050 | ·0483 | 55 | ·01578 | ·01588 | ·0090 |
| 28 | ·005379 | ·00058 | ·0461 | 56 | ·01684 | ·02175 | ..... |
| 29 | ·005575 | ·00067 | ·0440 | 57 | ·01799 | ·02763 | ..... |
| 30 | ·005792 | ·00078 | ·0420 | 58 | ·01922 | ·03350 | ..... |
| 31 | ·005938 | ·00090 | ·0400 | 59 | ·02065 | ·03938 | ..... |
| 32 | ·006079 | ·00103 | ·0380 | 60 | ·02226 | ·04525 | ..... |
| 33 | ·006214 | ·00117 | ·0361 | 61 | ·02403 | ·05113 | ..... |
| 34 | ·006378 | ·00132 | ·0342 | 62 | ·02598 | ·05700 | ..... |
| 35 | ·006580 | ·00148 | ·0325 | 63 | ·02812 | ·06300 | ..... |
| 36 | ·006832 | ·00165 | ·0309 | 64 | ·03031 | ·07000 | ..... |
| 37 | ·007098 | ·00182 | ·0293 | 65 | ·03257 | ·07800 | ..... |
| 38 | ·007335 | ·00200 | ·0278 | 66 | ·03493 | ·08800 | ..... |
| 39 | ·007561 | ·00219 | ·0264 | 67 | ·03742 | ·10000 | ..... |
| 40 | ·007729 | ·00239 | ·0250 | 68 | ·03998 | ·11500 | ..... |
| 41 | ·007865 | ·00260 | ·0239 | 69 | ·04301 | ·30000 | ..... |
| 42 | ·008050 | ·00282 | ·0228 | 70 | ..... | 1·00000 | ..... |

TABLE B.—LIFE AND LABOUR TABLE No. I.

| Age. $x$ | $l_x^{(1)}$ | $d_x^{(1)}$ | $r_x$ | $d_x^{(1)}+r_x$ |
|---|---|---|---|---|
| 15 | 100000 | 287 | ..... | 287 |
| 16 | 99713 | 328 | ..... | 328 |
| 17 | 99390 | 386 | ..... | 386 |
| 18 | 99004 | 474 | ..... | 474 |
| 19 | 98530 | 566 | ..... | 566 |
| 20 | 97964 | 620 | ..... | 620 |
| 21 | 97344 | 655 | ..... | 655 |
| 22 | 96689 | 662 | ..... | 662 |
| 23 | 96027 | 650 | ..... | 650 |
| 24 | 95377 | 633 | ..... | 633 |
| 25 | 94744 | 628 | ..... | 628 |
| 26 | 94116 | 629 | ..... | 629 |
| 27 | 93487 | 645 | 47 | 692 |
| 28 | 92795 | 665 | 54 | 719 |
| 29 | 92076 | 684 | 62 | 746 |
| 30 | 91330 | 705 | 71 | 776 |
| 31 | 90554 | 717 | 81 | 798 |
| 32 | 89756 | 727 | 92 | 819 |
| 33 | 88937 | 737 | 104 | 841 |
| 34 | 88096 | 749 | 116 | 865 |
| 35 | 87231 | 765 | 129 | 894 |
| 36 | 86337 | 786 | 142 | 928 |
| 37 | 85409 | 808 | 155 | 963 |
| 38 | 84446 | 826 | 169 | 995 |
| 39 | 83451 | 841 | 183 | 1024 |
| 40 | 82427 | 850 | 197 | 1047 |
| 41 | 81380 | 853 | 212 | 1065 |
| 42 | 80315 | 862 | 226 | 1088 |
| 43 | 79227 | 882 | 242 | 1124 |
| 44 | 78103 | 903 | 259 | 1162 |
| 45 | 76941 | 938 | 277 | 1215 |
| 46 | 75726 | 980 | 296 | 1276 |
| 47 | 74450 | 1020 | 319 | 1339 |
| 48 | 73111 | 1056 | 347 | 1403 |
| 49 | 71708 | 1091 | 381 | 1472 |
| 50 | 70236 | 1120 | 420 | 1540 |
| 51 | 68696 | 1145 | 464 | 1609 |
| 52 | 67087 | 1177 | 514 | 1691 |
| 53 | 65396 | 1216 | 601 | 1817 |
| 54 | 63579 | 1255 | 744 | 1999 |
| 55 | 61580 | 1295 | 978 | 2273 |
| 56 | 59307 | 1332 | 1290 | 2622 |
| 57 | 56685 | 1360 | 1566 | 2926 |
| 58 | 53759 | 1378 | 1801 | 3179 |
| 59 | 50580 | 1393 | 1992 | 3385 |
| 60 | 47195 | 1401 | 2136 | 3537 |
| 61 | 43658 | 1399 | 2282 | 3681 |
| 62 | 40027 | 1386 | 2282 | 3668 |
| 63 | 36359 | 1363 | 2291 | 3654 |
| 64 | 32705 | 1322 | 2289 | 3611 |
| 65 | 29094 | 1264 | 2269 | 3533 |
| 66 | 25561 | 1190 | 2249 | 3439 |
| 67 | 22122 | 1104 | 2212 | 3316 |
| 68 | 18806 | 1001 | 2163 | 3164 |
| 69 | 15642 | 897 | 4693 | 5590 |
| 70 | 10052 | ..... | 10052 | 10052 |

TABLE C.—LIFE AND LABOUR TABLE No. II.

| AGE. $x$ | $l_x^{(m)}$ | $d_x^{(m)}$ | $r_x$ | $d_x^{(m)}+r_x$ |
|---|---|---|---|---|
| 15 | 100000 | 215 | ..... | 215 |
| 16 | 99785 | 243 | ..... | 243 |
| 17 | 99542 | 290 | ..... | 290 |
| 18 | 99252 | 357 | ..... | 357 |
| 19 | 98895 | 426 | ..... | 426 |
| 20 | 98469 | 467 | ..... | 467 |
| 21 | 98002 | 494 | ..... | 494 |
| 22 | 97508 | 501 | ..... | 501 |
| 23 | 97007 | 492 | ..... | 492 |
| 24 | 96515 | 481 | ..... | 481 |
| 25 | 96034 | 477 | ..... | 477 |
| 26 | 95557 | 479 | ..... | 479 |
| 27 | 95078 | 492 | 48 | 540 |
| 28 | 94538 | 509 | 55 | 564 |
| 29 | 93974 | 524 | 63 | 587 |
| 30 | 93387 | 541 | 73 | 614 |
| 31 | 92773 | 551 | 83 | 634 |
| 32 | 92139 | 560 | 95 | 655 |
| 33 | 91484 | 569 | 107 | 676 |
| 34 | 90808 | 579 | 120 | 699 |
| 35 | 90109 | 593 | 133 | 726 |
| 36 | 89383 | 611 | 148 | 759 |
| 37 | 88624 | 629 | 161 | 790 |
| 38 | 87834 | 644 | 176 | 820 |
| 39 | 87014 | 658 | 191 | 849 |
| 40 | 86165 | 666 | 206 | 872 |
| 41 | 85293 | 671 | 222 | 893 |
| 42 | 84400 | 679 | 238 | 917 |
| 43 | 83483 | 697 | 255 | 952 |
| 44 | 82531 | 715 | 274 | 989 |
| 45 | 81542 | 746 | 294 | 1040 |
| 46 | 80502 | 781 | 315 | 1096 |
| 47 | 79406 | 816 | 341 | 1157 |
| 48 | 78249 | 847 | 372 | 1219 |
| 49 | 77030 | 880 | 409 | 1289 |
| 50 | 75741 | 906 | 453 | 1359 |
| 51 | 74382 | 930 | 508 | 1438 |
| 52 | 72949 | 960 | 559 | 1519 |
| 53 | 71430 | 997 | 657 | 1654 |
| 54 | 69776 | 1033 | 816 | 1849 |
| 55 | 67927 | 1072 | 1079 | 2151 |
| 56 | 65776 | 1108 | 1431 | 2539 |
| 57 | 63237 | 1138 | 1747 | 2885 |
| 58 | 60352 | 1160 | 2022 | 3182 |
| 59 | 57170 | 1181 | 2252 | 3433 |
| 60 | 53737 | 1196 | 2432 | 3628 |
| 61 | 50109 | 1204 | 2562 | 3766 |
| 62 | 46343 | 1204 | 2642 | 3846 |
| 63 | 42497 | 1195 | 2677 | 3872 |
| 64 | 38625 | 1171 | 2704 | 3875 |
| 65 | 34750 | 1132 | 2711 | 3843 |
| 66 | 30907 | 1080 | 2720 | 3800 |
| 67 | 27107 | 1014 | 2711 | 3725 |
| 68 | 23382 | 934 | 2688 | 3622 |
| 69 | 19760 | 850 | 5928 | 6778 |
| 70 | 12982 | ..... | 12982 | 12982 |

TABLE D.—LIFE AND LABOUR TABLE NO. III.

| AGE, $x$ | $l_x^{(3)}$ | $d_x^{(3)}$ | $r_x$ | $w_x$ | $d_x^{(3)}+r_x+w_x$ |
|---|---|---|---|---|---|
| 15 | 100000 | 287 | ... | 3220 | 3507 |
| 16 | 96493 | 313 | ... | 3580 | 3893 |
| 17 | 92600 | 360 | ... | 3899 | 4259 |
| 18 | 88341 | 423 | ... | 4179 | 4602 |
| 19 | 83739 | 481 | ... | 4405 | 4886 |
| 20 | 78853 | 499 | ... | 4605 | 5104 |
| 21 | 73749 | 496 | ... | 4632 | 5128 |
| 22 | 68621 | 470 | ... | 4392 | 4862 |
| 23 | 63759 | 431 | ... | 4004 | 4435 |
| 24 | 59324 | 394 | ... | 3471 | 3865 |
| 25 | 55459 | 368 | ... | 3023 | 3391 |
| 26 | 52068 | 348 | ... | 2661 | 3009 |
| 27 | 49059 | 339 | 25 | 2369 | 2733 |
| 28 | 46326 | 332 | 27 | 2136 | 2495 |
| 29 | 43831 | 326 | 29 | 1929 | 2284 |
| 30 | 41547 | 321 | 32 | 1745 | 2098 |
| 31 | 39449 | 312 | 36 | 1578 | 1926 |
| 32 | 37523 | 304 | 39 | 1426 | 1769 |
| 33 | 35754 | 296 | 42 | 1291 | 1629 |
| 34 | 34125 | 290 | 45 | 1167 | 1502 |
| 35 | 32623 | 286 | 48 | 1060 | 1394 |
| 36 | 31229 | 285 | 52 | 965 | 1302 |
| 37 | 29927 | 283 | 54 | 877 | 1214 |
| 38 | 28713 | 281 | 57 | 798 | 1136 |
| 39 | 27577 | 278 | 60 | 728 | 1066 |
| 40 | 26511 | 273 | 63 | 663 | 999 |
| 41 | 25512 | 268 | 66 | 610 | 944 |
| 42 | 24568 | 264 | 69 | 560 | 893 |
| 43 | 23675 | 264 | 72 | 514 | 850 |
| 44 | 22825 | 264 | 76 | 470 | 810 |
| 45 | 22015 | 269 | 79 | 429 | 777 |
| 46 | 21238 | 275 | 83 | 391 | 749 |
| 47 | 20489 | 281 | 88 | 354 | 723 |
| 48 | 19766 | 286 | 94 | 320 | 700 |
| 49 | 19066 | 290 | 101 | 288 | 679 |
| 50 | 18387 | 293 | 110 | 258 | 661 |
| 51 | 17726 | 296 | 120 | 231 | 647 |
| 52 | 17079 | 300 | 131 | 205 | 636 |
| 53 | 16443 | 306 | 151 | 181 | 638 |
| 54 | 15805 | 312 | 185 | 158 | 655 |
| 55 | 15150 | 319 | 241 | 136 | 696 |
| 56 | 14454 | 325 | 314 | ..... | 639 |
| 57 | 13815 | 332 | 382 | ..... | 714 |
| 58 | 13101 | 336 | 439 | ..... | 775 |
| 59 | 12326 | 340 | 486 | ..... | 826 |
| 60 | 11500 | 341 | 520 | ..... | 861 |
| 61 | 10639 | 341 | 544 | ..... | 885 |
| 62 | 9754 | 338 | 556 | ..... | 894 |
| 63 | 8860 | 332 | 558 | ..... | 890 |
| 64 | 7970 | 322 | 558 | ..... | 880 |
| 65 | 7090 | 308 | 553 | ..... | 861 |
| 66 | 6229 | 290 | 548 | ..... | 838 |
| 67 | 5391 | 269 | 539 | ..... | 808 |
| 68 | 4588 | 244 | 527 | ..... | 771 |
| 69 | 3812 | 219 | 1144 | ..... | 1363 |
| 70 | 2449 | ..... | 2449 | ..... | 2449 |

TABLE E.—SALARY SCALES.

| Completed Year of Service. $n$ | Salary Scale No. I. | | | | Salary Scale No. II. (Equal Increments). | | | | Completed Year of Service. $n$ |
|---|---|---|---|---|---|---|---|---|---|
| | Salary. | $B_n$ | $c_n$ | $c_{n+\frac{1}{2}}$ | Salary. | $B_n$ | $c_n$ | $c_{n+\frac{1}{2}}$ | |
| 1 | 25 | ... | ·25 | ·375 | 25 | ... | ·25 | ·2975 | 1 |
| 2 | 50 | ... | ·50 | ·625 | 34·5 | .. | ·345 | ·3925 | 2 |
| 3 | 75 | ... | ·75 | ·875 | 44 | ... | ·44 | ·4875 | 3 |
| 4 | 100 | ... | 1·00 | 1·085 | 53·5 | ... | ·535 | ·5825 | 4 |
| 5 | 117 | .. | 1·17 | 1·255 | 63 | ... | ·63 | ·6775 | 5 |
| 6 | 134 | ... | 1·34 | 1·420 | 72·5 | ... | ·725 | ·7725 | 6 |
| 7 | 150 | ... | 1·50 | 1·585 | 82 | .. | ·82 | ·8675 | 7 |
| 8 | 167 | ... | 1·67 | 1·755 | 91·5 | ... | ·915 | ·9625 | 8 |
| 9 | 184 | ... | 1·84 | 1·920 | 101 | ... | 1·01 | 1·058 | 9 |
| 10 | 200 | ... | 2·00 | 2·040 | 110·5 | ... | 1·105 | 1·153 | 10 |
| 11 | 208 | ... | 2·08 | 2·120 | 120 | ... | 1·20 | 1·248 | 11 |
| 12 | 216 | .. | 2·16 | 2·205 | 129·5 | ... | 1·295 | 1·343 | 12 |
| 13 | 225 | ... | 2·25 | 2·290 | 139 | ... | 1·39 | 1·438 | 13 |
| 14 | 233 | ... | 2·33 | 2·370 | 148·5 | ... | 1·485 | 1·533 | 14 |
| 15 | 241 | ... | 2·41 | 2·455 | 158 | ... | 1·58 | 1·628 | 15 |
| 16 | 250 | ... | 2·50 | 2·520 | 167·5 | ... | 1·675 | 1·723 | 16 |
| 17 | 254 | ... | 2·54 | 2·560 | 177 | ... | 1·77 | 1·818 | 17 |
| 18 | 258 | ... | 2·58 | 2·600 | 186·5 | ... | 1·865 | 1·913 | 18 |
| 19 | 262 | ... | 2·62 | 2·640 | 196 | ... | 1·96 | 2·008 | 19 |
| 20 | 266 | 60·3 | 2·66 | 2·680 | 205·5 | 38·4 | 2·055 | 2·103 | 20 |
| 21 | 270 | 64·8 | 2·70 | 2·725 | 215 | 42·0 | 2·15 | 2·198 | 21 |
| 22 | 275 | 69·3 | 2·75 | 2·775 | 224·5 | 45·7 | 2·245 | 2·293 | 22 |
| 23 | 280 | 74·0 | 2·80 | 2·825 | 234 | 49·6 | 2·34 | 2·388 | 23 |
| 24 | 285 | 78·8 | 2·85 | 2·875 | 243·5 | 53·7 | 2·435 | 2·483 | 24 |
| 25 | 290 | 83·6 | 2·90 | 2·925 | 253 | 57·9 | 2·53 | 2·578 | 25 |
| 26 | 295 | 88·5 | 2·95 | 2·975 | 262·5 | 62·3 | 2·625 | 2·673 | 26 |
| 27 | 300 | 93·5 | 3·00 | 3·025 | 272 | 66·8 | 2·72 | 2·768 | 27 |
| 28 | 305 | 98·6 | 3·05 | 3·075 | 281·5 | 71·5 | 2·815 | 2·863 | 28 |
| 29 | 310 | 103·8 | 3·10 | 3·125 | 291 | 76·4 | 2·91 | 2·958 | 29 |
| 30 | 315 | 109·0 | 3·15 | 3·175 | 300·5 | 81·4 | 3·005 | 3·053 | 30 |
| 31 | 320 | 114·3 | 3·20 | 3·225 | 310 | 86·5 | 3·10 | 3·148 | 31 |
| 32 | 325 | 119·8 | 3·25 | 3·275 | 319·5 | 91·8 | 3·195 | 3·243 | 32 |
| 33 | 330 | 125·3 | 3·30 | 3·325 | 329 | 97·4 | 3·29 | 3·338 | 33 |
| 34 | 335 | 130·8 | 3·35 | 3·375 | 338·5 | 103·0 | 3·385 | 3·433 | 34 |
| 35 | 340 | 136·5 | 3·40 | 3·425 | 348 | 108·8 | 3·48 | 3·528 | 35 |
| 36 | 345 | 142·3 | 3·45 | 3·510 | 357·5 | 114·8 | 3·575 | 3·623 | 36 |
| 37 | 357 | 148·2 | 3·57 | 3·630 | 367 | 120·9 | 3·67 | 3·718 | 37 |
| 38 | 369 | 154·4 | 3·69 | 3·755 | 376·5 | 127·1 | 3·765 | 3·813 | 38 |
| 39 | 382 | 160·7 | 3·82 | 3·880 | 386 | 133·6 | 3·86 | 3·908 | 39 |
| 40 | 394 | 167·3 | 3·94 | 4·000 | 395·5 | 140·2 | 3·955 | 4·008 | 40 |
| 41 | 406 | 169·8 | 4·06 | 4·060 | 406 | 143·3 | 4·06 | 4·060 | 41 |
| 42 | 406 | 172·2 | 4·06 | 4·060 | 406 | 146·4 | 4·06 | 4·060 | 42 |
| 43 | 406 | 174·5 | 4·06 | 4·060 | 406 | 149·3 | 4·06 | 4·060 | 43 |
| 44 | 406 | 176·7 | 4·06 | 4·060 | 406 | 152·0 | 4·06 | 4·060 | 44 |
| 45 | 406 | 178·8 | 4·06 | 4·060 | 406 | 154·7 | 4·06 | 4·060 | 45 |
| 46 | 406 | 180·8 | 4·06 | 4·060 | 406 | 157·2 | 4·06 | 4·060 | 46 |
| 47 | 406 | 182·7 | 4·06 | 4·060 | 406 | 159·6 | 4·06 | 4·060 | 47 |
| 48 | 406 | 184·5 | 4·06 | 4·060 | 406 | 161·9 | 4·06 | 4·060 | 48 |
| 49 | 406 | 186·3 | 4·06 | 4·060 | 406 | 164·1 | 4·06 | 4·060 | 49 |
| 50 | 406 | 187·9 | 4·06 | 4·060 | 406 | 166·3 | 4·06 | 4·060 | 50 |
| 51 | 406 | 189·6 | 4·06 | 4·060 | 406 | 168·3 | 4·06 | 4·060 | 51 |
| 52 | 406 | 191·1 | 4·06 | 4·060 | 406 | 170·3 | 4·06 | 4·060 | 52 |
| 53 | 406 | 192·7 | 4·06 | 4·060 | 406 | 172·2 | 4·06 | 4·060 | 53 |
| 54 | 406 | 194·1 | 4·06 | 4·060 | 406 | 174·0 | 4·06 | 4·060 | 54 |
| 55 | 406 | 195·5 | 4·06 | 4·060 | 406 | 175·7 | 4·06 | 4·060 | 55 |

TABLE F.—SUMMARY OF GROUPS.

| Year of Service. | No. of Members. | | Age at Entry. | Average Year of Service. | Average Age at Entry. |
|---|---|---|---|---|---|
| 1 | 35 | 280 | 18 | 3 | 18 |
| 2 | 55 | | 18 | | |
| 3 | 60 | | 18 | | |
| 4 | 70 | | 18 | | |
| 5 | 60 | | 18 | | |
| 6 | 55 | 190 | 18 | 8 | 18 |
| 7 | 20 | | 17 | | |
| 8 | 40 | | 19 | | |
| 9 | 30 | | 18 | | |
| 10 | 45 | | 18 | | |
| 11 | 60 | 230 | 19 | 13 | 19 |
| 12 | 40 | | 18 | | |
| 13 | 50 | | 19 | | |
| 14 | 30 | | 19 | | |
| 15 | 50 | | 20 | | |
| 16 | 25 | 180 | 21 | 18 | 20 |
| 17 | 20 | | 20 | | |
| 18 | 40 | | 20 | | |
| 19 | 20 | | 20 | | |
| 20 | 25 | | 19 | | |
| 21 | 5 | 60 | 19 | 23 | 20 |
| 22 | 10 | | 22 | | |
| 23 | 20 | | 20 | | |
| 24 | 15 | | 21 | | |
| 25 | 10 | | 18 | | |
| 26 | 5 | 39 | 19 | 28 | 21 |
| 27 | 8 | | 22 | | |
| 28 | 11 | | 23 | | |
| 29 | 6 | | 22 | | |
| 30 | 9 | | 19 | | |
| 31 | 5 | 31 | 18 | 33 | 18 |
| 32 | 7 | | 17 | | |
| 33 | 9 | | 20 | | |
| 34 | 4 | | 17 | | |
| 35 | 6 | | 18 | | |
| 36 | 8 | 23 | 20 | 38 | 21 |
| 37 | 6 | | 21 | | |
| 38 | 3 | | 20 | | |
| 39 | 4 | | 22 | | |
| 40 | 2 | | 22 | | |
| 41 | 7 | 17 | 19 | 43 | 21 |
| 42 | 4 | | 21 | | |
| 43 | 2 | | 23 | | |
| 44 | 1 | | 25 | | |
| 45 | 3 | | 17 | | |
| TOTAL, . | 1000 | 1000 | ... | ... | 19 |

## TABLE G.—SUMMARY OF VALUATION.

| Year of Service. | No. of Members. | (I.) Life and Labour Table No. I. Salary Scale No. I. Interest 4 %. | | | | (II.) Life and Labour Table No. I. Salary Scale No. I. Interest 3 %. | | | | (III.) Life and Labour Table No. I. Salary Scale No. II. Interest 4 %. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Value of Pension Benefit. | Value of Contribution of 3·488 %. | Net Liability per Member. | Total Net Liability. | Value of Pension Benefit. | Value of Contribution of 4·567 %. | Net Liability per Member. | Total Net Liability. | Value of Pension Benefit. | Value of Contribution of 3·776 %. | Net Liability per Member. | Total Net Liability. |
| 3 | 280 | 143·1 | 145·1 | −2·0 | ... | 227·7 | 231·1 | −3·4 | ... | 120·1 | 122·0 | −1·9 | ... |
| 8 | 190 | 180·0 | 158·5 | 21·5 | 4085·0 | 273·0 | 245·7 | 27·3 | 5187·0 | 151·1 | 139·3 | 11·8 | 2242·0 |
| 13 | 230 | 233·8 | 157·8 | 76·0 | 17480·0 | 335·2 | 239·8 | 95·4 | 21942·0 | 195·3 | 148·4 | 46·9 | 10787·0 |
| 18 | 130 | 303·9 | 147·5 | 156·4 | 20332·0 | 411·7 | 219·2 | 192·5 | 25025·0 | 252·6 | 147·6 | 105·0 | 13650·0 |
| 23 | 60 | 385·3 | 138·6 | 246·7 | 14802·0 | 498·8 | 202·3 | 296·5 | 17790·0 | 321·4 | 146·1 | 175·3 | 10518·0 |
| 28 | 39 | 497·0 | 122·0 | 375·0 | 14625·0 | 611·4 | 174·9 | 436·5 | 17023·5 | 416·0 | 133·3 | 282·7 | 11025·3 |
| 33 | 31 | 586·8 | 120·7 | 466·1 | 14449·1 | 708·3 | 171·7 | 536·6 | 16634·6 | 502·1 | 134·4 | 367·7 | 11398·7 |
| 38 | 23 | 793·0 | 80·4 | 712·6 | 16389·8 | 901·4 | 111·4 | 790·0 | 18170·0 | 678·6 | 88·4 | 590·2 | 13574·6 |
| 43 | 17 | 916·9 | 52·6 | 864·3 | 14693·1 | 1012·0 | 71·7 | 940·3 | 15985·1 | 797·6 | 57·8 | 739·8 | 12576·6 |
| TOTALS, | 1000 | 4039·8 | 1123·2 | 2918·6<br>−2·0 | 116856·0 | 4979·5 | 1667·8 | 3315·1<br>−3·4 | 137757·2 | 3434·8 | 1117·3 | 2319·4<br>−1·9 | 85772·2 |
| | | | | 2916·6 | | | | 3311·7 | | | | 2317·5 | |

TABLE G.—SUMMARY OF VALUATION—*continued.*

| Year of Service. | No. of Members. | (IV.) Life and Labour Table No. II. Salary Scale No. I. Interest 4 %. | | | | (V.) Life and Labour Table No. III. Salary Scale No. I. Interest 4 %. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Value of Pension Benefit. | Value of Contribution of 3·817 %. | Net Liability per Member. | Total Net Liability. | Value of Pension Benefit. | Value of Contribution of 2·182 %. | Net Liability per Member. | Total Net Liability. |
| 3 | 280 | 165·6 | 168·3 | −2·7 | ..... | 43·0 | 43·4 | −·4 | ..... |
| 8 | 190 | 206·5 | 183·2 | 23·3 | 4427·0 | 79·1 | 61·8 | 17·3 | 3287·0 |
| 13 | 230 | 265·9 | 182·8 | 83·1 | 19113·0 | 136·4 | 72·0 | 64·4 | 14812·0 |
| 18 | 130 | 341·7 | 170·6 | 171·1 | 22243·0 | 218·8 | 75·5 | 143·3 | 18629·0 |
| 23 | 60 | 428·4 | 160·3 | 268·1 | 16086·0 | 315·8 | 76·4 | 239·4 | 14364·0 |
| 28 | 39 | 545·9 | 141·2 | 404·7 | 15783·3 | 454·8 | 72·0 | 382·8 | 14929·2 |
| 33 | 31 | 639·4 | 139·7 | 499·7 | 15490·7 | 552·5 | 72·9 | 479·6 | 14867·6 |
| 38 | 23 | 845·7 | 92·8 | 752·9 | 17316·7 | 793·2 | 51·1 | 742·1 | 17068·3 |
| 43 | 17 | 966·3 | 60·3 | 906·0 | 15402·0 | 916·6 | 33·4 | 883·2 | 15014·4 |
| TOTALS, . | 1000 | 4405·4 | 1299·2 | 3108·9 | 125861·7 | 3510·2 | 558·5 | 2952·1 | 112971·5 |
| | | | | −2·7 | | | | −·4 | |
| | | | | 3106·2 | | | | 2951·7 | |

## DISCUSSION.

THE paper having been read by the Secretary, Mr. V. MARR,

The PRESIDENT (Mr. G. M. LOW) said—I am sure it is very gratifying to us all to have received this contribution from an old and valued member of the Faculty in a distant part of His Majesty's dominions. Mr. Carment, as you are aware, received his professional education here, and he is well known to some of us who were his contemporaries, but his experience as an actuary has for the most part been gained in quite another part of the world. It is very pleasing indeed that our members going out to the colonies and to other countries should continue to take an interest in the Faculty, and should assist in the proceedings at these meetings. I hope and expect that this paper is only the precursor of others from members of the Faculty in different parts of the world; and if these come up to the standard of this paper we shall welcome them very heartily.

Mr. N. B. GUNN.—It has been a matter of gratification to me also to peruse this paper by our old friend, though we have not seen him for a long time, but whose career in Australia we have followed with interest. The subject Mr. Carment has chosen is an extremely interesting one. I do not know if there is any problem that has to be dealt with by actuaries which brings us more in contact with the variations in the conditions of life, than those connected with pension funds. In life assurance we deal with men in the mass. But in pension funds, instead of looking at the mortality in the mass, we have to study in detail the mortality, and also the characteristics of small homogeneous groups. The bodies in this country that have adopted pension funds are numerous, and include banks, railway companies, shipping companies, operative engineers, and the like. I do not know to what extent they have been adopted abroad, but the other day I was asked to obtain materials from the rules of pension funds in this country, in order that a foundry firm in France, employing 4000 to 5000 men, might start a fund for the benefit of its employees. Mr. Carment has given us the benefit of his experience in Australia, and, as we have seen, it differs very considerably in some respects from our experience here. Of course the general principles of valuation must be the same, whatever the country or the class of lives. But the methods of extracting the statistics, and of treating and modifying them, must differ in each particular case. I have only one criticism to make with reference to the paper, and that is to suggest that a point Mr. Carment brings out should be modified. He remarks that it would be desirable, if the element of secession be reckoned at all, to assume a much more moderate rate than had been actually found to prevail in the past. Well, there is no doubt that in valuing a comparatively new fund he is correct, especially at ages approximating to that at which the pension is to begin. The withdrawals from the fund are, in that case, likely to be less than before the fund was started. But when it has been in existence for a long time, as in the case of some of the railway funds here, the actuary is quite justified in keeping fairly close to the actual rate of withdrawal which has been experienced by the fund. I have referred to the difficulties connected with the manipulation of the data; and, as an example, I may refer to one which has come under my notice

recently. The fund was in connection with the police of one of the large cities in Scotland, and statistics were furnished showing the withdrawals for ten years. It might have been expected that these withdrawals for such a long period could be safely taken as a guide for the future; but on looking over them it was found that for the years 1899, 1900, and 1901, the withdrawals were double the maximum number in any of the previous six years. The war in South Africa commenced in 1899, and during these three years a large number of reservists were withdrawn from the force, and went to join the army in South Africa. In addition to that, as the members of the force explained, in consequence of a number of labouring men leaving this country for the war, the price of labour was enhanced, and many men were tempted to leave the force and rejoin ordinary civil life. Mr. Manly has published some interesting facts as to withdrawals, which I understand were compiled from the experience of the pension funds of railway companies in this country. Mr. G. F. Hardy made an investigation into the experience of the whole police force in Scotland, and found that the rate of withdrawal approximated very closely to Mr. Manly's table drawn from the railway statistics; but in the city police fund to which I refer, I find that after making allowance for the three abnormal years mentioned above, the rate is considerably greater in this municipal police force than in the aggregate force, including both municipal and rural police—in the first ten years of service. This, I think, shows how carefully statistics must be examined. Mr. Carment refers to the question of mortality, and draws attention to Mr. Manly's theory that the mortality of those who retire before the pension age is much greater than that among pensioners who have retired at the usual retiring age under normal conditions. I may say that in connection with the scheme I have referred to, the mortality among pensioners who retired before the normal pension age is about twice the $H^M$ mortality; while the rate among those who retired at the normal pension age is one and a half times the $H^M$ mortality. The question of pension funds comes before the actuary in two different ways—he may be asked to institute a pension fund, or to value one. The labour of a valuation is of course much greater, but the questions to be considered are very much simpler. When a fund is to be started, there are generally great difficulties in the way of getting trustworthy data on which to base our calculations, and we have to proceed pretty much in the dark. In regard to withdrawals, I remember a case where a large ship-owning firm wished to start a pension fund. I found that not a single member of their staff had retired after five years' service. While the circumstance said a great deal for the company, it did not help one in starting a pension fund. The fact that there had been no retirements after five years had to be taken into consideration in calculating the contributions, and the firm was disappointed that these were very much greater than in one of the ocean steamship companies in which there had been what one might call a normal rate of withdrawal. Another point which has a considerable bearing on the valuation of pension funds, is the salary scale. In making such a valuation it should be kept in view that the salary-rate is tending upwards. The rate of expense of living has increased in the past, and is, I think, likely to increase in the future. It is therefore wise, in valuing pension funds, to make allowance for an increasing salary-rate. We must make allowance for the probability that in the fifty or sixty years for which we have to look forward, the salaries may be larger, and that consequently the pensions based on these salaries will also increase. I have had great pleasure in reading this paper; and I think that all who study it will find it a most useful one.

Mr. A. Hewat.—I have listened with much interest to this paper by Mr. Carment, especially because I happen to be one of those members of

the Faculty who have long known him personally. I have, however, had only a short time in which to study the paper, and, naturally, I have been anticipated by Mr. Gunn in a number of the remarks I would have made. There is, as we all know, a great distinction between a "scheme" and a "fund". In the one "you pay as you go", and in the other "you get the money before you pay". The actuary has always to keep in view which is actually wanted. The age of 55 has been mentioned by Mr. Carment as an age of retirement for clergymen in funds that have come under his observation. Many people in this country might not be sorry if a somewhat similar practice were adopted here. It is found that, when there is a good pension fund, they are apt to retire early; while if there is no pension fund, long-suffering congregations know how long they can hold on! More than twenty years ago I was asked to prepare a scheme for a bank where retirement was optional at 60, and compulsory at 65 unless the employee were specially asked to remain longer. I was curious enough to enquire recently whether my estimates were about right. It turned out that I had overestimated the cost during those twenty years by about 7 per cent; and a remark was made that I ought not to have been so far out. I replied that they might congratulate themselves on having a cautious actuary; but after all I was not to blame. At 65 a bank agent in the country may be more valuable than he was at 35, because of his position and influence in the district. There is no actual hard work, and he can better keep the customers connected with the bank. Such men had not been compelled to retire if in good health. Then, with regard to railway schemes and others of that kind, no estimate for the purposes of other funds can be formed from the results of these schemes. The large body of the employees are allowed to draw their full salaries as long as they can, and consequently do not live long to enjoy the pension after retirement. There is a great difference where there is no compulsory age for retirement. I agree entirely with the view that refunds of contributions in the event of withdrawal are an inducement to retire in the early contributing years; and they introduce the savings-bank element, which of course has to be paid for. Mr. Carment refers to 2½ per cent on salary as being "a moderate rate" of contribution, but it is generally a wholly inadequate rate. Further on, where 4 per cent was the rate, he said that rate was "lamentably deficient". I am aware of one very large fund, in London, with about 6000 employees, who in their wisdom thought proper to commence operations without consulting an actuary. The first valuation was made by an eminent London actuary. Instead of the £100 pension promised, the fund could only pay £13, 8s. My first valuation—three years thereafter—brought out £17, 10s. The rate of salary is also a very important element, and it is difficult to deal with. Is the pension to be based on the average salary for the whole period of service, or on that for the last five years only? The contribution element also comes in in connection with the salary, the percentage of salary and the pension acting on one another, so that it is very difficult to get "sufficient exactness" in making estimates. My practice has been to make a very careful estimate, going over every individual on the list, noting what I think might be suitable increases, and trying to find some sort of "law". Some of the higher salaries I deal with in an exceptional manner. Then I confer with people who are familiar with the circumstances; not necessarily with the employees themselves, because of course they are apt to think they deserve too frequent and large increases. At the end of my report I state clearly on what assumptions my figures have been arrived at. Generally, there is not much difference between my estimate and the accountant's statements made from the books. The quinquennial or septennial valuation gives the opportunity for revising former estimates as to pensions, salaries, and contributions; and that can be done in the light

have called the standard rate of pension, would produce the same or very similar results by my method; but in applying his formulas to the valuation of the fund, it seems that he takes no cognisance of the actual salary receivable, or the amount of salary received in the past, both of which form a very large element in estimating the actual pension the fund has to provide for: in fact, it would appear that, no matter at what age the member entered, it is assumed that he started with a salary of £25 a year, increasing according to scale. This is a proposition I am unable to consent to. Many of my professional friends have found some difficulty in agreeing to my proposition that all salaries taken together are subject to the same increase according to age. I think they would find greater difficulty in accepting Mr. Carment's proposition that all salaries and pensions are the same according to years of service. His methods seem to apply very well to the example he has given in Table F, because in that fund they do all apparently enter at the same age; but even in that case there must surely be some at the higher ages who are receiving large salaries. In the case of a bank, or assurance company, it is not at all an uncommon thing for a man to be appointed from another office, who comes in at the age of 30 or 35 and starts at a high salary; and such cases are of course numerous in comparatively new companies, where they must start with men of mature years at good salaries. Surely it would not be correct to assume that these men enter at a salary of £25 a year, and to calculate the values of their contributions and retiring pensions on such a scale. It is indeed the highly-paid officials who often cause the greatest anxiety with regard to the future of these funds. Now, by my method, I base the reserves upon the actual salaries receivable, and the total salaries which have been received in the past where the pension is based on the average salary; and so I claim that I get much nearer the correct reserve than Mr. Carment does. I think that Mr. Carment would have added much to the value of his paper if he had valued his fund by my method, for the purpose of comparison; but as he has not given the necessary particulars, I am unable to make a valuation myself. Although I feel unable altogether to agree with Mr. Carment, I fully appreciate the ingenuity he has shown in the treatment of the subject, and I congratulate the members of the Faculty upon the opportunity Mr. Carment has afforded them of studying the subject under a different aspect.

Mr. V. Marr.—Mr. Carment mentions that in using Lubbock's summation formula for the calculation of pension benefits, it is often found that the more orders of differences used, the less accurate the results. The same thing has been recently brought under the Faculty's notice by Mr. Sprague in his paper "On a method of calculating the force of mortality in select life tables" (*T.F.A.*, i, 56). Mr. Carment says that negative values must be eliminated; but I do not think that they should always be excluded. If, for instance, in a fund in connection with a bank or a public body where membership is compulsory, negative values appear at ages, say, from 20 to 25, and members between these ages resign, it is pretty certain that their successors will be of practically the same ages; and while we do not actually get the contributions of those who leave, we get the contributions of their successors, and therefore the negative values may be safely included.

The President.—It only remains for us to record our very hearty thanks to Mr. Carment for this valuable contribution to our proceedings. The paper is in many respects admirable and interesting, both as to the working of pension funds in Australia, and as to the neat formulas which are furnished. Whether the tables themselves will serve for practical use in this country, where the age of retirement differs so much from that common in Australia, I cannot say; but we cannot fail to profit by the

study of the methods given by Mr. Carment. I have much pleasure in proposing a hearty vote of thanks to him.

## MR. CARMENT'S REPLY.

Having been allowed to peruse the report of the discussion which took place on the reading of my paper along with the appended communication from Mr. Manly, I will now endeavour to reply briefly to such points as seem to require notice. I am much gratified to find that the paper has been so favourably received by my old friends in the Faculty, and that it has created such an interesting discussion.

Mr. Gunn has referred to the rate of withdrawal, and seems to advocate the use of the actual rate which has been experienced in the past by a fund in making a valuation of its liabilities—at least where it has been a long time in existence. For my own part, I would generally prefer to ignore the withdrawal element, at least if the fund were strong enough to permit of this being done.

As to the labour of a valuation being greater than that involved in starting a fund in the first instance, I have had experience of more than one where the reverse has been the case; for the promoters of the funds in question invented so many complicated benefits and so many varied conditions, that a great amount of consideration had to be given to a number of difficult problems, and a vast quantity of preliminary tables had to be constructed, before the relation of the benefits and the payments was fixed at all. The subsequent valuation of the fund was a much easier and simpler matter.

As to the average rate of salary being higher nowadays than in former years, I am not sure that this has been proved to be the case in Australia. Although no doubt the salaries of the higher officials in any given service may be continually on the increase, this is probably balanced by the rank and file containing a larger proportion of boys and youths whose remuneration is necessarily on a very moderate scale. In the case of a fund formed fourteen years ago among the staff of a large Australian life office, the average salary of the whole staff in the first year of the fund was £213, and now it is £202.

Mr. Hewat has apparently somewhat misunderstood what I said as to the average age of retirement in a fund which I mentioned, where the actual average age of retirement in the past was as low as 55, but there was no *compulsory* age in the case of the fund referred to. The fact appears to have been that there was considerable leniency on the part of the management in granting pensions to men who had probably not in reality become permanently incapacitated. Mr. Hewat also quotes me as referring to 2½ per cent on salary as a moderate rate of contribution, but says it is generally an inadequate one; and certainly under most circumstances this would be so, but the case I referred to was one of a fund where practically the *only* benefit is a pension in old age, and where retirement, though optional at 60, is not compulsory till 70. So far no one, while still in good health, has retired before the compulsory age, and retirements on account of permanent breakdown before that age have been extremely rare. I agree with him that in the event of the recovery of a man who has retired, provision should be made for his re-employment in the service in some capacity; but as a general rule it should be possible to make reasonably certain, when placing a man on the pension list, that he has really become permanently incapacitated for any further useful work. Mr. Hewat also states that his own practice is to go over every individual on the list of any fund he is valuing, and try to forecast his future salary. Surely, however, in the case of a large fund this would be

almost impossible, besides being really unnecessary; although as I pointed out in my paper, it will nearly always be desirable for each of the higher officials in accordance with the merits of his particular case.

I must also differ from Mr. Hewat where he says that if the management expenses are to be provided for by the surplus interest earned, the rate assumed, then the valuation is not made at the assumed rate. take an example:—Suppose a fund is earning 4 per cent, *clear of expenses* and that it appears probable that this rate of interest will be maintained for a considerable time to come; then I should consider it right and proper to value the fund at say 3½ per cent, thus leaving a margin for adverse contingencies. Possibly Mr. Hewat may have somewhat misunderstood my reference to this point in the paper.

I am pleased to find that Mr. Manly has contributed to the discussion although he differs from me on the point of taking the salary scale according to "years of service" instead of "years of age". He represents me as assuming that all the members of the fund enter at one age, but a reference to Table F will show that in the case there supposed the average ages at entry in the different individual groups vary from 17 to 25, and in the *quennial* groups to which they are reduced in the last column they vary between 18 and 21. The actual ages of separate individuals might vary between still greater limits. It seems to me that, unless you are to value each member of the fund separately, some method of averaging is absolutely necessary, except as regards the case of a few of the higher-salaried officials which I expressly stated would have to be treated individually. This being so, it would probably not make very much difference to the result whether years of service or years of age are taken; and as the *pension* also progresses according to years of service, it seems natural to treat the salary also as a function of the year of service. No doubt if a large proportion of the members entered at high salaries and at comparatively advanced ages the case would assume a somewhat different aspect, but generally speaking most services are recruited by young men commencing at a small salary. Consequently as regards the great bulk of the members, always excepting the few men with salaries much above the average, it will generally be found safe to make the assumptions I have done, which it must be remembered should be founded in the case of each individual fund upon the data supplied by itself as to salaries, ages, and years of service. No doubt when once we have compiled a set of tables on Mr. Manly's principles the resulting valuation would be performed with comparative ease, but the preparation of the tables in the first instance would be a work of considerable labour seeing that those already published by him would not be directly available if the age of compulsory retirement were postponed until 70, or if a salary scale or a rate of retirement were assumed different from those which he has incorporated in his figures.

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## A Method of printing Particulars on Premium Receipts and Notices.

At the same meeting a brief note was read by Mr. James Chatham as to the use which could be made of a machine called the "Addressograph", for printing particulars on premium receipts and notices, at the rate of about five hundred an hour. The machine was exhibited in operation; and a vote of thanks was given to Mr. Chatham for his communication.

No. 10. Vol. I

# TRANSACTIONS

OF THE

# FACULTY OF ACTUARIES.

CONSTITUTED 1856.

INCORPORATED 1868.

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With Abstract of Discussion thereon.

Law Reports.

Joint Mortality Investigation.

LONDON:
CHARLES & EDWIN LAYTON,
FARRINGDON STREET.
1893.

*Price 1s. 6d.*

# THE FACULTY OF ACTUARIES IN SCOTLAND

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*Fellows*

[illegible]

[illegible]

[illegible]

[illegible]

# TRANSACTIONS

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TRANSACTIONS

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FACULTY OF ACTUARIES.

*Notes on Mortality in India and some other Tropical Countries.* By SPENCER CAMPBELL THOMSON, B.A., *Manager and Actuary of the Standard Life Assurance Company.*

[Read before the Faculty, 27 April 1903.]

BEFORE bringing to your notice the short memorandum which I have prepared on the subject of mortality in the tropics, I feel that I owe the Faculty an apology for submitting to them so fragmentary a paper as the present.

I had hoped, by recapitulating the history of mortality statistics in India and some other tropical countries from the earliest date at which these became available, to have traced down to the present day the progress that has been made and the lives which have been saved in consequence of improved manner of living and enlarged knowledge and better treatment of disease; but other duties intervened which prevented my fulfilling my purpose, and my intended paper has contracted into these few notes exhibiting the experience of my own company.

One reason which has specially induced me to submit the paper in its present shape is that this is the last meeting of the Faculty for the season, and I am anxious without further delay to ventilate the whole question, in view of the very varied rates and rules which have been adopted and are being acted upon by life offices for the determination of the extra premiums for India

and other tropical residence, and which I would like to see harmonised.

The rate of mortality in the British Isles, and in other countries falling within temperate limits, has now been ascertained with great accuracy, and is exhibited by various tables embracing the whole population or some portion thereof, or by the combined experience of the principal life offices. Tables of the fundamental premiums required to cover the risk for assurance of all classes of contingency have also been deduced. But once the boundary line into the semi-tropical or tropical regions of the world is crossed, the whole becomes chaos; and it is for the purpose of drawing attention to this state of matters and with the view of making a first step towards the removal of this reproach from the actuarial profession, and not with the more ambitious aim of accurately determining what the extra premium charged in the case of each country should be, that I have prepared the present memorandum for the notice of my professional brethren.

The reasons why in the past the subject has not been more sufficiently dealt with by actuaries are probably two. *First*, the want in most cases of the requisite facts, through the backwardness or remoteness of the more recently developed countries, by reason of which the study of statistics has been neglected and life assurance likewise has only been taken advantage of to a small extent. *Secondly*, the constant change for the better in the rate of mortality which has been going on in most tropical countries during the last fifty years, whereby the data of a few years previous are after a short time no longer accurate.

In *India* more than any other country an improvement has been going on, and the death-rate has materially decreased, so that the old tables of Woolhouse, Samuel Brown, and others are no longer applicable to the circumstances of the present day. The causes which have contributed to this improvement are pretty well known, namely,—better sanitation—more frequent change home or to the hills as the result of improved means of communication—simpler and sounder modes of life with a more moderate use of stimulants —the more frequent presence of ladies in the country—and most recently the destruction of malarial germs, or rather of the insects which play so large a part in carrying the infection from them. But though these facts are recognised, there is no general consensus of opinion as to what the extra for Indian residence ought really to be, with the consequence that the rates charged vary very considerably.

The statistics of any one office in India must of course be limited, more especially when the lives are classified severally as Europeans born and brought up in Europe, Natives, Eurasians or persons of

mixed blood, and country-born Europeans; and for this reason I would strongly recommend that under the joint care of the Faculty of Actuaries and the Institute of Actuaries, all the offices established in India and transacting a business of any importance there should be asked to combine their individual experience and form a standard Mortality Table which would now, I think, hold the field for a good number of years, and with certain adjustments be more or less applicable, failing a better, to some other Eastern countries. And I think the present time a very suitable one at which to undertake the task; for though it can hardly be said with any certainty that the lowest point has been reached in the fall of the death-rate from the high figure at which it stood fifty or more years ago, I believe that the descent will in the years to come be much more gradual, whilst the level of the more favourable returns for Great Britain and other temperate climates will, I venture to say, never be attained to.

The causes of death and incidence of mortality in India will also always vary from what these are at home, and it must especially be borne in mind that the value of selection is not in India so potent a factor as at home in reducing the death-rate during the early years of the assurance. Doubtless a sound and healthy constitution free from taint of disease will in many cases tell favourably; but a number of diseases which prevail in India, and which on the most sanguine computation it will take many years to stamp out—such as cholera, malaria, and other fevers—may attack the strongest, while fatal accidents are also probably more common than in the United Kingdom, and none of these causes can be eliminated by selection. War, too, accounts yearly for a certain number of deaths, though in this case a special extra is generally charged, as it is also sometimes for residence in an unusually malarious district.

The time is opportune, too, for such a task as I propose, inasmuch as the more active members of the two actuarial bodies are now nearly freed from the strain imposed upon them by the great work of the new combined experience in which they have been lately engaged.

But as the combined experience which I have suggested cannot, if undertaken, be compiled and reported without a considerable delay, it has occurred to me that it might be interesting and instructive if I were to fill up the gap meantime by the exhibition of some statistics derived from the experience of the company with which I am myself connected, which has had business relations with India (either under its present name or as the Colonial Life Assurance Company, which was transferred to the Standard) since 1846, and by a comparison of these with certain other tables. And at this part of my notes I must indicate that the carrying

out of the calculations which I exhibit has been the work almost wholly of my coadjutors in actuarial work, Mr. Sievwright, Mr. Gunn, and Mr. Cameron, names which vouch for the work having been done with carefulness.

The first exhibit which I will offer you is a table showing, in quinquennial groups of ages, the actual and expected mortality of lives assured with the Standard Company between 1846 and 1900 under all classes of policies.

TABLE 1.—*Indian Mortality Experience, including Natives, Eurasians, etc.*, 1846-1900.

| Age. | Exposed to Risk. | Expected Deaths. | | Actual Deaths. |
|---|---|---|---|---|
| | | English No. 1, Males. | Standard E. I. Experience 1870-1885. | |
| 19 | 157·08 | 1·19 | 2·17 | ... |
| 20-29 | 17221·53 | 160·19 | 213·71 | 214 |
| 30-39 | 35387·09 | 407·17 | 463·00 | 446 |
| 40-49 | 24773·21 | 362·64 | 482·36 | 417 |
| 50-59 | 8746·01 | 172·73 | 305·33 | 255 |
| 60-69 | 1545·88 | 62·86 | 90·91 | 80 |
| 70-79 | 237·67 | 20·02 | 26·52 | 19 |
| 80 and upwards | 13·16 | 2·06 | 2·63 | 1 |
| Total. | 88081·63 | 1188·86 | 1586·68 | 1432 |
| Actual deaths to 100 expected | | 120·45 | 90·26 | ..... |

The deaths recorded in the experience table named at the head of the fourth column are included in the "actual deaths", and the ratio of the total actual deaths to the expected by the experience from 1870 to 1885 indicates the improvement which has taken place since that date, so that the addition of the deaths since 1885, notwithstanding that the deaths from 1846 to 1870 are included in the column of actual deaths, has had the effect of improving the rate by about 10 per cent, the improvement having for the most part taken place between ages 30 and 60.

The next tables exhibit the same facts for five years only, from 1895-1900:—

TABLE 2.—*Indian Mortality Experience, including Natives, Eurasians, etc.*, 1895-1900.

| Age. | Exposed to Risk. | Expected Deaths. | | Actual Deaths. |
|---|---|---|---|---|
| | | English No. 1, Males. | Standard E. I. Experience 1870-1885. | |
| 19 | 18·75 | ·14 | ·26 | ... |
| 20-29 | 4318·42 | 40·14 | 53·49 | 87 |
| 30-39 | 8682·08 | 99·70 | 113·52 | 87 |
| 40-49 | 6085·25 | 89·02 | 118·62 | 89 |
| 50-59 | 2278·83 | 45·51 | 80·65 | 61 |
| 60-69 | 465·50 | 18·81 | 27·26 | 15 |
| 70-79 | 83·25 | 7·54 | 9·97 | 10 |
| 80 and upwards | 7·67 | 1·15 | 1·47 | ... |
| Total, | 21929·75 | 302·01 | 405·24 | 299 |
| Actual deaths to 100 expected | | 99·00 | 73·78 | ..... |

Here it will be observed that the actual deaths, 1895-1900, are less than 75 per cent of the deaths expected by the experience table 1870 to 1885, and even slightly below the English Life Table.

TABLE 3.—*Natives only*, 1895-1900.

| Age. | Exposed to Risk. | Expected Deaths. | | Actual Deaths. |
|---|---|---|---|---|
| | | English No. 1, Males. | Standard E. I. Experience 1870-1885. | |
| 17-19 | 3·42 | ·08 | ·05 | ... |
| 20-29 | 677·42 | 6·28 | 8·42 | 7 |
| 30-39 | 1552·17 | 17·86 | 20·32 | 14 |
| 40-49 | 1060·75 | 15·48 | 20·36 | 18 |
| 50-59 | 156·25 | 2·38 | 5·27 | 4 |
| 60-69 | 30·25 | 1·84 | 1·89 | 2 |
| 70 and upwards | 3·00 | ·25 | ·34 | 1 |
| Total, | 3483·26 | 44·17 | 56·65 | 46 |
| Actual deaths to 100 expected | | 104·14 | 81·20 | ..... |

TABLE 4.—*Eurasians only*, 1895-1900.

| Age. | Exposed to Risk. | Expected Deaths. | | Actual Deaths. |
|---|---|---|---|---|
| | | English No. 1, Males. | Standard E.I. Experience 1870-1885. | |
| 18-19 | 2·08 | ·02 | ·03 | ... |
| 20-29 | 60·83 | ·56 | ·76 | ... |
| 30-39 | 267·25 | 3·11 | 3·53 | ... |
| 40-49 | 288·00 | 4·23 | 5·66 | 5 |
| 50-59 | 141·50 | 2·96 | 5·21 | 7 |
| 60-69 | 56·42 | 2·35 | 3·39 | 1 |
| 70 and upwards | 17·17 | 1·56 | 2·07 | ... |
| Total, | 833·25 | 14·79 | 20·65 | 13 |
| Actual deaths to 100 expected | | 87·83 | 62·95 | ..... |

TABLE 5.—*Country-born Europeans only*, 1895-1900.

| Age. | Exposed to Risk. | Expected Deaths. | | Actual Deaths. |
|---|---|---|---|---|
| | | English No. 1, Males. | Standard E. I. Experience 1870-1885. | |
| 18-19 | ·67 | ·00 | ·01 | ... |
| 20-29 | 285·83 | 2·68 | 3·53 | 3 |
| 30-39 | 900·08 | 10·36 | 11·78 | 10 |
| 40-49 | 715·42 | 10·51 | 14 09 | 10 |
| 50-59 | 309·08 | 6·15 | 10·88 | 5 |
| 60-69 | 36·58 | 1·45 | 2·12 | 1 |
| 70-79 | 11·67 | 1·04 | 1·37 | ... |
| 80 and upwards | 2·08 | ·30 | ·38 | ... |
| Total, | 2261·41 | 32·49 | 44·16 | 29 |
| Actual deaths to 100 expected | | 89·26 | 65·67 | ..... |

Here it is seen that among Natives, Eurasians, and Country-born Europeans, as compared with the period to 1885, a very considerable improvement has taken place.

From the last three tables, so far as they go—and owing to the paucity of numbers the conclusions must not be pressed too far—it would appear that among the three classes, of Natives, Eurasians, and Country-borns, the death-rate is at least as favourable as amongst Europeans; but it must be fully borne in mind that the lives in question were very carefully selected under European supervision, and that in other circumstances the death-

rate would in all probability be considerably greater, both because the lives would be of an inferior class and also on account of the moral risk—that is, the risk from fraud, which is always likely to arise in India if the selection of lives is not carefully guarded.

As regards the comparison with the English Life Table No. 1, it must be borne in mind that the normal experience of the home risks of a life company as compared with this table should aggregate not more than 80 per cent of the expectation, when a fair proportion of newly-selected lives are included in the observations.

It has further to be observed that the deaths recorded in these tables took place—with a very few exceptions—in India, and that the rate of mortality at home is swollen by the return of invalids to this country, a matter to be kept in mind in determining Indian premiums which are reducible to home rates on the life coming to reside in Europe. I am not aware of any full observations which have been made on the mortality of retired Indians, but the following table, founded upon a small number of observations for the years 1846-1885, indicates that the extra risk is considerable.

TABLE 6.—*Mortality Experience among lives resident in the United Kingdom or other Temperate Climate who had previously lived in India for not less than* 12 *consecutive months*, 1846-1885.

| Age. | Exposed to Risk. | Expected Deaths. | | Actual Deaths. |
|---|---|---|---|---|
| | | English No. 1, Males. | Carlisle. | |
| 19-29 | 375·17 | 3·55 | 3·15 | 2 |
| 30-39 | 2165·33 | 25·20 | 23·42 | 23 |
| 40-49 | 2493·58 | 36·08 | 35·48 | 40 |
| 50-59 | 1364·68 | 27·96 | 24·13 | 28 |
| 60-69 | 376·75 | 15·36 | 14·58 | 13 |
| 70-78 | 45·50 | 3·59 | 3·26 | 4 |
| Total, | 6820·41 | 111·74 | 104·02 | 110 |
| Actual deaths to 100 expected | | 98·44 | 105·75 | ... |

Two tables of Indian Mortality have been framed by the Standard Office for its own use, and premiums deduced therefrom—the first on the experience of fifteen years, from 1870 to 1885, and the more recent on the experience from 1885 to 1900. All classes of policies are included, as well as all classes of lives.

Seeing that the experience is scanty at the higher ages, owing

to the great majority of the lives retiring to this country after completion of their term of service, it was found necessary in both cases to continue the curves by a junction with the $H^M$ table, with certain additions to the rate of mortality as shown by that table, and both were adjusted by the graphic method.

The first of these tables is based on 26,848 years of life and 472 deaths. The rate of mortality at central ages for quinquennial groups is as follows:—

TABLE 7.—*Standard Office Indian Experience Table*, 1870-1885.

| Age. | Exposed to Risk. | Deaths. | | Adjusted $q_x$ at Central Age. | $q_x$ $H^M$. | Ratio, $\frac{\text{Standard}}{H^M}$. |
|---|---|---|---|---|---|---|
| | | Actual. | Expected by 1885 Experience. | | | |
| 20-24 | 895·75 | 10 | 11·58 | ·0132 | ·0068 | 1·94 |
| 25-29 | 3475·39 | 44 | 42·54 | ·0122 | ·0069 | 1·77 |
| 30-34 | 5460 45 | 68 | 68·28 | ·01245 | ·0081 | 1·54 |
| 35-39 | 5947·12 | 87 | 81·48 | ·0136 | ·0095 | 1·44 |
| 40-44 | 4864·59 | 83 | 82·83 | ·01695 | ·0107 | 1 58 |
| 45-49 | 3230·25 | 68 | 74·21 | ·0232 | ·0137 | 1·70 |
| 50-54 | 1820·12 | 63 | 57·81 | ·0322 | ·0175 | 1·84 |
| 55-59 | 784·57 | 31 | 31·04 | ·0430 | ·0240 | 1·79 |
| Total, | 26428·24 | 449 | 449·22 | ... | ... | ... |

The second table, 1885-1900, is based upon 52,219 years of life and 756 deaths. The rate of mortality in quinquennial groups is as follows:—

TABLE 8.—*Standard Office Indian Experience Table*, 1885-1900.

| Age. | Exposed to Risk. | Deaths. | | Adjusted $q_x$ at Central Age. | $q_x$ $H^M$. | Ratio, $\frac{\text{Standard}}{H^M}$. |
|---|---|---|---|---|---|---|
| | | Actual. | Expected by 1885 Experience. | | | |
| 20-24 | 2150·14 | 22 | 21 80 | ·01025 | ·0068 | 1·51 |
| 25-29 | 7662·09 | 74 | 74·11 | ·00966 | ·0069 | 1·40 |
| 30-34 | 10124·75 | 81 | 98·85 | ·00972 | ·0081 | 1·20 |
| 35-39 | 10089·25 | 139 | 110·47 | ·01090 | ·0095 | 1·15 |
| 40-44 | 8553·76 | 94 | 117·18 | ·01371 | ·0107 | 1·28 |
| 45-49 | 6449·51 | 123 | 113·40 | ·01765 | ·0137 | 1·29 |
| 50-54 | 4063·12 | 94 | 94·41 | ·02346 | ·0175 | 1·34 |
| 55-59 | 1724·66 | 56 | 54·21 | ·03200 | ·0240 | 1·34 |
| Total, | 50826·38 | 683 | 684·33 | — | ... | ... |

The mortality experience of Ceylon has throughout, as might be expected, shown a death-rate lower than that experienced by the Standard Company in India during the same period of observation.

Tables of net premiums have also been calculated from some of the above data; but as it would not be safe, without considerable modification, to base office premiums upon them, owing to the paucity of numbers and the somewhat select character of the lives forming the subject of the observations, I do not think it desirable to give them to the public at the present time, lest it should convey the impression that I consider the rates, as they stand, to be a fair and safe foundation on which to base office premiums to be charged in India and Ceylon.

The mortality experience of the Standard Office has also been ascertained for other places in the East, such as China, Japan, the Straits Settlements, and Mauritius, but as the lives at risk and the deaths are few, I do not trouble you with the figures. I will only add that all these countries show a tendency to improvement in healthiness of recent years, though in Mauritius and the Straits Settlements the experience is considerably less favourable than in India.

In the *West Indies*, which includes most of the larger islands and also the mainland colony of Demerara, the death-rate has also improved of recent years, according to the Standard experience, as the following tables indicate; but it must be understood that the islands and mainland colonies and possessions still vary considerably from one another in respect of healthiness and rate of mortality, and did so even more in years past before attention had begun to be paid to sanitation and the stamping out of yellow and other fevers.

The following tables exhibit the Standard Office experience.

TABLE 9.—*West Indian Mortality Experience*, 1846-1900.

| Age. | Exposed to Risk. | Expected Deaths. | | Actual Deaths. |
|---|---|---|---|---|
| | | English No. 1, Males. | Standard West Indian Experience to 1885. | |
| 19 | 101·67 | ·77 | 1·37 | ... |
| 20-29 | 4[illegible]55·08 | 43·29 | 52·86 | 46 |
| 30-39 | 13041·08 | 139·59 | 198·81 | 178 |
| 40-49 | 11643·75 | 171·95 | 304 84 | 275 |
| 50-59 | 6796·17 | 140·96 | 274·25 | 234 |
| 60-69 | 2738·17 | 116·32 | 169·72 | 170 |
| 70-79 | 718·08 | 62·44 | 82·18 | 72 |
| 80 and upwards | 75·08 | 12 39 | 15·51 | 18 |
| Total, | 38769·08 | 687·61 | 1099·54 | 993 |
| Actual deaths to 100 expected | | 144·41 | 90·31 | ... |

TABLE 10.—*West Indian Mortality Experience*, 1895-1900.

| Age. | Exposed to Risk. | Expected Deaths. | | Actual Deaths. |
|---|---|---|---|---|
| | | English No. 1, Males. | Standard West Indian Experience to 1885. | |
| 19 | 19·83 | ·15 | ·27 | ... |
| 20-29 | 1096·42 | 10·09 | 12·46 | 12 |
| 30-39 | 2203·42 | 25·41 | 36·15 | 24 |
| 40-49 | 1895·25 | 27·85 | 49·46 | 38 |
| 50-59 | 1099·83 | 22·82 | 44·83 | 26 |
| 60-69 | 416·67 | 17·35 | 25·59 | 25 |
| 70-79 | 196·08 | 17·48 | 23·11 | 20 |
| 80 and upwards | 16·75 | 2·70 | 3·39 | 8 |
| Total, | 6948·75 | 123·85 | 195·26 | 153 |
| Actual deaths to 100 expected | | 123·54 | 78·35 | ... |

The following table contrasts the death-rate amongst coloured persons with that of the whole lives assured:—

TABLE 11.—*West Indies—Mortality Experience amongst Coloured Persons only*, 1846-1885 (*included in Table* 9).

| Age. | Exposed to Risk. | Expected Deaths. | | Actual Deaths. |
|---|---|---|---|---|
| | | English No. 1, Males. | Standard West Indian Experience, 1846-1885. | |
| 14-19 | 8·92 | ·06 | ·12 | ... |
| 20-29 | 679·50 | 6·40 | 7·69 | 4 |
| 30-39 | 1952·50 | 22·57 | 32·31 | 34 |
| 40-49 | 2052·17 | 30·13 | 53·66 | 47 |
| 50-59 | 1014·33 | 20·51 | 40·40 | 49 |
| 60-69 | 219·92 | 8·81 | 13·21 | 16 |
| 70-84 | 36·08 | 3·49 | 4·58 | 5 |
| Total, | 5963·42 | 91·97 | 151·97 | 155 |
| Actual deaths to 100 expected | | 168·53 | 101·99 | ... |

## DISCUSSION.

MR. D. DEUCHAR (Vice-President), who occupied the chair, said—We are much indebted to Mr. Thomson for his valuable paper. If he had done nothing more than make the suggestion for a general investigation of Indian risks under the supervision of the Faculty and the Institute, we should still be indebted to him. I think the suggestion is a very valuable one. I have not the knowledge of this subject which some others have, but such a paper, coming as it does from the manager of the *Standard*, must have very great weight. As usual, the paper has been specially considered by two of our members, and I now call upon them to open the discussion.

Mr. T. WALLACE.—The remarks which I have to make will be confined entirely to the mortality in India, but I would preface them by giving expression to a feeling of great gratification, which I believe will be shared by every one present, that Mr. Thomson has seen his way to submit to us and to the profession the valuable data contained in his paper. He has supplied us with reliable material of a kind which we have long desiderated, and without which we have been groping, not perhaps in absolute darkness, but in a very uncertain and insufficient light. By means of these data we are now able to determine for assurance purposes, with a degree of accuracy hitherto unattainable, the value of European and Native lives in India. We have had in the past Indian mortality tables, prepared by eminent actuaries; but from the nature of the experience on which they were based—for the most part the records of officials whose occupation and environment differ materially from those of insured lives—there were always grave doubts as to their suitability for assurance purposes. A comparison of the *Standard* experience with any of those will show that these doubts were not unfounded. Apart altogether from climate, life assurance in India is conducted under conditions different from those that prevail at home. In many cases it is not easy to get reliable information as to health and habits. The medical examination may not be so thorough or so trustworthy as it should be; a report by the private medical attendant may not be procurable when required, on account of the medical practitioner's having removed to another district or returned home; and in the case of natives, evidence of age is frequently of a very indefinite and unsatisfactory character; while the opportunities for impersonation and other forms of fraud are much in evidence. In India self-selection, as exercised by the assured, is more pronounced; and selection, as exercised by the office, less effective than in this country. All these considerations point to the importance of having the actual experience of offices to guide us, and to the value of the data now submitted. Not the least valuable portion of the statistics before us is that relating to the mortality—which is shown to be heavy—that prevails after permanent return to temperate zones. The heavy mortality occurs in the age-groups 30-39 and 40-49. At ages 50-59 the death-rate is nearly the same as by the $H^M$ Table; while at the younger and older ages, 19-29 and 60-80 (where, however, the numbers exposed to risk are comparatively small), the mortality is lighter.

We might have expected to see the heavy death-rate continued at the older ages; and had the observations at these ages been more extensive, it probably would have been so continued. A population table like the English Life Table, which includes in its observations persons who have resided in tropical climates, includes this extra mortality. But when a valuation is made by such a table as the $H^M$, the *Standard* experience clearly points to the necessity of setting aside a suitable additional reserve for such cases. It has been stated that in India selection, as exercised by the office, is less effective than it is in this country. Mr. Chatham, in a paper read before the International Congress of Actuaries two years ago, pointed this out, and gave some statistics bearing on the subject. At present we can only form a very rough idea of the extent to which selection affects the rate of mortality. In the event of a joint investigation being made, this is a matter that should, and no doubt would, receive due attention. I had occasion recently to investigate the mortality experience of a comparatively small group of assured lives in India, of which I have the result before me. A large proportion of the business was effected within recent years, and the results show a remarkable agreement with the *Standard* experience 1895-1900, up to age 60, beyond which the data become so attenuated as to be of little importance. The rates of mortality per cent are :—

| Age. | Europeans and Natives together. | Natives only. |
|---|---|---|
| 25 | ·98 | ·86 |
| 35 | 1·11 | ·73 |
| 45 | 1·37 | 1·26 |
| 55 | 2·53 | 3·66 |

Compared with the *Standard*, the mortality of Europeans and Natives together is only slightly higher at the younger ages, and slightly lower at the older ages. For Natives only, however, the agreement is not so close; and in that case my results show a somewhat lower rate of mortality up to age 49, and a much higher rate at older ages. Comparing the death-rates of Natives and Europeans, the former is lower up to age 50, but very much higher afterwards. This investigation was made for a special purpose which did not necessitate the elimination of periodical residence in this country, and therefore for Europeans the observations extend over the whole period of the policy. Had the periods of residence in this country been eliminated, the results would not have been materially affected; and I am confirmed in this opinion by a comparison of Tables 1 and 6 of the *Standard* experience. I may say that the total observations embraced 11,883 years of life and 217 deaths; and for Natives alone 3422 years of life and 68 deaths. These are not of sufficient magnitude to give much weight to the results, but they tend to strengthen Mr. Thomson's plea for an early investigation into the combined experience of offices doing business in India. He has given several good reasons why the time is opportune for making such an investigation. I venture to suggest another. The amount of life business yearly effected in India has been rapidly increasing, and it is very generally believed that it will continue to grow, especially among natives. Every year adds to the number of offices that enter the field there. The competition is becoming almost as keen as it is at home; and some offices, in their eagerness for business, have reduced the rates of premium to a point which some of us consider to be totally inadequate. The sooner they have the searching light of experience shed upon them the better.

Mr. W. Smith Nicol.—At first sight Mr. Thomson's plea for a combined tabulation of foreign risks seems most reasonable, but on second thoughts I

am a little dubious. The interest in the matter is not general, but is confined to particular companies. Presumably all offices would be asked to share the cost; but why should offices, for example, which do a purely home business, be asked to concern themselves with foreign experience? The value of the work done would very much depend on the lines on which the investigation proceeded. Take, for instance, the separating of Europeans from Natives. Unless the proposal form contains a specific question as to the race of the applicant, the names are a very unsafe guide. A proposer may have the very Scottish name of Colin Roderick Dhu Lochiel Campbell, and yet be a native. An all-important point is the care exercised in selecting the business. Mr. Thomson attributes the *Standard's* favourable experience largely to skilled European selection, and I am quite sure he is right. His office has been doing business in India for nearly sixty years, and has steadily been gaining experience in selection which must be invaluable to it; and I venture to think that a combined experience such as he suggests would not really be much of a help to any one office. My argument depends on the weight to be given to the value of selection, and in my opinion it is impossible to exaggerate this factor. If I am right, I think it follows that, in dealing with special risks such as those for foreign residence, the hope of framing a general scale of extra premiums is doomed to disappointment. If one office finds that it can, owing to its special knowledge of the field, do a paying business at one-half or two-thirds of the extra charged by another company, I cannot see why it should be asked to charge more, nor can I see why the second office should be expected to charge less. Tariff is not everything. Something more than the mere knowledge of the rates of premium is wanted—in particular, the kind of business to be avoided. Further, what has followed in the past on combined tabulation of life companies' experience in special classes, does not make me more anxious to have a fresh one. Take, for instance, liquor-sellers. How many offices charge the full extras shown, by the joint experience taken out a few years ago, to be called for? Assuming, however, that I am wrong, and that a joint investigation is expedient, let me mention some of the existing data. There is first and foremost the *Standard* experience itself. Then there is the rapidly-increasing number of British companies which have done and are doing business in the East. In regard to native lives, there are two native companies in Bombay. Then, also, a considerable mass of data will ere long be in the possession of the American and Canadian companies. Mr. Thomson's remarks afford a brief but, as might have been expected, a most comprehensive commentary on Indian business, and one which is full of value and suggestiveness. The very brevity and compactness of his statements will, however, I fear, be only fully realised after practical experience of the distinctive features of the business. In the administration of an Indian business, questions arise of which we have no counterpart at home. First, there is the preservation of the continuity of the company's representation in an unhealthy climate, five or six thousand miles from the base. Is the representative to have an establishment of his own? or is he to work under a firm? In the former case a good man costs money, and in the latter case the risk will probably take the form of undesirable business. Having got a man who possesses all the qualifications for obtaining and scrutinising business, and dealing on his own responsibility with all questions that the various departments at head office have distributed among them, the company never knows when the secretary may be invalided home. A few years ago, the question of exchange was very much in evidence, and in several instances companies had to write off large sums in consequence of the fall in the value of the rupee. Owing to the closing of the mints and the making of the sovereign legal tender in India

for fifteen rupees, exchange has for some time been practically constant, but there are those who seriously question the permanency of this condition. It may be asked—What has this to do with mortality? Not very much, but it has to do with the fixing of the premium and the keeping of the reserves, unless the rupee assets and liabilities are always made to correspond. This may seem a simple thing to achieve, but in reality the range of rupee investments such as most companies would consider suitable, is very limited indeed. Finally, there are legal questions, but these and many other details can best be learned by actual experience. I would only say, in conclusion, that the doing of Indian life business is a very complex affair. It abounds in possibilities, some of which have already emerged. The weight of these can be fairly estimated; but there may be entirely fresh ones in the future, and my decided conviction is that an Indian business of any magnitude should only be done at rates of premium which leave a considerable margin for contingencies.

Mr. J. Chatham.—Mr. Thomson has enumerated the causes of the improvement in the mortality in tropical countries—such as better sanitation, and destruction of the insects carrying infection. The results of this improvement are shown in Tables 1 and 2, where the mortality in the years 1895-1900 is shown to be much better than for 1846-1900. In the earlier period the actual deaths exceed the expected by the English Life Table in every case between ages 20 and 69, and the excess over all is about 20 per cent. In 1895-1900 the actual deaths between ages 20 and 69 are, with one exception, less than the expected, and over all are slightly better than that table. In Tables 3, 4, and 5, Mr. Thomson gives particulars for Natives, Eurasians, and Country-born Europeans, and says that, as compared with the period to 1885, a very considerable improvement has taken place; but he does not give the particulars of the earlier tables with which he makes the comparison. I find that an investigation made by the *Oriental* Office, for the years 1874-1891, corroborates his statistics so far as Natives are concerned, there being 566 deaths against 469 expected by the $H^M$ Table; but in the case of Eurasians it is rather different, 66 deaths having occurred against 76 expected by that table. I observe that Mr. Thomson has no ambition to fix a complete scale of extra premiums; but he has been good enough to indicate what the extra premium for India should be. It is quite a common thing nowadays to limit the payment of extra premiums to a specified number of years; and I think there are more than two offices—in fact quite a number—where the period is ten years. On this basis, I estimate that the extra premium would be about 14s. per cent at age 20, and about 24s. at 55. Mr. Thomson has given several tables on West Indian mortality. I have had something to do with a West Indian insurance company which has made two investigations into its mortality; and in the second period there is a considerable improvement over the first. I would draw attention to the incidence of the mortality in Table 10, which relates to the period 1895-1900. Down to age 59 the actual deaths are less than the expected by the West Indian experience to 1885; at ages 60-69 they are practically the same; and at higher ages they are greater. That is exactly what I found in the experience I have referred to. The improvement took place up to age 50, but after that the mortality was higher than before. It has been suggested that, as only a small number of offices do business in India, the proposed investigation would not be taken up generally by the offices; but I feel that the investigation proposed should not be limited to offices doing Indian business. What we want to know is the extra premium to charge to persons proceeding from this country to reside in India, and to enable us to ascertain it correctly we must know the mortality among such persons after their return to this

country. I think the proposed investigation would be more likely to be taken up by the Faculty and the Institute, and the results more important, if all the offices were asked to join in it and to combine their experience, although there might be included in it an investigation into purely Indian mortality.

Mr. J. J. McLauchlan.—Mr. Thomson refers to the fact that the rate of mortality at home is swollen by the return of invalids to this country. A short time ago I had occasion to investigate the mortality in the navy, as shown by the official reports on the health of the navy, which give not only the numbers of those who die but the number discharged as invalided. I found a very interesting relation between these two sets of statistics. For instance, in the East Indian station (which includes the East Coast of Africa as far south as Madagascar, and also Burmah) I found that the extra mortality from 1886 to 1890 was 2·7 per 1000, and from 1891 to 1895 4·1 per 1000, showing an apparent large increase. On the other hand, the extra rate of invaliding was much higher in the former period, being 43·4 per 1000 in the first and only 16·5 in the second. The question arose, how to get an intelligible result from these figures, and I was enabled to do this by the application of a rule which I had previously investigated, namely, that, in order to get the rate of mortality for insurance purposes, we must add to the number of actual deaths on the station 10 per cent of those sent home as invalided. Applying this rule to the above example, we get an extra rate of mortality of 7·0 per 1000 in the first and 5·8 per 1000 in the second period, figures that are quite intelligible and consistent with each other. In connection with insurances on the lives of natives, a case came under my notice, where a native of Lagos wrote saying that the "Registrar-General" showed that, while the yearly rate of mortality among natives in Lagos was *only* 49 per 1000, it was 190 for Europeans, and natives ought therefore to be charged much lower premiums. Not having seen such a report as he referred to, I asked him to send it, and in due course received a very interesting report, prepared and issued in the colony, which showed that my correspondent was right as to the facts. The mortality among Europeans was extraordinarily high. Out of 150 at risk, it was 28 in 1896 and 23 in 1897. I think these local government mortality reports would be of assistance to us in coming to conclusions as to extra premiums.

Mr. D. Paulin.—I should like to add my acknowledgment to those that have already been made, to the great value of this paper to the profession, and also to the value of the suggestion that Mr. Thomson has made that the experience of the offices doing business abroad should be collated by the Faculty and the Institute. Mr. Smith Nicol has expressed a doubt whether there is sufficient interest on the part of the great body of insurance companies to incur the trouble and expense of a joint investigation into Indian mortality, as many offices pride themselves on the fact that their business is entirely a home business. I understand this phrase simply to mean that they have no branch offices and no agents in foreign countries. I am not aware that there is any office in this country which refuses, on terms, to accept lives that are going to reside in India and other tropical countries. We may thus find that every company is interested in the result of such an enquiry as Mr. Thomson suggests, to enable us to arrive scientifically at the appropriate extra premiums for residence in India and other tropical countries. Two things Mr. Thomson in this paper has made clear to us—(1) that in recent years there has been a very great improvement in the mortality in India and the West Indies, and (2) that, notwithstanding that betterment, a very substantial extra is still required for such risks, so that no injustice may be done to the home

policyholders. The excessive competition of recent years has led some offices to limit their extra premiums to a very few years, and one London institution takes such cases up to £1000, without any extra at all. Mr. Thomson's paper makes it clear that such procedure is giving an unjust preference to one set of policyholders, to the detriment of the others. Among the causes which Mr. Thomson records as having contributed to the greater longevity of policyholders, we are glad to find a place given to the more frequent presence of ladies in the country. I have no doubt this factor has contributed to two of the others, namely, sounder modes of life and the more moderate use of stimulants.

The CHAIRMAN.—Gentlemen, I think it will be your unanimous wish that I should, in your name, convey the thanks of the Faculty to Mr. Thomson for his excellent paper. I think I may say that the Council will consider carefully the suggestion made by Mr. Thomson for a joint investigation, as well as the difficulties that have been raised.

Mr. THOMSON, in reply, said—I have to thank you for the kind way in which my paper has been received, and for the suggestive remarks made by the several gentlemen who have spoken upon it. Mr. Smith Nicol has put before you all that can be said against my proposal; but I think he has taken too pessimistic a view. As he suggests, a large margin is necessary for fluctuations. I do not think there would be any insuperable difficulties in connection with joint action. Every office might be asked to contribute its experience, and not merely those doing business in India locally. The more experience we can obtain, the better; and if other offices are willing and able, by all means they should be asked to contribute. I have only brought up the subject, as I have told you, in order to ventilate it before the Faculty and the Institute, and I hope that they will take it up seriously, discussing the objections and advantages. I think that the medical selection in India is quite as good as in this country. Many of those who go out are the very pick of the population, undergoing medical examination before they get their appointments, and the soldiers who go are also more or less select lives. This elimination of the unfit will counterbalance any laxity in the medical selection of lives in India, though I cannot say I have suffered from anything of the sort. As regards the record of past health and habits of proposers, it is very easy to get at the true state of affairs—at least in the case of Europeans—by enquiring in the locality where they reside.

# *Law Reports.*

THE Council of the Faculty, believing that it will be useful to place on record decisions of the Scottish Law Courts in cases connected with actuarial questions or the business of life assurance, have arranged to print occasionally in the *Transactions* reports of such cases. These reports will be furnished by Mr. William Harvey, LL.B., Advocate, whose valuable "Lectures on Insurance Law", delivered to the late Actuarial Society of Edinburgh, appeared in vol. iii of that Society's *Transactions* (pp. 261-338). It will be of much assistance if any of our readers will inform us of proceedings on matters of the above kind in any of the Scottish Courts.

## INSURABLE INTEREST.

*Simcock* v. *Scottish Imperial Insurance Company.*

(Court of Session, 20 August 1902.)

This was an action for recovery of a sum of £250, the amount contained in a policy of life insurance effected by Simcock, a pork butcher, upon the life of his foreman, Hough, with the Scottish Imperial Insurance Company.

The Company pleaded in defence that Simcock had no insurable interest in his foreman's life, or, in any event, that if he had such an interest, it was fully covered by a prior policy, also for the sum of £250, upon which it was admitted Simcock had, prior to the present action, recovered the amount insured.

The circumstances under which the question arose are fully explained in the following judgment of Lord Pearson, in which his Lordship sustained the alternative defence stated for the Company:—

LORD PEARSON—I proceed to consider the original defence as to insurable interest, which comes first in order. This is based on the Statute 14 Geo. III, cap. 48, being an Act "for regulating insurances upon lives, and for prohibiting all such insurances except in cases where the persons insuring shall have an interest in the life or death of the persons insured". The statute proceeds on the preamble that it has been found by experience

that the making insurances on lives or other events wherein the assured shall have no interest, has introduced a mischievous kind of gaming. Section 1 enacts that from and after the passing of the Act "no insurance shall be made by any person or persons, bodies politic or corporate, on the life or lives of any person or persons, or on any other event or events whatsoever, wherein the person or persons for whose use, benefit, or on whose account such policy or policies shall be made, shall have no interest, or by way of gaming or wagering; and that every assurance contrary to the true intent and meaning hereof shall be null and void to all intents and purposes whatsoever". And by section 3 it is provided "that in all cases where the insured hath interest in such life or lives, event or events, no greater sum shall be recovered or received from the insurer or insurers than the amount or value of the interest of the insured in such life or lives, or other event or events".

The defenders contend that section 1 applies to render this insurance null and void, in respect that at the date of the policy the pursuer had no interest in Hough's life within the meaning of that section. They contend, *separatim*, that any insurable interest he had, has been far more than satisfied by the recovery of a sum of £250 under a similar policy from the Provident Life Office, and that section 3 applies to disentitle him to recover more. The facts on which these questions depend are as follows.

The pursuer and Hough had known one another for twenty-three years. Both belonged to the district of Altrincham, near Manchester; and both were in the employment of a pork-butcher there for seven years, learning the trade. Two years later, the pursuer began business on his own account in Manchester; and about three years afterwards, Hough entered his employment, and lodged and slept with the pursuer before either of them married. He served at first as a workman, at 22s. a week; and, as time went on, he became foreman, at a standing wage of 36s. 6d., and occasionally 40s. a week, with allowances. In winter, which is the busy time, the pursuer employs seven men. The work, including the foreman's work, is hard; and Hough, as foreman, took his part in the work of killing pigs, carrying and scalding the carcases, and cutting them up. The hours are long, from six in the morning until nine or ten at night.

Hough was a clever workman and a good foreman, and one whom it would be and was difficult to replace. He knew the pursuer's recipes for the seasoning and the making-up of the various products, such as pork-pies and sausages; and he had some good recipes of his own, of which he made common stock with his employer. The pursuer was in the practice of keeping his staff together by allowing them a wage during illness or holidays, and Hough shared in this benefit. But he was a weekly servant, and could leave or be dismissed on one week's notice, beyond which there was no contract between them. There was, however, an understanding between them that they would not part, and that in the event of the pursuer giving up business he would make Hough an allowance for life, but no sum was fixed.

It appears that in these circumstances Mr. Gresty, an insurance agent and broker in Manchester, suggested to the pursuer to insure his foreman's

life, the insurable interest being estimated at £500, or about one year's profit of the pursuer's business. Application was accordingly made by the pursuer in November 1896 to the Provident Life Office for an insurance on Hough's life for this sum. After the usual enquiries, the proposal was accepted for £250, for which a policy was issued on 21 November.

A year later, on 30 November 1897, a proposal was made to the defenders' company for a similar insurance of £250, and as the result this policy now sued on was issued on 2 February 1898.

On these facts, the question is whether the pursuer had at the date of this policy an interest in Hough's life, within the meaning of section 1 of the statute above quoted; and if so, what was the amount or value of that interest, in order to ascertain whether the provision of section 3 applies to the case. The statute is a British statute; and the policy of it has been reaffirmed by the legislature so recently as 1866, when it was extended simpliciter to Ireland (29 & 30 Vic., cap. 42). It has not, I believe, been the subject of decision in Scotland, except in the case of *Turnbull* after referred to. But there are some points upon which the English decisions are instructive—the cases to which I was referred being *Halford* v. *Kymer* (1830, 10 Barnewall & Cr. 724); *Hebdon* v. *West* (1863, 3 Best & Smith 579); *Branford* v. *Saunders* (1877, 25 Weekly Repr. 650); *Barnes* (1892, 1 Q.B. 864). The interest must be a pecuniary interest—a view adopted by Professor Bell (*Principles of the Law of Scotland*, § 520). It must be in a reasonable sense capable of valuation in money, seeing that by section 3 the "amount or value" of the interest is assigned as the limit of the sum which can be recovered. Further, the 3rd section provides that no greater sum than such amount or value shall be "recovered or received from the insurer or insurers"; which has been held to mean that if the policyholder has received the full amount from one insurer, he is precluded by the terms of this section from recovering or receiving any more from any other insurer.

The leading case is *Hebdon* v. *West*. Hebdon had been for twenty years a clerk in a bank, and had received advances from the bank to the amount of £4700; and the managing partner had assured him that he would not be called on to pay so long as he (the managing partner) lived. Hebdon's salary having been increased from £200 to £600 a year under a contract for seven years, he insured the managing partner's life for £5000. A year later, his debt to the bank having meanwhile increased to £6000, he insured the managing partner's life in another office for an additional sum of £2500. The managing partner having died and the bank having stopped payment, Hebdon recovered the £5000 on the first policy, and handed it over towards payment of his indebtedness to the bank. He then sued for £2500 on the second policy. His claim was disallowed, on the ground (1) that the managing partner's promise that Hebdon would not be called on to pay up the debt did not create a pecuniary or appreciable interest in his life; (2) that while the contract for seven years' employment at a salary of £600 created an insurable interest to the extent of the period unexpired at the date of the policy, yet the £5000 recovered under the first policy being more than the "amount or value" of that interest,

he was precluded from recovering any additional sum under the second policy.

In the present case, the only engagement was from week to week; and the understanding that the parties should stick together, and that the pursuer would provide for Hough, was obviously incapable of valuation. It is true that the statute cannot be carried out quite logically; for it is settled that the point of time to be regarded is the date of the insurance, irrespective of what may happen afterwards. Thus a creditor may insure his debtor's life; but he insures against the debtor's death at any time, and not merely against his death before payment; the theory being, I presume, that his chance of payment depends upon the debtor remaining alive. A master may insure a valuable servant's life up to the value of his present interest in it; and the validity of the policy will not be affected by the subsequent departure or dismissal of the servant, or even (it would appear) by the master's death. But the value of the interest as at the date of the policy must be calculated with some reference to the legal relation subsisting between the parties. A contract of service affords a fairly sure basis of valuation. If there is no contract, the whole matter is thrown loose. It is impossible to appreciate in money the "amount or value" of an interest whose endurance rests on sentiment or good feeling or mutual advantage. There might be an assessment of it as a jury question; and in the present case I have little doubt that the pursuer could have honestly said he would rather lose £250 (or possibly even £500) than Hough's services. But the Courts have never undertaken such a valuation, and it does not appear to me to satisfy the requirements of the statute, having regard to the enactment itself and to the decisions which have followed upon it.

In my opinion, therefore, the utmost limit of the insurable interest here was the value to the pursuer of one week's employment; and as the pursuer had a prior policy with the Provident Life Office, upon which he has recovered £250 in respect of Hough's death, the action fails on the ground (1) that he had no insurable interest in the life which was not already covered at the date of the policy sued on; and (2) that in any view the value of any possible interest has been recovered from the other office.

The statute came up for consideration in the Scotch case of *Turnbull & Co.*, 27 July 1896 (34 S.L.R. 146). But the present questions did not arise for decision; for in that case the Leith merchants, who insured the life of their foreign agent through whom they carried on a lucrative business, obtained a policy which bore on the face of it—"no further proof of their interest will be required when this policy becomes a claim". It was in these circumstances "not disputed that if the pursuers' connection with their agent had continued to be the same as it was when the proposal was made, the defenders could not and would not have disputed the pursuers' insurable interest or the amount of it; and I think" (adds Lord Kincairney) "it is not for the Court to consider whether the interest would not have been such as to satisfy the requirements of the Act". The decision turned on a different question, namely, whether an alleged change of circumstances (to wit, the resignation of the agent on three

months' notice, given after the date of the proposal but before that of the policy) let in a plea that there was no interest at the date of the policy, or that the interest was materially different from what it had been explained to be. And "it was not contended that the amount of the sum payable under the policy could be effected by the change of circumstances, nor is there any plea to that effect".

Counsel for Simcock—Wilson, K.C., and W. Thomson. Counsel for the Insurance Company—Henry Johnston, K.C., and Deas.

*Note by Mr. Harvey.*—In some of the states of America the theory which prohibits insurance without interest in the life insured is carried out logically, and the assignee of an "own-life" policy can only recover to the extent of his interest; but this extension of the principle of the Act of George III has never prevailed in this country.

## MEMBERSHIP IN MUTUAL INSURANCE COMPANY.

[It will be noticed that the following case relates to fire insurance. As, however, it is theoretically possible that a question of the kind might arise with a life assurance office similarly constituted, it has been thought desirable to print the report here, although it is believed that no existing life assurance company—mutual or proprietary—has a constitution resembling that of the company named.]

*The Star Fire and Burglary Insurance Company Limited* v. *C. Davidson and Sons Limited.*

(Court of Session, 21 October 1902.)

This was a case on appeal from the Sheriff Court at Aberdeen to the Second Division of the Court of Session. The question raised was whether an applicant for insurance against fire who had accepted an offer by a company (whose name contained no indication that it was a mutual company) to insure premises, was bound to accept, as in implement of the agreement, a policy containing a condition that the insured should become a member of the company, a condition of which he had no notice in the offer by the company to insure.

The Star Fire and Burglary Insurance Company Limited, the company in question, had by letters dated 1 and 6 December 1900 offered to insure the Mugiemoss Paperworks, Aberdeenshire, belonging to C. Davidson and Sons Limited, against fire to the extent of £5000, at the same rate of premium as the Messrs.

Davidson were paying to other offices. On 10 December the Messrs. Davidson, through an agent, wrote accepting this offer.

The Insurance Company on 22 December forwarded a fire policy in implement of this agreement, asking payment of the first premium. The policy proceeded on the narrative that the Messrs. Davidson had, "by a proposal and declaration dated the 10th day of December in the year of our Lord 1900, and which is the basis of this contract, applied to be admitted a member of the Star Fire and Burglary Insurance Company Limited, by effecting a policy for insuring against loss or damage by fire the property" in question, and the said company had accepted the proposal of the member.

The policy incorporated the memorandum and articles of association, by which every member was liable to contribute to the assets of the company in the event of its being wound up during the time he was a member or within one year after he had ceased to be a member. As already stated, the Insurance Company's offer to accept the risk contained no reference to the condition that policyholders were to be members of the company. The policy was in the ordinary form issued by the company.

The Messrs. Davidson refused to accept the policy or to pay the premium, and thereafter the Insurance Company brought this action against them to recover the premium. Messrs. Davidson lodged defences, and pleaded that they were not bound to accept the policy, as they had not agreed to become members of the Company.

On 2 August 1901 the Sheriff-substitute (Duncan Robertson), holding that the defence was irrelevant, found that the Messrs. Davidson were liable for the premium. The Sheriff (Crawford) affirmed this judgment against the Messrs. Davidson, on the ground that it was their duty to inform themselves as to the constitution of the Company when entering into a contract with it. Both the Sheriff and Sheriff-substitute relied on the case of *The General Accident Insurance Corporation* v. *Cronk*, 31 January 1901, 17 T.L.R., 233.

The Second Division recalled these judgments, and found that the Messrs. Davidson were not bound to accept the policy or pay the premium. Their Lordships delivered the following opinions:—

LORD JUSTICE-CLERK—Were it not for the decisions of the two Sheriffs I should have thought this a very clear case. The pursuers offered to accept a fire risk for £5000 over buildings belonging to the defenders, and the defenders intimated through an agent that they were willing to give them the insurance. But on receiving from the pursuers a policy embodying the arrangement, the defenders found to their astonishment that the policy which they had asked for was made out in their favour on condition of

their becoming members of a limited liability company. They were under no obligation to become members of a limited liability company, and were in my opinion entitled to declare the whole matter to be off. There was here no *consensus in idem placitum*, and I think the judgments of the two Sheriffs should be recalled and the defence sustained.

LORD YOUNG—I am of the same opinion. There is no contract here except what is to be derived from the letters. If it had been the case that the defenders knew that the pursuers' company was a mutual company I think the contract would have been established; but the case is presented by both Sheriffs on the footing that the defenders were under a complete misapprehension on that subject, and did not understand that it was a mutual company. The moment they discovered that, they returned the policy, and that without any detriment to the pursuers. That being so, I hold, on the authority of many cases, that they were entitled to be off and not bound to go on with the contract on that footing.

LORD TRAYNER—I am of the same opinion. The defenders agreed to enter into a contract of insurance with the pursuers—nothing more. But the policy which the pursuers tendered contained a contract of insurance *plus* a contract of copartnery, which was a thing the defenders never contemplated and never agreed to. I think the defenders cannot be made partners of the pursuers' company against their wish, or without having given their deliberate consent to such a contract. Never having consented to become the pursuers' partner, they were entitled to refuse the policy tendered to them, and are consequently entitled now to be assoilzied from the pursuers' claim.

LORD MONCREIFF was absent.

Counsel for the Insurance Company—Salvesen, K.C., and Graham Stewart. Counsel for Messrs. Davidson—Campbell, K.C., and Macfarlane.

# *Joint Mortality Investigation of the Institute of Actuaries and the Faculty of Actuaries.*

*Official Symbols adopted by Joint Committee,* 12 *June* 1903.

A desire having been expressed that, in order to assist in securing uniformity on the part of future writers making use of the experience, an authorised set of symbols should be adopted to indicate the different sections of the Select and Aggregate

experience dealt with in the Investigation, the following symbols were approved by the Joint Committee at their meeting on 12 June 1903. These appear on page v of the Select Assurance Tables recently published.

*THE BRITISH LIFE OFFICES TABLES*, 1893.

FULL AGGREGATE TABLES—Males :—

| | |
|---|---|
| Whole-Life Participating Assurances, | $O^{M}$ |
| Whole-Life Non-Participating Assurances, | $O^{NM}$ |
| Endowment Assurances, | $O^{EM}$ |
| Assurances with Limited Payments, | $O^{LM}$ |
| Assurances with Increasing Scale of Premiums, | $O^{IM}$ |
| Temporary Assurances, | $O^{TM}$ |
| Contingent Assurances, | $O^{CM}$ |
| Joint Assurances, | $O^{JM}$ |

"TRUNCATED" AGGREGATE TABLES—Males :—

| | |
|---|---|
| Whole-Life Participating Assurances (excluding the first five years of Assurance), | $O^{M(5)}$ |
| &c., &c., &c., | &c. |

SELECT TABLES—Males :—

| | |
|---|---|
| Whole-Life Participating Assurances, | $O^{[M]}$ |
| &c., &c., &c., | &c. |

*THE BRITISH OFFICES LIFE ANNUITY TABLES*, 1893.

| | |
|---|---|
| Male Annuitants' Tables (Aggregate), | $O^{am}$ |
| Male Annuitants' Tables (Select), | $O^{[am]}$ |

*Female Lives.*

F instead of M, and *f* instead of *m* throughout.

ERRATA.

*British Offices* $O^{M}$ *Life Tables*, 1863-1893: Page 63—Temporary Annuities. 3 per cent, $a_{41:\overline{36}|}$. For 19·652, read 16·652.

*Unadjusted Data—Endowment Assurances and Minor Classes of Assurance*: Introduction, page ix. Joint Life Assurances—Female Lives. Years of Risk "After Ten Years". For 41,090, read 41,093.

# NOTICE.

Binding Cases for Vol. I. have been prepared, and will be supplied by Messrs. T. and A. Constable, OR CHARLES & EDWIN LAYTON. 11 Thistle Street, Edinburgh, at the price of 1s. 6d., or will be sent post-free on receipt of 1s. 8d.

Members of the Faculty and others can have their copies of the volume bound in the official cover by sending them to Messrs. Constable with OR CHARLES & EDWIN LAYTON. a remittance of 2s. 6d. if resident in Edinburgh, or 2s. 10d. if elsewhere in the United Kingdom, including return postage.

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[illegible]

[illegible]

[illegible]

# TRANSACTIONS

OF THE

# FACULTY OF ACTUARIES.

THE ROYAL CHARTER incorporating the Faculty having been long out of print, and some enquiries having recently been made for copies, it has been thought desirable to reprint it in the *Transactions*. The opportunity has been taken to include the present Rules (as fixed at a Special Meeting of the Faculty held on 28 March 1904) and Bye-Laws, together with the last Annual Report by the Council.

## ROYAL CHARTER OF INCORPORATION.

Victoria, by the Grace of God of the United Kingdom of Great Britain and Ireland Queen, Defender of the Faith; To all to whom these presents shall come, Greeting: Whereas, We considering that an humble petition has been presented to Us by Donald Lindsay, William Thomas Thomson, Samuel Raleigh, William Spens, John M'Gregor M'Candlish, David Maclagan, George Ramsay, David Smith, William Smith, James Watson, Andrew Hugh Turnbull, William Wood, Robert Balfour, George Auldjo Esson, Charles Fox Griffith, Charles Pearson, Kenneth Mackenzie, David Chisholm, Robert Christie junior, William Dickson, Thomas Goldie Dickson, William Finlay, David Clunie Gregor, James Howden, James Maclean Macandrew, John Ogilvie,

Robert Rainie, Alexander Weir Robertson, Edward Sang, Henry Ambrose Smith, George Todd, John Watson, James Meikle, Walter Brown, George Auldjo Jamieson, David Robertson Williamson Huie, James Wilkie, David Pearson, James John Philip Anderson, Frederick Hayne Carter, Frederic James Hallows, Hunter Douglas Prain, Thomas Robertson, Alexander Stables junior, Andrew Wood Stewart, John Stott, Charles Prentice, David John Surenne, Spencer Campbell Thomson, David Deuchar, James Ritchie Macfadyen, and Alexander Henderson Whytt, all members of the Faculty of Actuaries in Scotland; **Setting forth**; That the profession of Actuary, to which the petitioners belong, has been called into existence mainly by the requirements of Life Assurance Institutions, the sound management and prosperity of which depend to a great extent on correct observations and calculations in regard to the duration of human life, and the probable rate of interest to be realised over a prolonged tract of future time: That the business of Life Assurance has been extensively and successfully carried on in Scotland, and from the time of its first introduction, in the beginning of the century, to the present time, it has continued steadily to develop, and a very large amount of the savings of the people of the United Kingdom, for the future benefit of their families, is entrusted to such Institutions: That, apart from the schemes which have been in existence for many years for the benefit of the Widows of Members of the different learned professions, there are Assurance Institutions established in Scotland whose aggregate obligations to the public now probably exceed Eighty Millions Sterling, and whose invested funds amount to upwards of Twenty Millions: That it is mainly in connection with the proper regulation of the affairs of these Institutions the skill and services of an Actuary are called into requisition; his special duties being, FIRSTLY, to take care that the Institution under his charge, or which may at any time desire his opinion and advice, is founded on a safe basis, both as regards the rate of Mortality assumed for any particular country, class, or sex, and the rate of Interest at which it may be calculated the money entrusted to the care of such Institution can be safely improved; SECONDLY, to ascertain from time to time as the Institution makes progress, by appropriate calculations, whether the rate of Mortality actually experienced, and the rate of Interest realised, are in accordance with the data assumed; for the performance of these duties it is evident that not only a sound knowledge of mathematical principles is required, but also the practical application of financial judgment and experience: That, in addition to the requirements of Life Assurance Institutions, the profession of Actuary is largely called into requisition, in the same manner as that of Advocate or Barrister, in advising and

directing the public in regard to a great variety of pecuniary interests, frequently involving interests of large amount: That the responsibility attaching to the profession of an Actuary is consequently great, requiring the services of well-educated and trustworthy men, specially trained in Actuarial business and calculations, more especially as the subject of their professional study is one with which the general public are almost entirely unacquainted, and in which, therefore, they must be wholly dependent upon the skill and integrity of the Actuary employed: That the Actuaries of Scotland, in the year One thousand eight hundred and fifty-six, formed themselves into an Association or Faculty, with the object of uniting in one body those practising the profession, and of promoting the studies necessary thereto, and generally of furthering the objects in which, as Members of the same profession, they have a common interest: And that the petitioners humbly conceive that it would tend greatly to promote and facilitate the attainment by the Members of their profession of the qualifications which are essential to the proper performance of their important duties; and consequently, that it would conduce much to the benefit of the public, if the Petitioners who form the existing Body of Actuaries in Scotland were United into a Body Corporate and Politic, having a Common Seal, with power to make Rules and Bye-Laws for the qualification and admission of Members and otherwise: And the Petitioners therefore humbly prayed that We would be graciously pleased to grant them a Royal Charter, incorporating them, and such persons as may hereafter be duly admitted Members, into one Body Corporate and Politic, by the name, style, and title of "The Faculty of Actuaries in Scotland", with perpetual succession and a Common Seal, and power to acquire and hold Property, and to make Rules and Bye-Laws, and with such other powers, privileges, and authorities as are usually given to other Bodies Politic and Corporate, of the like nature, in such manner as to us in our Royal Wisdom shall seem proper: And Whereas such petition has been referred to the Lord Advocate of Scotland to consider thereof, and report his opinion what may properly be done therein: And We having taken the said Petition and Report thereon into Our Royal consideration, and being satisfied that the intentions of the Petitioners are laudable and deserving of encouragement: Therefore, We have constituted, erected, and incorporated, as We by Our prerogative Royal and of Our special grace, certain knowledge, and mere motion, by these presents, for Us and Our Royal Successors, Constitute, erect, and incorporate the said Donald Lindsay, William Thomas Thomson, Samuel Raleigh, William Spens, John M'Gregor M'Candlish, David Maclagan, George Ramsay, David Smith, William Smith, James Wat-

son, Andrew Hugh Turnbull, William Wood, Robert Balfour, George Auldjo Esson, Charles Fox Griffith, Charles Pearson, Kenneth Mackenzie, David Chisholm, Robert Christie junior, William Dickson, Thomas Goldie Dickson, William Finlay, David Clunie Gregor, James Howden, James Maclean Macandrew, John Ogilvie, Robert Rainie, Alexander Weir Robertson, Edward Sang, Henry Ambrose Smith, George Todd, John Watson, James Meikle, Walter Brown, George Auldjo Jamieson, David Robertson Williamson Huie, James Wilkie, David Pearson, James John Philip Anderson, Frederick Hayne Carter, Frederic James Hallows, Hunter Douglas Prain, Thomas Robertson, Alexander Stables junior, Andrew Wood Stewart, John Stott, Charles Prentice, David John Surenne, Spencer Campbell Thomson, David Deuchar, James Ritchie Macfadyen, and Alexander Henderson Whytt, and such other persons as shall hereafter be admitted as Members of the said Society, into one Body Politic and Corporate by the name of "The Faculty of Actuaries in Scotland", unto which name they shall have perpetual succession, and shall have a Common Seal, with power to alter and renew the same at discretion; and shall by the same name sue and be sued, implead and be impleaded, and answer and be answered in all Our Courts: As also, We will and ordain that the said Society shall be capable in law to take, purchase, and hold, to them and their successors, any goods, chattels, or personal property whatsoever, and shall also be capable in law to take, purchase, and hold, in the said Corporate name, such lands, buildings, and heritages as may be necessary for the purposes of the Society, with power to alienate, dispone, and dispose of all or any such lands, buildings, and heritages, goods, chattels, or personal property, and also to raise and receive such sums of money for the purposes of the Society, as they may think necessary, by annual contributions, fees on Entrants, or otherwise from the Members thereof; and to do all other acts or things incidental or appertaining to a Body Corporate: Declaring that all Deeds and other Writings whatsoever affecting heritable or moveable property shall be valid and effectual in all respects, if conceived in the name of the Corporation, and sealed with the Seal and subscribed by three Members of Council, and by the Secretary and Treasurer of the Corporation for the time being: And We also, for Ourselves and Our Royal Successors, give and grant to the Petitioners and to those persons who shall hereafter compose the said Society, full power and authority, at their ordinary General Meetings as after appointed, to constitute, make, and ordain such Bye-laws, Rules, and Regulations as to the majority of the Society present at such Meeting shall seem proper, provided that the same are not inconsistent with this Charter, or contrary to the laws of the Realm; and to alter and abrogate the said

Bye-Laws, Rules, and Regulations as to the majority of the Society present at such Meeting shall seem proper: And We will and ordain that such Bye-Laws, Rules, and Regulations shall be duly kept, observed, and obeyed: And We hereby will and ordain that a stated General Meeting of the Corporation shall be held once in every year in Edinburgh, in the month of January, or at such other time and place as the Society shall from time to time determine, with the power of adjournment as to the said General Meeting shall seem expedient, and that General Meetings of the Society may also be held at such other times and places as may from time to time be fixed, and that Special Meetings may also be held (provided the same are duly called in terms of the Bye-Laws, Rules, and Regulations for the time) at such times and places as may be necessary or expedient; and that at each Annual General Meeting, or at any adjournment thereof, the Society shall choose out of the Members thereof a Council, Secretary, and Treasurer (the same person being eligible for both of the last-mentioned offices), and such other Officers as the Society may find hereafter to be necessary and proper: And We will and ordain that the Society shall have power from time to time, and in such manner as may be fixed by the Bye-Laws, Rules, and Regulations, to constitute and appoint a Committee of Examiners for the purpose of conducting and regulating such examination of Entrants and others as the Corporation may from time to time direct, and in such manner as they may appoint, in furtherance of the objects of the Society; and that the course of Education to be pursued, and the amount of general and professional acquirements to be exacted from such Entrants, shall be such as the Corporation shall from time to time fix: And We further hereby will, grant, and declare that the present Council, Secretary, and Treasurer of the said Faculty shall hold their offices and discharge their functions respectively as Council, Secretary, and Treasurer of the Society hereby incorporated, until the stated Annual General Meeting in January One thousand eight hundred and sixty-nine, and that they and their successors in office to be chosen according to the Bye-Laws, Rules, and Regulations of the Society, shall have full power to manage, direct, order, and appoint in all matters and things touching and concerning the said Society, in terms of, and conform to, the Bye-Laws, Rules, and Regulations thereof; And We, for Ourselves and Our Royal Successors, declare that this our present Charter shall be in and by all things valid and effectual in law, according to the true intent and meaning of the same; and it shall be accepted and understood in the sense most favourable and beneficial to the said Corporation, notwithstanding any misrecital, defect, uncertainty, or imperfection in the same. In witness whereof, We have ordered the Seal appointed by the

Treaty of Union to be kept and made use of in place of the Great Seal of Scotland, to be appended hereto.

Given at Our Court at Saint James's the twenty-first day of September in the year One thousand eight hundred and sixty-eight, and in the thirty-second year of Our Reign.

Per Signaturam manu S. D. N. Reginæ supra script.

Written to the Seal and registered the fifteenth day of October 1868.

(Signed) JAMES P. HALLEY,
*Deputy Director of Chancery.*

Sealed at Edinburgh the fifteenth day of October in the year One thousand eight hundred and sixty-eight.

(Signed) COLIN MACKENZIE,
*Deputy Keeper of the Seal.*

£LXXX. Scots.

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## OBJECTS OF THE FACULTY.

The Faculty of Actuaries in Scotland was constituted in 1856 for the purpose of associating professionally those gentlemen who are engaged in the management of Life Assurance Institutions, or who are otherwise following the profession of an Actuary; and for promoting the study of the Doctrine of Probabilities, of Vital Statistics and Statistics in general, of Finance as bearing on fluctuations in the value of money, and of all cognate subjects, a knowledge of which is essential to the efficient discharge of the duties of a Life Assurance Manager or of an Actuary.

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## RULES.

### *Members.*

1. The Faculty shall consist of Fellows, Honorary Fellows, and Associates.

2. The Manager, or other the principal Officer, of any Scottish Life Assurance Institution, may be elected a Fellow of the Faculty by the Council, provided at least three-fourths of the Members of Council for the time being shall concur in such election. Every such election shall be reported to the first General Meeting of the Faculty thereafter.

3. Any Associate of the Faculty may, on application, be admitted a Fellow, subject to the approval of the Council.

His application shall be considered at a meeting of the Council of which seven days' previous notice, specifying his name, shall have been given; and if a majority of those present approve of his admission he shall be declared duly admitted, and his admission shall be reported to the first General Meeting of the Faculty thereafter.

4. Any gentleman who possesses qualifications which render his admission to the Faculty desirable, may be elected a Fellow on the nomination of the Council. Such nomination must be concurred in by at least three-fourths of the Members of Council present at a meeting of which seven days' previous notice, specifying the nominee, shall have been given. The nomination shall be submitted to the first General Meeting of the Faculty thereafter. A ballot shall then be taken, and if a majority of votes be in favour of the admission of the nominee, he shall be held duly elected.

5. Any gentleman of distinguished attainments in mathematical, statistical, or financial subjects, or who has rendered important service in promoting the objects of the Faculty, may be elected an Honorary Fellow, at any General Meeting of the Faculty, upon the recommendation of the Council. It shall also be in the power of the Council, if they see fit, to recommend to the Faculty that a Fellow who has retired from the profession after long service should be elected an Honorary Fellow.

6. Any Student of the Faculty (see Rule 9) who shall have attained the age of twenty-one, and shall have passed, to the satisfaction of the Council, three Examinations, at such intervals and on such subjects as the Council may prescribe, shall be admitted an Associate.

7. Any Student who shall have taken a Degree at one of the Universities of Great Britain or Ireland, mathematics having been one of the special subjects of examination for such Degree, being twenty-one years of age, may, in the discretion of the Council, be exempted from the first of the Examinations under Rule 6.

8. Any Student who is a Fellow (by examination) of the Institute of Actuaries may, in the discretion of the Council, be exempted from the first and second of the Examinations under Rule 6.

*Students.*

9. Any gentleman desirous of prosecuting his studies in connection with the Faculty, may be enrolled as a Student, on the recommendation of two Fellows of the Faculty, and subject to the approval of the Council.

*Office-Bearers.*

10. The Faculty, at the Ordinary General Meeting to be held each year as hereinafter provided, shall elect by ballot from

among the Fellows a President, not more than three Vice-Presidents, an Honorary Secretary, an Honorary Treasurer, and not more than fifteen other Members to act along with them as the Council of the Faculty for the ensuing year.

11. Of the ordinary Members of Council the four whose names are highest on the list as adjusted at the last Ordinary General Meeting, or such smaller number than four as, taken along with the number of vacancies by death, resignation, or other causes, will make four in all, shall each year become ineligible for re-election till after the expiry of one year, except to fill the office of President, Vice-President, Honorary Secretary, or Honorary Treasurer.

12. When a vacancy occurs among the Office-Bearers or other Members of the Council, by death, resignation, or otherwise, in the interval between one annual election and another, the Council, if they see fit, may make an interim appointment to fill such vacancy.

13. The Council shall manage and direct all matters which concern the Faculty, in terms of the Rules and subject to the control of the Faculty. They shall have power to appoint an acting Secretary, a Librarian, and such other Officers as may be required, with such remuneration, if any, as the Council may see fit, and also to appoint Committees for any purpose, and to delegate any of their powers to such Committees.

14. The Council shall have power to make and alter, from time to time, such Bye-Laws, not inconsistent with these Rules nor with the Charter of Incorporation, as may be found necessary. Any new or altered Bye-Laws shall remain in force until the next General Meeting of the Faculty, when they shall be either confirmed, amended, or annulled.

15. The Council shall prescribe the Examinations to be undergone before admission to the Faculty. They shall appoint Examiners to conduct such Examinations, and, on the report of the Examiners, shall decide on the admission or rejection of candidates.

16. The Council shall watch over all parliamentary and other proceedings affecting Life Assurance or the interests of the Profession, and shall report thereon to a General Meeting of the Faculty when such proceedings appear of sufficient importance.

17. The Honorary and acting Secretaries shall conduct the correspondence of the Faculty under the direction of the Council, and shall attend the Meetings of the Faculty and of the Council, and preserve minutes thereof.

18. The Honorary Treasurer, or the acting Treasurer if there be one, shall receive all moneys due to the Faculty. He shall keep an account of his receipts and disbursements, which account, made up to 31st December in each year, shall be audited by an

Auditor to be named by the Council, and shall be submitted to the Faculty at the next Ordinary General Meeting.

*Meetings.*

19. Ordinary and Special General Meetings of the Faculty shall consist of Fellows only—eight to be a quorum. At Meetings of the Council, five Members shall form a quorum.

20. There shall be one Ordinary General Meeting of the Faculty each year, in the month of June, upon such day as may be fixed by the Council. Intimation of such Meeting shall be given by circular addressed to each Fellow, and by advertisement in at least one Edinburgh newspaper, not less than seven days previous to the day of meeting.

21. Special General Meetings of the Faculty for the transaction of business may be summoned by the Council at such times as they may consider advisable, upon seven days' notice. Upon a requisition addressed to the Council, signed by not less than ten Fellows, and stating the subject to be brought forward, the Council shall be bound to call a Special General Meeting of the Faculty, but for consideration only of the said subject, fourteen days' notice being given.

22. The President, or in his absence one of the Vice-Presidents, shall take the chair at all Meetings of the Faculty and of the Council. In the absence of the President and of all the Vice-Presidents, the Meeting shall elect a Chairman.

23. The Chairman of any Meeting shall have a deliberative vote, and shall also have a casting vote in the event of equality.

24. Sessional Meetings of the Faculty shall be held at such times as seem fit to the Council, for the purpose of hearing papers and communications upon subjects of interest to the profession, and of engaging in discussion thereon, such papers and communications having previously received the approval of the Council. Associates and Students shall be entitled to attend the Sessional Meetings, to contribute papers (subject always to the approval of the Council as aforesaid), and to take part in the discussions; and the like privileges shall be extended to gentlemen other than Fellows, Associates, or Students, who on 31st December 1900 were members of the Actuarial Society of Edinburgh, and who shall pay a contribution of 2s. 6d. on the first day of January in each year. Provided always that at such Sessional Meetings it shall not be competent to enter upon any business of the Faculty other than as aforesaid. The Council shall have power to print and publish, at the expense and for behoof of the Faculty, any communications read at the Sessional Meetings, reports of the discussions thereon, and any other matters which

to them may seem calculated to be of use to the profession. For the purpose of carrying out the objects of this rule the Council shall also have power to associate with themselves, in such manner as they may see fit, any Members of the Faculty whose assistance may be thought likely to be of service.

*Funds.*

25. Students shall pay upon enrolment a fee of £1, 1s., and thereafter, on the first day of January in each year, a subscription of 2s. 6d. On each occasion after the first of giving in his name for examination, a Student shall pay a fee of £1, 1s.

26. Students, on becoming Associates, shall pay an entrance fee of £2, 2s., in addition to the annual subscription for the year then current.

27. Associates, on becoming Fellows, shall pay a further entrance fee of £2, 2s., in addition to the excess of the annual subscription of a Fellow over that of an Associate for the year then current.

28. Gentlemen, other than Associates, on becoming Fellows, shall pay an entrance fee of £10, 10s., in addition to the annual subscription for the year then current.

29. Fellows admitted after 1st January 1889, and Fellows admitted before that date who are resident or who have their place of business within a radius of twenty-five miles from Edinburgh, shall pay an annual subscription of £2, 2s.; Fellows admitted before 1st January 1889 whose residence and place of business are beyond the above limit, and Associates, shall pay an annual subscription of £1, 1s.; the subscription to be payable in each case on the first day of January in each year in advance.

30. Any Fellow or Associate whose subscription shall be in arrear for six months shall be applied to in writing by the Treasurer, and shall be incapacitated from exercising his rights as a Member till such subscription be paid. If it be not paid within one year from the date on which it became due, the name of the defaulting Member shall be removed from the roll; but the Council shall have power to repone him at any time, on such terms as they may see fit, if they consider that the circumstances warrant this.

31. On any Member attaining the age of seventy, his annual subscription shall cease; and it shall be in the power of a Member at any time to commute his annual subscription by a single payment calculated on the basis of the $O^M$ Table at 3½ per cent interest, subject to such regulations as may from time to time be made by the Council: provided that in the case of a

Member who has attained the age of sixty and has retired from business, the commutation money shall not exceed £10, 10s.: and provided also that Fellows residing at a distance from Edinburgh who may commute the modified subscription payable in terms of Rule 29 shall, in the event of their coming within the distance limit, be liable to pay or commute, as from that date, the difference between such modified subscription and the full subscription payable by other Fellows. The sums received for commutation of subscriptions shall be carried to a separate account, and the income only from the investments forming such account shall be employed for the ordinary purposes of the Faculty.

32. Until a Member shall have paid up all his arrears and given to the acting Secretary written notice of resignation, he shall be liable for all subscriptions due.

33. After payment of current expenses, the Funds of the Faculty shall, from time to time, be applied by the Council in promoting the objects of the Faculty, or be invested for like purpose in such manner as they may see fit.

34. The Council may, with the consent of a General Meeting, and under such conditions as the Faculty may prescribe, devote a sum, or annual sum, to any object tending, in the opinion of the Council, to promote the study of Actuarial or cognate subjects.

*Library.*

35. The Faculty shall maintain a Library of professional works for consultation and reference, the arrangements in regard to which shall be entrusted to the Council, subject to the approval and control of the Faculty.

36. Students shall have access to the Library with consent of the Council, and under such regulations as they may from time to time resolve on; and the Council may also continue the like privilege to gentlemen who on 31st December 1900 were members of the Actuarial Society of Edinburgh, and who shall have regularly paid the annual contribution provided by Rule 24.

*Faculty Hall.*

37. For the purpose of meetings, and for the preservation of the Faculty's Library, it shall be competent to the Council to rent suitable premises.

*General Regulations.*

38. The Rules of the Faculty may be repealed, altered, or added to, at an Ordinary or Special General Meeting, provided at least

fourteen days' notice shall have been given by circular specifying the proposed changes.

39. In the event of any impropriety of conduct on the part of any Member of the Faculty, the Council shall be specially summoned to consider the case; and if they shall be of opinion that his remaining a Member would be discreditable to the Faculty, their decision shall be reported to the next General Meeting of the Faculty, and a ballot shall then be taken among all the Fellows present. If three-fourths of the votes then given are for his expulsion, it shall be declared that he is no longer a Member, and his name shall be erased from the list of Members; but the power of expulsion shall not be exercised unless at least one-third of the Fellows resident in Scotland be present at the meeting when the ballot shall take place.

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## BYE-LAWS.

### *Examinations.*

1. The Examinations of candidates for admission as Associates shall take place annually in the month of April, and shall extend over two days, five hours being allowed each day.

2. At least one month's public notice shall be given of the days on which the Examinations are to take place.

3. Candidates shall give the acting Secretary not less than fourteen days' notice of their intention to come forward for examination.

4. The Examinations may be conducted in writing, or *viva voce*, or both, at the discretion of the Examiners. No means of information shall be allowed to candidates, except such as the Examiners may think proper to supply.

5. The Subjects of Examination shall be as follows:—

### First Examination.

Arithmetic.
Equations.
Series.
Permutations and Combinations.
Binomial Theorem.
Theory and use of Logarithms.
Elements of the Calculus of Finite Differences.
Theory of Probabilities.

### Second Examination.

Interest and Annuities-certain, with Construction of Monetary Tables.
Probabilities of Life and of Survivorship.

Theory of Life Contingencies, including Annuities and Assurances, with Construction of relative Tables.
History and Characteristics of Mortality Tables.
Application of the Calculus of Finite Differences to Life Contingencies.
Elements of the Differential and Integral Calculus.

THIRD EXAMINATION.

Mortality, Marriage, and Sickness Investigations, including Construction of Tables from actual or from hypothetical data.
Graduation of Tables.
Formulas for Summation and Interpolation.
Application of the Differential and Integral Calculus to Life Contingencies.
Life Assurance Finance and Practice, namely:—
- Calculation of Premiums.
- Valuation of Assets and Liabilities.
- Distribution of Surplus.
- Surrender and Conversion of Policies and Bonuses.
- Book-keeping and Accounts.
- Investments.
- Miscellaneous Questions.

Law of Life Assurance.
Reversions, Life-Interests, and other Contingencies.
Widows' Funds, Superannuation Funds, and Friendly Societies,—Calculation of Rates, and Valuations.

6. Students shall take the Examinations in the order in which they are prescribed. Unless for cause shown to the satisfaction of the Council, a Student must appear for examination within three years from the date of his enrolment, and subsequently at intervals of not more than three years; failing which his name shall be removed from the roll.

*Sessional Meetings and Publications.*

7. The Council shall annually nominate not fewer than three Fellows or Associates, who are not Members of Council, to act along with them, or with any Committee appointed by them, in carrying out the objects contemplated by Rule 24.

8. Except as may be otherwise arranged by the Council, a Sessional Meeting shall be held on one Monday in each month from November to March inclusive, at 4.30 P.M.

9. Every paper or other communication intended to be read at a Sessional Meeting shall be given in at least fourteen days before the date of such Meeting; and the Council shall determine whether

or not it shall be published or issued in printed form at the expense and for behoof of the Faculty.

10. Every paper or communication given in to be read at a Sessional Meeting, or to be included in the published *Transactions* of the Faculty, shall, in the absence of any previous agreement to the contrary, be the property of the Faculty: provided that the Council shall, if required by the Author, decide within three months from the date when such paper or communication is given in whether or not the same is to be published or issued in printed form by the Faculty; and, if their decision is in the negative, or if in any case a paper or communication so given in is not published or issued in printed form by the Faculty within twelve months from the said date, all right and property in the same shall be held to have been relinquished by the Faculty.

11. In the absence of any special arrangement, the Author of any paper shall be entitled, free of charge, to twenty-five copies of the part of the *Transactions* of the Faculty containing his paper, when issued; and to additional copies at half the published price.

12. Every Fellow and every Associate shall be entitled to receive a copy of each current part of the *Transactions* of the Faculty free of cost, and one additional copy at half the published price.

13. Every Student, and every Member of the late Actuarial Society of Edinburgh who is a contributor to the funds of the Faculty in terms of Rule 24, shall be allowed one copy of each current part of the *Transactions* of the Faculty at half-price.

14. One copy of any part of the *Transactions* of the late Actuarial Society of Edinburgh may be obtained by a Fellow, Associate, or Student, or by a contributing Member of the late Society, at the reduced price formerly charged to Members of the Society, provided that in the opinion of the Council there shall be in hand a sufficient stock of the particular part required.

### *Subscriptions.*

15. Members desirous of commuting their annual subscriptions shall produce evidence of age to the satisfaction of the Council. In calculating the commutation money, age shall be reckoned as at the last birthday preceding the date at which the commutation is to be made, and the annuity-value shall be adjusted to the quarter-day (being the first day of January, April, July, or October) next succeeding that date. The decision of the Council shall be final as to the sum payable in any case.

16. In the event of any Student not having paid his annual subscription by the thirtieth day of June, his name shall be removed from the roll.

*Library.*

17. The Library shall be open daily from 10.30 till 4 o'clock (Saturdays 10.30 till 1 o'clock), except during the month of August, when it shall be closed for the annual revision of the Catalogue.

18. No one shall be allowed to borrow more than three volumes at one time, except with the sanction of the Council.

19. Books shall not be lent for a longer period than one month; but when a book is returned it may be re-borrowed, provided it has not been bespoken in the meantime.

20. Books shall be lent only on the receipt of the borrower in a book to be kept in the Library for the purpose, or on the receipt of a messenger bearing a written order signed by the borrower.

21. A borrower losing, injuring, or defacing any book shall supply another copy, or shall pay such sum as the Council may determine.

22. Cyclopædias, Works of Reference, and unbound numbers of Scientific Journals and Periodicals (except the *Transactions* of the Faculty, or the *Journal* of the Institute of Actuaries) shall not be lent; but they may be consulted in the Hall during the hours mentioned above.

## REPORT BY THE COUNCIL TO THE ORDINARY GENERAL MEETING HELD ON 29 JUNE 1903.

THE Council have pleasure in submitting their Report for the period from 1st January 1902. Owing to the alteration of the date of the Ordinary General Meeting from January to June the present Report has reference to a period of seventeen months in place of one year as formerly. The Accounts submitted are for the year ended 31st December 1902.

At a Special Meeting of the Faculty, held on 15th December last, Dr. THOMAS BOND SPRAGUE, M.A. (Cantab.), and Mr. JAMES MEIKLE, were unanimously elected Honorary Members, under *Rule 3, in recognition of the eminent services they had rendered to the actuarial profession. It was reported at that meeting that the following six gentlemen, all Associates of more than one year's standing, had been admitted Fellows of the Faculty, under Rule 2 (*b*), since the last Ordinary Meeting held on 29th January 1902:—

WILLIAM ADAMSON, Scottish Accident Life and Fidelity Insurance Company, Edinburgh.

* Since the date of this Report the Rules have been revised and renumbered.

WILLIAM BANNATYNE, Scottish Temperance Life Assurance Company, Glasgow.

JOHN BOWIE, Scottish Temperance Life Assurance Company, Glasgow.

JAMES BUCHANAN, M.A. (Cantab.), D.Sc., F.I.A., Scottish Widows' Fund Life and Assurance Society, Edinburgh.

PHILIP CAMPBELL M'KEAN, Scottish Widows' Fund Life and Assurance Society, Edinburgh.

PHIPPS TURNBULL, 1 Royal Circus, Edinburgh.

The following four gentlemen have since been admitted Fellows of the Faculty, in terms of the same Rule:—

ARCHIBALD M'INTOSH, Scottish Equitable Life Assurance Society Edinburgh.

ROBERT MACKAY, Northern Assurance Company, Aberdeen.

JOHN ADAM RANKIN, Edinburgh Life Assurance Company, Edinburgh.

FREDERICK ALEXANDER ROSS, care of Messrs. Moncreiff & Horsbrugh, C.A., Edinburgh.

The following nine gentlemen, viz.:—LAWRENCE MACLAGAN CATHLES, PETER LORIMER CATTANACH, Jun., M.A., AUSTYN JAMES CLAUDE FYFE, GEORGE GRAHAM, CHARLES WILLIAM STEELE JAMIESON, WILLIAM RICHARD M'ILVENNA, GEORGE CUNNINGHAM M'LAREN, EDMUND FORBES PRICE, and GORDON WILLIAM THOMSON, having passed the Final Examination, and having the necessary qualification as to age, have been admitted Associates and received the Faculty's Diploma.

Mr. FRANK SANDERSON, M.A., Actuary, Canada Life Assurance Company, Toronto, was elected a Fellow at last Ordinary Meeting in terms of Rule 2 (*c*).

The Council regret to report three deaths among the Fellows, viz.:—Mr. HUGO JAMES NAHMER, Dublin, who was admitted a Fellow in 1887; Mr. FRANCIS LAING, F.I.A., Actuary of the Northern Assurance Company, London, who was elected a Fellow in 1892; and Mr. HENRY G. S. LAWSON, M.A., Actuary of the Scottish Accident Life and Fidelity Insurance Company, Edinburgh, who was admitted a Fellow in 1894. They have also to report that two Fellows have retired from Membership.

The total Membership is now 174, viz.:—5 Honorary Fellows, 154 Fellows, and 15 Associates.

Since 1st January 1902, 41 Students have been admitted, the total number on the roll at present being 176.

The Annual Examinations of Students of the Faculty were held on 24th and 25th April 1902, and 23rd and 24th April 1903. On the former occasion 67 Candidates presented themselves for examination, namely, 27 for the First, 18 for the Second, and 22 for the Final Examination. On the latter occasion 63 presented themselves, namely, 23 for the First, 14 for the Second, and 26 for the Final Examination.

The following is a list of the successful Candidates arranged in order of merit:—

FIRST EXAMINATION, 1902.

1. C. KEITH GRANGER, City of Glasgow Life Assurance Company, Glasgow.
2. D. L. S. DOUGLAS, Standard Life Assurance Company, Glasgow.
3. JOHN T. SMITH, 119 Plantation Street, Accrington.
4. STANLEY D'E. COLAM, Edinburgh Life Assurance Company, Edinburgh.
5. JOHN M. ANDERSON, Scottish Union and National Insurance Company, Edinburgh.
6. JAMES W. RUSSELL, Northern Assurance Company, Edinburgh.
7. THOMAS FRAZER, Jun., Scottish Union and National Insurance Company, Edinburgh.

8 and 9. { ALEXANDER M'LAREN, Scottish Life Assurance Company, Edinburgh. / DAVID M. ROBERTSON, Scottish Metropolitan Life Assurance Company, Edinburgh. } *Equal.*

SECOND EXAMINATION, 1902.

1. JAMES C. BOYD, North British and Mercantile Insurance Company, Edinburgh.
2. FREDERICK W. ROBERTSON, Standard Life Assurance Company, Edinburgh.
3. J. MURRAY LAING, National Mutual Life Association of Australasia, London.
4. JOSEPH G. PEARSON, Scottish Union and National Insurance Company, Edinburgh.
5. JOHN C. DENMEAD, M.A., Somerset House, London.
6. HARRY A. FAIRMAN, Scottish Equitable Life Assurance Society Edinburgh.
7. JOHN W. THOMSON, Scottish Life Assurance Company, Edinburgh.

8. George Brown, Scottish Equitable Life Assurance Society, Edinburgh.
9. Alex. S. Sellar, M.A., Scottish Imperial Insurance Company, Glasgow.

FINAL EXAMINATION, 1902.

1. Frederick A. Ross, care of Messrs. Moncreiff & Horsbrugh, C.A., Edinburgh.
2. Peter L. Cattanach, Jun., M.A., care of Messrs Lindsay, Jamieson, & Haldane, C.A., Edinburgh.
3. Robert Mackay, Northern Assurance Company, Aberdeen.
4. Archibald M'Intosh, Scottish Equitable Life Assurance Society, Edinburgh.
5. George Graham, North British and Mercantile Insurance Company, Edinburgh.
6. John Adam Rankin, Edinburgh Life Assurance Company, Edinburgh.
7. Charles William Steele Jamieson, Scottish Amicable Life Assurance Society, Glasgow.

FIRST EXAMINATION, 1903.

1. David N. Strathie, 3 Victoria Terrace, Glasgow, W.
2. James G. Kyd, Northern Assurance Company, Aberdeen.
3. Walter Borland, Scottish Temperance Life Assurance Company, Glasgow.
4. Angus D. Macphail, Scottish Life Assurance Company, Edinburgh.
5. George G. Blackwood, Edinburgh Life Assurance Company, Edinburgh.
6. John P. P. Oliver, North British and Mercantile Insurance Company, Edinburgh.

SECOND EXAMINATION, 1903.

1. Hugh F. Cowan, Edinburgh Life Assurance Company, Edinburgh.
2. John M. Geoghegan, care of Messrs. Barstow & Millar, C.A., Edinburgh.
3. John H. Robertson, M.A., Scottish Amicable Life Assurance Society, Glasgow.
4. John T. Smith, 119 Plantation Street, Accrington.
5. John H. Imrie, M.A., 29 Queen Street.

FINAL EXAMINATION, 1903.

1. ROBERT RAYNAL BRODIE, Scottish Provident Institution, Edinburgh.
2. THOMAS FREDERIC ANDERSON, F.I.A., Royal Exchange Assurance Corporation, London.
3. WILLIAM RICHARD M'ILVENNA, Scottish Equitable Life Assurance Society, Edinburgh.
4. LAWRENCE MACLAGAN CATHLES, Scottish Metropolitan Life Assurance Company, Edinburgh.
5. AUSTYN JAMES CLAUDE FYFE, Northern Assurance Company, Aberdeen.
6. EDMUND FORBES PRICE, Standard Life Assurance Company, Edinburgh.
7. JAMES COLVILLE BOYD, North British and Mercantile Insurance Company, Edinburgh.

8. and 9. { GEORGE CUNNINGHAM M'LAREN, South African Mutual Life Assurance Society, Cape Town. / GORDON WILLIAM THOMSON, Life Association of Scotland, Edinburgh. } *Equal.*

The Honorary Treasurer's Accounts for the year ended 31st December 1902, with relative state of the Funds, audited by Mr. David A. Clapperton, C.A., are herewith submitted. The Receipts for the year amounted to £881, 1s. 2d., and the Expenditure to £664, 2s. 11d. The Funds in hand at the close of the year amounted to £2068, 4s. 8d. It will be observed that the sum of £250, advanced in 1901 on account of the New Mortality Experience Tables, has been repaid. A sum of £100, advanced for the same purpose in 1902, has also been repaid.

Donations to the Library have been received from the Institute of Actuaries, the Scottish Widows' Fund and Life Assurance Society, the Society of Accountants in Edinburgh, the Washington Insurance Company, New York, the Spectator Company, New York, Mr. James Chisholm, Mr. R. P. Hardy, Mr. Archibald Hewat, Mr. Arthur Hunter, Mr. James Meikle, Dr. T. B. Sprague, Mr. T. E. Streeter, Mr. R. T. Thomson, and others, to all of whom the thanks of the Council have been conveyed.

Appended to this Report will be found a List * of the principal Books and Pamphlets that have been added to the Library since the publication of the Supplement to the Catalogue in 1902. There is kept

* Not reprinted here.

in the Hall, for the convenience of Members, a list of all recent additions to the Library.

The Sessional Meetings for 1901-1902 were completed by two meetings, held on 3rd February and 10th March 1902, when papers entitled "Some Practical Notes on Reversions, with Results of an Investigation into the Mortality among Life-Tenants," by Mr. Neil Campbell, and "On Premiums deduced from the Mortality Experience (1863-1893) of British Life Offices," by Mr. James Chatham, were read and discussed.

During the Session 1902-3 the following Papers were read:—

10th November 1902. "On Policy Values deduced from the Mortality Experience (1863-1893) of British Life Offices." Mr. James Chatham.

15th December 1902. "On the Book-keeping of a Life Office." Mr. James J. M'Lauchlan.

16th February 1903. "On an Investigation to show the Rates of Mortality and Marriage among Daughters of Peers and of Heirs-Apparent, with Values of certain Benefits depending on Marriage and Death." Mr. M. M. Lees.

23rd March 1903. "Some Practical Notes on the Valuation of Pension Funds." Mr. David Carment.

27th April 1903. "Notes on Mortality in India and other tropical countries." Mr. Spencer C. Thomson.

At the Meeting on 23rd March Mr. James Chatham gave a demonstration of a method of printing particulars on premium receipts and notices by means of a machine with endless chains consisting of links fitted with india-rubber type.

All the above papers have been or are in course of being published as *Transactions* of the Faculty.

The Council have made arrangements with Mr. WILLIAM HARVEY, LL.B., Advocate, to furnish for insertion in the *Transactions*, reports of cases in the Scottish Law Courts connected with actuarial questions or the business of life insurance. It is hoped that Members will assist in this matter by giving the Honorary Secretary of the Faculty notice of any cases of which they may have knowledge, with a view to their being brought before Mr. Harvey.

In consequence of Mr. J. M. Warden and Mr. James Watt, W.S., having been elected Members of Council at the last Ordinary General Meeting, the Council nominated Mr. Vyvyan Marr and Mr. Hugh

Brown to take their places as Members of the Faculty elected to assist the Committee on Sessional Meetings.

In connection with the Investigation of the New Collective Mortality Experience, the two volumes which were in course of preparation at the date of last Report were duly published. These consisted of Mortality and Monetary Tables deduced from the graduated Data, and comprised (1) Aggregate Tables based upon the Experience of Whole Life Participating Assurances, Male Lives, $O^M$ and $O^{M(5)}$, and (2) Tables for Single and Joint Lives, Males and Females, set out in the form of Select or Extended Tables, based upon the Experience of Life Annuitants. The calculation of the Mortality and Monetary Tables, founded upon Mr. G. F. Hardy's Graduation of the Experience of Whole Life Participating Assurances on Male Lives in the form of SELECT TABLES, $O^{[M]}$, is now completed, and the Tables are in the printers' hands with a view to publication in what will be the seventh volume published under the authority of the Joint Committee. It is intended also to publish an account of the Processes and Methods followed throughout the Investigation in dealing with the Data and the Graduation thereof, and in the calculation of the Tables.

The Council remitted to a special Committee to consider and report on the question of Commutation of Annual Subscriptions. Appended hereto is a copy of their Report which has been adopted by the Council with the following addition:—

"That Members residing at a distance from Edinburgh who may commute the modified subscription payable by them in terms of Rule 26 shall, in the event of their coming to reside within the distance limit, be liable to pay or commute as from the date of their change of residence, the difference between such modified subscription and the full subscription payable by Members resident within the said limit."

Resolutions in accordance with this Report will be submitted to the Meeting.

A proposal will also be submitted that Fellows of the Institute of Actuaries who may be elected Fellows under Rule 5 shall pay on admission an entrance fee of £4, 4s. in place of the fee of £10, 10s. required from other persons, not Associates, on admission as Fellows.

*By order of the Council,*

GEO. M. LOW, *President.*

EDINBURGH, 8*th June* 1903.

# APPENDIX No. I.

## RECEIPTS AND EXPENDITURE FOR YEAR 1902.

**Dr.**

| | | | | | | |
|---|---|---|---|---|---|---|
| Amount of Funds, 31st December 1901, | | | | | £1,851 | 6 | 5 |
| Entrance Fees— | | | | | | | |
| Fellows, | | | £27 | 6 | 0 | | |
| Associates, | | | 12 | 12 | 0 | | |
| Students, | | | 15 | 15 | 0 | | |
| | | | £55 | 13 | 0 | | |
| Examination Fees, | | | 35 | 14 | 0 | | |
| Annual Subscriptions, | £316 | 19 | 6 | | | | |
| Less Subscriptions returned to Students on payment of Examination Fees, | 0 | 7 | 6 | | | | |
| | | | 316 | 12 | 0 | | |
| Interests and Dividends, | | | 57 | 11 | 7 | | |
| Rent received, | | | 15 | 0 | 0 | | |
| Repayment of Sums advanced on account of Mortality Investigation, | | | 350 | 0 | 0 | | |
| Proceeds of *Transactions* sold, | | | 50 | 10 | 7 | | |
| | | | | | | 881 | 1 | 2 |
| | | | | | | £2,732 | 7 | 7 |

**Cr.**

| | | | | | | |
|---|---|---|---|---|---|---|
| Cost of *Transactions*, | | | | | £232 | 13 | 4 |
| Library— | | | | | | | |
| Purchase and Binding of Books, etc., | | | | | 20 | 13 | 6 |
| Faculty's Hall— | | | | | | | |
| Rent, less Taxes, | £142 | 19 | 4 | | | | |
| Rates, Taxes, and Fire Insurance, | 35 | 9 | 3 | | | | |
| Furniture (written off for depreciation), | 16 | 17 | 3 | | | | |
| Sundries, | 43 | 19 | 6 | | | | |
| | | | | | 239 | 5 | 4 |
| Examination, Management, and Miscellaneous Expenses— | | | | | | | |
| Examination Expenses, | £37 | 16 | 0 | | | | |
| Stationery, Printing, etc., | 20 | 5 | 1 | | | | |
| Return to Life Members of the late Actuarial Society, | 8 | 12 | 6 | | | | |
| Miscellaneous Expenses, Postages, etc., | 4 | 17 | 2 | | | | |
| | | | | | 71 | 10 | 9 |
| Mortality Investigation, temporary advance, | | | | | 100 | 0 | 0 |
| | | | | | £664 | 2 | 11 |
| Amount of Funds, 31st December 1902, as per Statement subjoined, | | | | | 2,068 | 4 | 8 |
| | | | | | £2,732 | 7 | 7 |

Edinburgh, 1st June 1903.—I have examined the Accounts (of which the foregoing is an Abstract) of Colin M'Cuaig, C.A., Honorary Treasurer to the Faculty of Actuaries in Scotland, for the year ending 31st December 1902, and have found them to be correctly stated and sufficiently vouched and instructed. I have also examined the Statement of Funds and have found it correct, and the Security Documents have been exhibited to me.

D. A. CLAPPERTON, C.A., *Auditor.*

## STATEMENT OF FUNDS

### AS AT 31ST DECEMBER 1902.

| | | | |
|---|---|---|---|
| £750 Caledonian Railway 4 per cent. Consolidated Preference Stock, No. 1 (Cost), | £892 | 17 | 9 |
| £300 Highland Railway 4¼ per cent. Debenture Stock (Cost), | 389 | 2 | 9 |
| £220 Victoria Government 3½ per cent. Inscribed Stock (Cost), | 196 | 13 | 0 |
| £200 North British Railway 3 per cent. Debenture Stock (Cost), | 208 | 7 | 2 |
| £150 Great Central Railway 4½ per cent. Preference Stock (Cost), | 195 | 12 | 9 |
| Value of Furniture in Faculty's Rooms, | 151 | 15 | 4 |
| Sum on Account Current with British Linen Company Bank, | 28 | 16 | 9 |
| In hands of Hon. Treasurer, | 4 | 19 | 2 |
| | £2,068 | 4 | 8 |

NOTE.—No value has been put upon the Library, which contains a large and valuable collection of Books and Pamphlets; nor upon the stock of *Transactions* in hand.

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## APPENDIX No. II.

### REPORT *of* COMMITTEE *appointed at the* COUNCIL MEETING *on 20th October* 1902 *to consider and report as to a Scheme of Commutation of Subscriptions.*

Your Committee have carefully considered the remit made to them, and beg to report as follows:—

When the Faculty was founded in 1856, the annual subscription was fixed at one guinea, and it continued at that figure until 1888. It was then increased to two guineas, and has since remained at that amount. The practice of allowing annual subscriptions to be commuted is becoming more common in professional bodies, and the services of members of the Faculty are occasionally requisitioned for

the purpose. The Committee are of opinion that the Faculty might allow the annual subscription to be commuted on fair and equitable terms; but before considering these it will be convenient to see what other professional bodies have done in this direction, and a list of a number of them is appended. This list is not intended to be in any way complete, but it is to be looked upon rather as typical of the terms usually charged. These bodies may, roughly speaking, be divided into two classes—those who examine, and those who do not. The Institute of Actuaries may be taken as an example of the former, and the Royal Scottish Geographical Society as an example of the latter. The list does not include the various examining legal bodies in Scotland, as they do not at present allow the annual subscription to be commuted, although one or two of them have had the matter under consideration. One Society—the Writers to the Signet—has no annual subscription. They generally, however, allow the annual subscription to their widows' funds to be commuted. Another very important examining body not included in the list is the Society of Chartered Accountants in Edinburgh, the reason being that the members pay an entrance fee and no annual subscription.

The hold on the members of the bodies in the non-examining class is, as a rule, small, and accordingly we find the terms of commutation liberal, as the rate of withdrawal will be considerable. They are generally recruited from men of all ages, and the terms must be sufficiently attractive for all. A common annual subscription is one guinea, and the commutation of it generally runs from ten to twenty guineas. Only one Society in the list does not allow commutation for Fellows resident in Scotland; but it is believed that even in their case the question has been under consideration.

It is different, however, with the members of the other class. These bodies are recruited by examination from the ranks of young men; and though membership is not absolutely essential to the carrying on of the profession, yet it is very desirable. Consequently we find the terms are higher, and in the case of the Institute of Actuaries graduated according to age. The annual subscription is generally two or three guineas, and the commutation of it runs from twenty to forty guineas.

The Faculty of Actuaries is an examining body, analogous to the Institute of Actuaries, and the Committee think that the commutation should be graduated according to age. There is no age limit at present to the annual subscription charged by the Faculty, and the first question to be determined in considering the terms upon which commutation should be allowed is whether one should be introduced. This question has been considered in connection with a letter by Mr.

Hugh Blair of 2nd December last to the Secretary to the Faculty calling attention to the desirability of introducing a limit. It has been the practice to elect, in terms of the rules, members who have retired from official connection with an Assurance Office after long service as Honorary Fellows; but some of the members equally entitled on other grounds to be freed from the annual subscription do not come under the rule, and the result is that they have been called upon to pay the annual subscription long after they have retired from business and attained an age at which they might fairly expect to be freed from the contribution. The present position therefore is, as Mr. Blair says, somewhat anomalous. The Committee have very carefully considered this point, and recommend that the age limit should be fixed at 70, and that all those who have now attained that age, or who may attain it in future, should be freed from the annual subscription. This age may seem somewhat advanced, but there is a fair proportion of the members between 60 and 70, and the Committee feel that the introduction of an earlier age would entail a considerable loss upon the Faculty. They will revert to this point later on. If the limit is fixed at age 70, the Committee believe that four or five members would be at once freed from the annual subscription.

Assuming 70 to be the limiting age for the calculations, the next point to be considered is the rate of interest to employ in the commutation. The Committee think that 3½ per cent would be a fair rate to adopt, as that is the rate at which Insurance Offices are generally willing to discount future premiums, and the assured to commute. It is also the rate, at all events at present, at which any money received could be invested in good securities. Another reason for employing that rate is, that it is not proposed to make any allowance for withdrawals, although that factor is not entirely absent. It is also a rate much below that adopted by the Institute of Actuaries in their terms for commutation, although possibly they may have made allowance for withdrawals. If that rate is employed, the following are the values of temporary annuities of an annual subscription of two guineas payable in advance, according to the undermentioned tables. A column has been added showing the values of whole life annuities of the annual subscription payable in advance by the $O^M$ table, which indicate the difference the proposed limiting of the subscription will make. It is of course chiefly in favour of the older members.

The Committee recommend that the $O^M$ table with 3½ per cent interest should be adopted for the commutation of annual subscriptions, but that if a Fellow retires from official and private practice

after he has attained the age of 60, then he should be at liberty to commute for the sum of ten guineas.

| Age. | Carlisle. | English Life. | $H^M$. | $O^M$. | $O^M$. Whole Life. |
|---|---|---|---|---|---|
| 30 | £38 7 0 | £37 14 0 | £39 2 0 | £39 14 0 | £41 10 0 |
| 35 | 35 18 0 | 35 3 0 | 36 8 0 | 36 17 0 | 39 1 0 |
| 40 | 33 0 0 | 32 5 0 | 33 6 0 | 33 12 0 | 36 7 0 |
| 45 | 29 8 0 | 28 18 0 | 29 15 0 | 29 19 0 | 33 7 0 |
| 50 | 26 0 0 | 25 1 0 | 25 13 0 | 25 16 0 | 30 2 0 |
| 55 | 21 2 0 | 20 12 0 | 21 0 0 | 21 2 0 | 26 13 0 |
| 60 | 15 9 0 | 15 8 0 | 15 11 0 | 15 12 0 | 23 1 0 |
| 65 | 9 1 0 | 8 18 0 | 8 19 0 | 9 0 0 | 19 9 0 |

The present is a convenient time for applying the principle of commutation. The annual subscription now is such that any increase would, it is feared, be regarded as a burden by the junior members of the profession, and it is unlikely that any increase would be made for many years to come. Again, it is likely that a number of the older members will take advantage of the proposed table to commute their subscriptions, and if so, the money could be put out to considerable advantage at the present time. Your Committee are strongly of opinion that the commuted subscriptions should be carried to a separate account, and that the income only from the money representing that account should be employed for the ordinary purposes of the Faculty. The interest derived from this account would of course be less than the amount of the subscriptions so commuted, and in this respect the commutations would at first be a source of weakness to the Faculty. The expenditure, however, is well within the income, and no fear need be apprehended on this score if the precautions above recommended are adopted. The commutations would of course ultimately be a source of strength to the Faculty.

In conclusion, the recommendations of the Committee may be briefly summarised as follows:—

(1) That the age at which subscriptions should cease be fixed at 70.

(2) That the $O^M$ Table at $3\frac{1}{2}$ per cent interest be employed in the commutation of subscriptions.

(3) That upon retirement after age 60 the commutation should not exceed ten guineas.

(4) That the sums received for commutation be carried to a separate account, and the income only from the investments forming such account employed for the ordinary purposes of the Faculty.

(Signed) C. M'CUAIG.
JAMES CHATHAM.
JAMES WATT.

EDINBURGH, 19*th February* 1903.

## LIST REFERRED TO IN PRECEDING REPORT.

| | Entrance Fee. | Annual Subscription. | Commutation. |
|---|---|---|---|
| SOCIETY OF ANTIQUARIES OF SCOTLAND, . . . | £2 2 0 | £1 1 0 | ..... |
| At date of admission, . . | ..... | ..... | 21 0 0 including entrance fee |
| After 5 annual contributions, . | ..... | ..... | 15 15 0 |
| ,, 10 ,, ,, . | ..... | ..... | 10 10 0 |
| ROYAL SCOTTISH GEOGRAPHICAL SOCIETY, . . . | ..... | 1 1 0 | ..... |
| Under 10 years' standing, . | ..... | ..... | 20 0 0 |
| Over 10 and under 20, . . | ..... | ..... | 11 5 0 |
| ,, 20 ,, ,, 30, . . | ..... | ..... | 10 0 0 |
| ,, 30 years' standing, . . | ..... | ..... | 5 0 0 |
| PHILOSOPHICAL INSTITUTION, . | ..... | 1 5 0 | 12 12 0 |
| ROYAL STATISTICAL SOCIETY, . | ..... | 2 2 0 | 21 0 0 |
| ROYAL SOCIETY, . . . . | 10 0 0 | 3 0 0 | 60 0 0 |
| ROYAL SOCIETY OF EDINBURGH, | 2 2 0 | ..... | ..... |
| For 1st 10 years, . . . | ..... | 3 3 0 | None for Fellows resident in Scotland. |
| Thereafter for 15 years, . . | ..... | 2 2 0 | ..... |
| INSTITUTE OF ACTUARIES, . . | ..... | 3 3 0 or 2 2 0 | ..... |
| Ages: 40–45, . . . . | ..... | ..... | 37 16 0 |
| 45–50, . . . . | ..... | ..... | 31 10 0 |
| 50–55, . . . . | ..... | ..... | 25 4 0 |
| 55–60, . . . . | ..... | ..... | 18 18 0 |
| 60 and upwards, . . | ..... | ..... | 15 15 0 |
| Retirement after 60, . | ..... | ..... | 10 10 0 |
| | | | The above rates apply to both annual subscriptions. |

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